

Secondary

MATHEMATICS

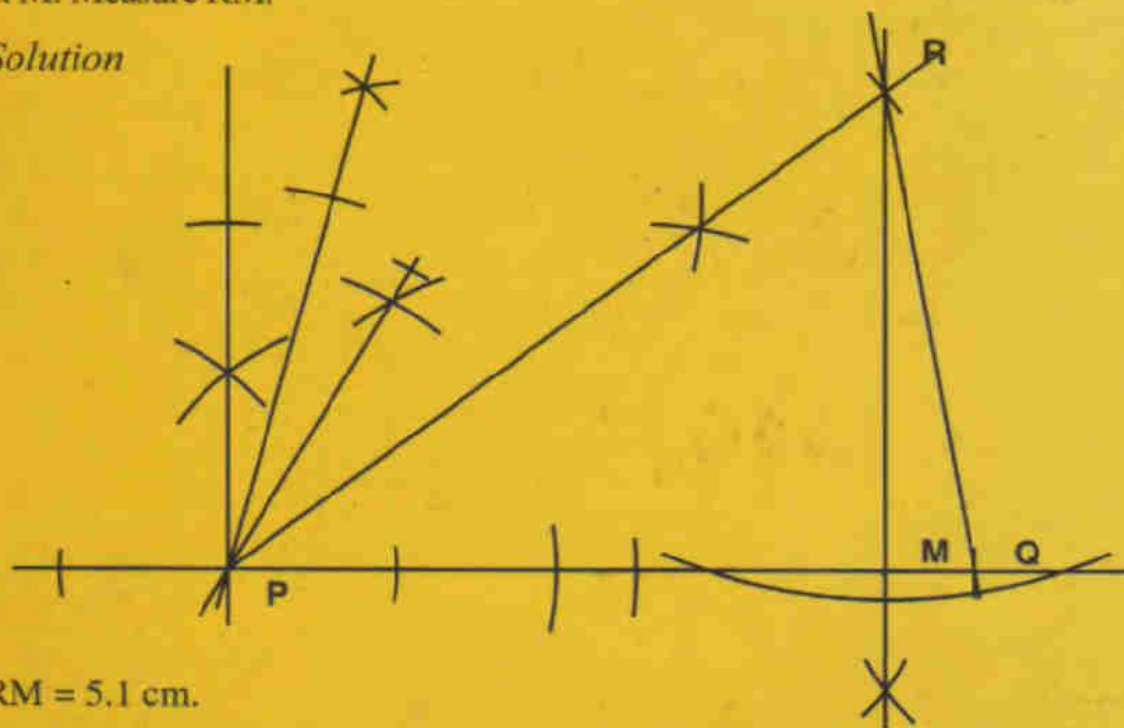
Students' Book One

Third Edition

Question

Without using a protractor, construct a triangle PQR such that $PQ = 7.5$ cm, $PR = 8.3$ cm and angle $RPQ = 37.5^\circ$. Drop a perpendicular from R to cut PQ at M. Measure RM.

Solution



RM = 5.1 cm.



KENYA LITERATURE BUREAU

Approved by the
Ministry of Education

Secondary

MATHEMATICS

Students' Book One

(Third Edition)



KENYA LITERATURE BUREAU
NAIROBI

KENYA LITERATURE BUREAU
P. O. Box 30022 – 00100, Nairobi
Website: www.klb.co.ke
E-mail: info@klb.co.ke

© Ministry of Education

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transcribed, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior written permission of the publisher.

ISBN 9966-44-472-6

First published 1987

Reprinted 1988, 1989, 1990, 1991, 1992

Second Edition 1992

*Reprinted 1994, 1995, 1996 (twice), 1997, 1998(twice), 1999 (twice)
2000(twice), 2001 (twice), 2002*

Third Edition 2003

*Reprinted 2004 (thrice), 2005 (thrice), 2006, 2007(twice), 2008, 2009(twice)
2010(four times), 2011(four times), 2012, 2013(six times), 2014, 2015(twice) **2016***

KLB 12974 20M 2016

Published and printed by Kenya Literature Bureau

PROLOGUE

The new mathematics syllabus for the Kenya Certificate of Secondary Education was developed in accordance with the objectives of the secondary cycle of the 8-4-4 system of education. Secondary Mathematics Students' Book One has been written to match the objectives as spelt out in the syllabus.

The main thrust of the book is on meeting mathematical needs of the cross-section of learners found in our secondary schools. This has been ably done by the team of writers that consists of mathematics educators with a vast experience in the teaching of the subject at different levels. The team was drawn from classroom teachers, inspectors and curriculum developers.

I am grateful to all those who have participated at various levels in the writing and revision of this title.

THE MANAGING DIRECTOR,
Kenya Literature Bureau

CONTENTS

	<i>Page</i>
<i>Prologue</i>	<i>v</i>
<i>Acknowledgements</i>	<i>xiii</i>
<i>Introduction</i>	<i>xv</i>
<i>Chapters</i>	
1. NATURAL NUMBERS	1
Place Value	1
Billions	2
Rounding Off	2
Operations on Whole Numbers	4
Word Problems	7
Even Numbers	9
Odd Numbers	9
Prime Numbers	9
2. FACTORS	10
3. DIVISIBILITY TEST	12
Divisibility Test for 2	12
Divisibility Test for 3	13
Divisibility Test for 4	14
Divisibility Test for 5	15
Divisibility Test for 6	16
Divisibility Test for 8	17
Divisibility Test for 9	18
Divisibility Test for 10	19
Divisibility Test for 11	20
4. GREATEST COMMON DIVISOR	22
5. LEAST COMMON MULTIPLE	24
Multiples	24
L.C.M. of Two or More Numbers	24
6. INTEGERS	28
Introduction	28
The Number Line	29
Operations on Integers	30

	Order of Operations	37
7.	FRACTIONS	41
	Introduction	41
	Comparing Fractions	42
	Operations on Fractions	45
	Order of Operations on Fractions	54
8.	DECIMALS	58
	Introduction	58
	Conversion of Fractions into Decimals	58
	Recurring Decimals	61
	Decimal Places	63
	Standard Form	64
	Operations on Decimals	65
	Order of Operations	71
9.	SQUARES AND SQUARE ROOTS	73
	Squares	73
	Squares from the Tables	73
	Square Roots	76
	Square Root by Factorisation	76
	Square Roots from Tables	78
10.	ALGEBRAIC EXPRESSIONS	80
	Symbolic Representation	80
	Simplification of Algebraic Expressions	82
	Brackets	85
	Factorisation by Grouping	89
	Algebraic Fractions	90
	Simplification by Factorisation	91
	Substitution	92
	Mixed Exercise 1	94
11.	RATES, RATIO, PROPORTION AND PERCENTAGE	96
	Rates	96
	Ratio	97
	Increase and Decrease in a given Ratio	99
	Comparing Ratios	100

	Distributing a Quantity in a given Ratio	101
	Direct and Inverse Proportion	102
	Percentages	105
	Percentage Increase and Decrease	106
12.	LENGTH	110
	Introduction	110
	Significant Figures	111
	Perimeter	113
	The Circle	116
13.	AREA	123
	Units of Area	123
	Area of Regular Plane Figures	124
	Area of a Circle	129
	Surface Area of Solids	135
	Area of Irregular Shapes	141
14.	VOLUME AND CAPACITY	143
	Introduction	143
	Conversion of Units of Volume	143
	Volumes of Cubes, Cuboids and Cylinders	143
	Capacity	146
15.	MASS WEIGHT AND DENSITY	148
	Mass	148
	Weight	149
	Density	149
16.	TIME	153
	Units of Time	153
	The 12 and 24-Hour Systems	154
	Travel Timetables	156
17.	LINEAR EQUATIONS	160
	Introduction	160
	Word Problems	162
	Linear Equations in Two Unknowns	165
	Solution by Substitution	168

18.	COMMERCIAL ARITHMETIC	171
	Introduction	171
	Currency Exchange Rates	172
	Profit and Loss	175
	Discount	177
	Commission	178
	Mixed Exercise 2	179
19.	CO-ORDINATES AND GRAPHS	182
	Co-ordinates	182
	The Graph of a Straight line	185
	Graphical Solutions to Simultaneous Linear Equations	188
	General Graphs	189
20	ANGLES AND PLANE FIGURES	197
	Introduction	197
	Types of Angles	197
	Angles on a Straight Line	201
	Angles at a Point	201
	Angles on a Transversal	206
	Angle Properties of Polygons	211
21.	GEOMETRICAL CONSTRUCTIONS	229
	Construction of a Line	229
	Construction of Perpendicular and Parallel Lines	229
	Construction of Parallel Lines	235
	Construction of Regular Polygons	237
	Construction of Irregular Polygons	239
22.	SCALE DRAWING	248
	The Scale	248
	Scale Diagrams	249
	Bearings and Distances	251
	Angles of Elevation and Depression	256
	Simple Survey Techniques	262
	Areas of Irregular Shapes	267
23.	COMMON SOLIDS	271
	Introduction	271

Sketching of Solids	275
Nets of Solids	277
Models of Solids	281
Surface Area of Solids from Nets	284
Distance Between two Points on the Surface of a Solid	286
Mixed Exercise 3	289
<i>Revision Exercise 1</i>	294
<i>Revision Exercise 2</i>	295
<i>Revision Exercise 3</i>	296
<i>Revision Exercise 4</i>	298
<i>Revision Exercise 5</i>	298

ACKNOWLEDGEMENTS

The Managing Director, Kenya Literature Bureau, would like to thank the following writers who participated in the revision of this book:

- | | | |
|------------------|---|---|
| Justus Abuko | – | <i>Mukumu Girls' High School, Khayega</i> |
| Luke M'masi | – | <i>Eregi Girls' High School, Kakamega</i> |
| Kithikii Kasungu | – | <i>St. Charles Lwanga Secondary School, Kitui</i> |
| Anne Opiyo | – | <i>Gagra Secondary School, Madiany</i> |

INTRODUCTION

This is the first students' title of the KLB Secondary Mathematics series and is intended for use at Form One level. Although the book is written mainly for the 8-4-4 secondary school mathematics syllabus, it will prove useful to students pursuing similar courses both within and outside Kenya.

In this book, mathematics is presented in a simple and precise manner. Elaborate examples aimed at illustrating specific mathematical ideas are offered as precursor to wide-ranging exercises. This ensures that varying abilities and interests of student are adequately catered for. The mixed exercises further give the students an opportunity to consolidate the mathematical ideas met, so that an all-round positive result is attained.

It is expected that the book will provide a smooth transition from primary to secondary school mathematics for each learner.

Chapter One

NATURAL NUMBERS

1.1: Place Value

Natural numbers, which are also called **counting numbers**, consist of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

A digit may have a different value because of its position in a number. The position of a digit in a number is called its **place value**. On the other hand, the **total value** is the product of the digit and its place value. For example, in the number 478, the place value of 8 is ones and its total value is 8, the place value of 7 is tens and its total value is 70, while the place value of 4 is hundreds and its total value is 400.

The table below shows the place values of digits in some numbers.

Number	HUNDRED MILLIONS	TEN MILLIONS	MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
3 4 5 6 7 8 9 0 1	3	4	5	6	7	8	9	0	1
7 6 9 3 0 1 8 5 4	7	6	9	3	0	1	8	5	4
9 0 2 3 5 0 4 0 9	9	0	2	3	5	0	4	0	9

A place value chart can be used to identify both place value and total value of a digit in a number. The place value chart is also helpful when writing numbers in words. The above numbers in words are:

- Three hundred and forty five million, six hundred and seventy eight thousand, nine hundred and one.
- Seven hundred and sixty nine million, three hundred and one thousand, eight hundred and fifty four.
- Nine hundred and two million, three hundred and fifty thousand, four hundred and nine.

1.2: Billions

A **billion** is **one thousand million**, written **1 000 000 000**. There are ten places in a billion. Thus, 23 289 438 001 is read as twenty three billion, two hundred and eighty nine million, four hundred and thirty eight thousand and one.

Express the following numbers in words:

- (i) 3 468 729 165
- (ii) 64 587 492 379
- (iii) 745 389 576 244

Draw a place value chart and show the figures.

Example 1

What is the place value and total value of each of the digits underlined below:

- (a) 47 397 263 402
- (b) 389 410 000 245

Solution

- (a) The place value for 6 is ten thousands. Its total value is 60 000.
The place value for 7 is billions and its total value is 7 000 000 000.
- (b) The place value for 0 is thousands. Its total value is zero.
The place value of 3 is hundred billions. Its total value is 300 000 000 000.

Exercise 1.1

1. What is the place value and total value of each of the digits underlined below:
 - (a) 74 379 652 137
 - (b) 48 677 395
 - (c) 3 486 789
 - (d) 98 374 803 041
2. Write the numbers in question 1 in words.
3. Write the following numbers in symbols:
 - (a) Forty million, six hundred thousand and six.
 - (b) Five hundred and ninety million, seven hundred thousand, five hundred.
 - (c) Thirty five billion, nine hundred thousand and ten.
 - (d) Eighty billion, forty five thousand.
4. Draw a chart to show the numbers in question 3.

1.3: Rounding Off

Suppose the number of students in an institution is 5 349. The figure could be estimated as 5 000, 5 300 or 5 350, depending on the level of accuracy expected.

In the first case (5 000), 5 349 has been rounded off to the nearest thousand. The digit in the hundreds place (3) is less than 5, therefore the digit in the thousands place (5) is retained. All the other digits are replaced by zeros.

The second rounding off (5 300) has been done to the nearest 100. In this case, the digit in the tens place (4) is less than 5 and 3 is therefore retained in the hundreds place. The last two digits are replaced by zeros.

The third rounding off (5 350) is to the nearest 10. The digit in the ones place (9) is greater than 5 and therefore one is added to the digit in the tens place. The ones place is replaced by zero.

Example 2

Round off each of the following numbers to the nearest number indicated in the bracket:

- (a) 473 678 (100)
- (b) 524 239 (1000)
- (c) 2 499 (10)
- (d) 38 679 (10 000)

Solution

- (a) 473 678 is 473 700 to the nearest 100.
- (b) 524 239 is 524 000 to the nearest 1 000.
- (c) 2 499 is 2 500 to the nearest 10.
- (d) 38 679 is 40 000 to the nearest 10 000.

Exercise 1.2

1. Round off the following numbers to the nearest number indicated in the bracket:
 - (a) 379 (10)
 - (b) 89 365 (100)
 - (c) 37 468 592 (10 000)
 - (d) 89 123 564 (1 000 000)
 - (e) 348 506 279 438 (1 000 000 000)
 - (f) 89 232 113 214 (1 000 000 000)
2. Round off each of the following numbers to the nearest number indicated in the bracket:
 - (a) Thirty eight million, seven hundred and thirty nine thousand, six hundred and nineteen. (10 000)
 - (b) Eight hundred thousand, four hundred and ninety. (100)
 - (c) Two hundred and ninety two thousand, four hundred and forty four. (1 000)
 - (d) Fourteen million, eight hundred and fifty thousand and sixty nine. (1 000 000)

- (e) Three hundred and forty eight thousand, six hundred and six. (10), (100), (1 000).
- (f) Thirty eight billion, three hundred and nine million, eight thousand and two. (10 000 000 000)
3. A company was reported to have made a profit of sh. 93 678 563. Two daily newspapers gave the figure, one to the nearest 1 000 000 and the other to the nearest 10 000. What was the difference between the rounded off figures?
4. A number was rounded off to the nearest 10 000 and given as 500 000. Which of the following numbers was likely to have been rounded off?
- (a) 498 382 (b) 508 462 (c) 489 693

1.4: Operations on Whole Numbers

Addition

Example 3

Work out: (a) $98 + 6\,734 + 348$ (b) $6\,349 + 259 + 79\,542$

Solution

- (a) Arrange the numbers in vertical forms as follows:

$$\begin{array}{r}
 98 \\
 6\,734 \\
 + 348 \\
 \hline
 7\,180
 \end{array}$$

(b)

$$\begin{array}{r}
 6\,349 \\
 259 \\
 + 79\,542 \\
 \hline
 86\,150
 \end{array}$$

Subtraction

Example 4

Work out: $73\,469 - 8\,971$

Solution

The numbers should be arranged in vertical form and digits lined up in their correct place values.

$$\begin{array}{r}
 73\,469 \\
 - 8\,971 \\
 \hline
 64\,498
 \end{array}$$

Multiplication

The result of multiplying two or more numbers is called their **product**.

Example 5

Work out: 469×63

Solution

$$\begin{array}{r}
 469 \\
 \times 63 \\
 \hline
 1\,407 \\
 + 28\,140 \\
 \hline
 29\,547
 \end{array}
 \rightarrow 469 \times 3 = 1\,407 \quad \text{or} \quad
 \begin{array}{r}
 469 \\
 \times 63 \\
 \hline
 28\,140 \\
 + 1\,407 \\
 \hline
 29\,547
 \end{array}
 \rightarrow 469 \times 60 = 28\,140$$

$$\begin{array}{r}
 469 \times 3 = 1\,407 \\
 469 \times 60 = 28\,140 \\
 \hline
 29\,547
 \end{array}$$

The expression may also be written as:

$$\begin{aligned}
 469 \times (60 + 3) &= 469 \times 60 + 469 \times 3 \\
 &= 28\,140 + 1\,407 \\
 &= 29\,547
 \end{aligned}$$

Division

When a number is divided by another, the result is called the **quotient**. Sometimes a quotient and a remainder is obtained. The quotient is taken to be the whole number part. The number being divided is called the **dividend** and the number dividing is called the **divisor**.

Example 6

Work out: $6\,493 \div 14$

Solution

The dividend is 6 493 and the divisor is 14. It is convenient to use the long division method.

$$\begin{array}{r}
 463 \text{ rem } 11 \\
 14 \overline{) 6493} \\
 \underline{-56} \\
 89 \\
 \underline{-84} \\
 53 \\
 \underline{-42} \\
 11
 \end{array}$$

In the above example, 463 is the **quotient** and 11 the **remainder**.

Note:

$$6\,493 = (463 \times 14) + 11$$

In general, **dividend = quotient x divisor + remainder**.

This relation can be used to check whether division has been carried out correctly.

Exercise 1.3

1. Work out:

- (a) $9 + 348 + 2\,960 + 38$
- (b) $536\,810 + 8\,725 + 473\,602$
- (c) $492\,375\,600 + 572\,041 + 4\,789\,561$
- (d) $903\,672 + 29 + 683\,426 + 8\,736$

2. Work out:

- (a) $476\,350\,300 - 97\,230\,575$
- (b) $67\,325 - 59\,322$
- (c) $496\,262 - 89\,320$
- (d) $941\,600 - 93\,226$
- (e) $293\,658 - 87\,254$
- (f) $487\,241 - 91\,396$

3. Work out:

- (a) 365×15
- (b) 472×25
- (c) $3\,729 \times 36$
- (d) $4\,926 \times 47$
- (e) $52\,342 \times 64$
- (f) $60\,493 \times 78$

4. Work out:

- (a) $1\,288 \div 23$
- (b) $1\,764 \div 38$
- (c) $1\,184 \div 21$
- (d) $5\,194 \div 54$
- (e) $81\,479 \div 69$
- (f) $76\,183 \div 36$

5. When a number is divided by 15, the quotient is 18 and remainder 5. What is the number?

6. Work out the following:

(a) $3(75 + 32) + 5(35 + 60)$

(b) $142 + 258 \div 6$

(c) $1\,305 \div (670 - 235) + 6 \times 780 \div 13$

(d) $975 \div 15 \times 8 + 420$

(e) $970 - (435 + 324) + 6(480 - 350)$

(f) $1\,444 + (16 \times 180) \div 90$

7. Work out:

(a)
$$\begin{array}{r} 672 \times 480 \\ \hline 96 \end{array}$$

(b)
$$\begin{array}{r} 252 \times 266 \\ \hline 42 \times 38 \end{array}$$

(c)
$$\begin{array}{r} 240 + 144 \\ \hline 48 \end{array}$$

(d)
$$\begin{array}{r} 648 - 243 \\ \hline 81 \end{array}$$

1.5: Word Problems

To work out word problems, one must read the given information carefully, identifying the operations required. The solution should be presented in an orderly and logical manner.

Example 7

Otieno had 3 469 bags of maize, each weighing 90 kg. He sold 2 654 of them.

(a) How many kilograms of maize was he left with?

(b) If he added 468 more bags of maize, how many bags did he end up with?

Solution

(a) One bag weighs 90 kg.

$$3\,469 \text{ bags weigh } 3\,469 \times 90 = 312\,210 \text{ kg}$$

$$2\,654 \text{ bags weigh } 2\,654 \times 90 = 238\,860 \text{ kg}$$

$$\begin{aligned} \text{Amount of maize left} &= 312\,210 - 238\,860 \\ &= 73\,350 \text{ kg} \end{aligned}$$

Alternatively

$$\begin{aligned} \text{Amount of maize left} &= (3\,469 - 2\,654) \times 90 \\ &= 815 \times 90 \\ &= 73\,350 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{(b) Number of bags} &= 815 + 468 \\ &= 1\,283 \end{aligned}$$

Exercise 1.4

1. Four coffee factories had 4 687, 3 891, 2 356 and 679 bags of cleared coffee.

(a) How many bags of coffee did they have altogether?

(b) If each bag weighed 62 kg, what was the total mass of the coffee?

2. A businessman had 1 860 bales of maize flour in his store. He sold 425 of them.
 - (a) How many bales were left?
 - (b) If each bale contained 12 packets, how many packets remained in the store?
3. The average mass of students in a class of 45 was 41 kg at the beginning of a term. At the end of the term, they had each gained 3 kg. Calculate:
 - (a) their total mass at the end of the term.
 - (b) the difference between their total mass at the start of the term and end of the term.
4. A vegetable vendor had 1 348 cabbages. He sold 750 on the first day and 240 on the second day. He added 462 to the remaining stock on the third day.
 - (a) How many cabbages did he have at the end?
 - (b) If he sold all the cabbages at an average cost of sh. 12, how much money did he collect?
5. A bookstore had 30 816 exercise books which were packed in cartons. Each carton contained 24 exercise books. The mass of an empty carton was 2 kg and a full carton 12 kg.
 - (a) How many cartons were there?
 - (b) What was the total mass of the empty cartons?
 - (c) What was the total mass of the books alone?
6. A car uses 1 litre of petrol for every 6 kilometres. The car was to travel 480 kilometres and had 15 litres at the beginning of the journey. Each litre costs sh. 58.00.
 - (a) How much more petrol did the car need in order to just complete the journey?
 - (b) How much did the fuel cost?
7. A *matatu* charges sh. 120 as fare from town A to town B. It has a capacity of 18 passengers. It can however carry 5 more passengers, but will have to pay a penalty of sh. 100 at each of 8 checkpoints. The distance between A and B is 84 km and the cost of petrol is sh. 58 per litre. If the *matatu* consumes 1 litre for every 7 kilometres, calculate:
 - (a) how much is gained if the *matatu* does not overload.
 - (b) how much is lost if the *matatu* overloads.
8. Five companies employed 2 340, 3 455, 675, 960 and 1 350 workers. The first two companies laid off 1 worker for every 5 while the other three recruited 2 new workers for every 3.

- (a) What was the total number of workers at the beginning?
 - (b) How many people:
 - (i) lost jobs?
 - (ii) got jobs?
 - (c) What was the total number of workers finally?
9. A minibus had 23 passengers at the beginning of a journey. Twelve passengers alighted at the first stop while 9 boarded. Six of those who boarded at the first stop alighted at the second stop and 12 got in. The minibus did not stop again up to the final destination. The charges from the starting point were sh. 50 up to the first stop, sh. 70 up to the second stop and sh. 85 up to the final destination.
- (a) How many passengers alighted at the final destination?
 - (b) How many passengers were ferried by the minibus through the journey?
 - (c) How much money was collected during the trip?

1.6: Even Numbers

A number whose last digit is 0, 2, 4, 6 or 8 is called an even number.

For example, 3 658, 2 430, 642 and 876 are even numbers.

1.7: Odd Numbers

Any number which ends with the digit 1, 3, 5, 7 or 9 is an odd number.

For example, 471, 1 263, 1 197 and 7 129 are odd numbers.

1.8: Prime Numbers

A prime number is a number that has only two factors, that is, 1 and itself.

For example, 2, 3, 5, 7, 11, 13, 17 and 19 are prime numbers.

Note:

- (i) 1 is not a prime number.
- (ii) 2 is the only even number which is a prime number.

Exercise 1.5

1. List down all the even numbers between 50 and 100.
2. List down all odd numbers between 10 and 50.
3. How many even numbers are there between 1 and 100?
4. List the prime numbers between 640 and 650.
5. Which of the following numbers are even? Which ones are odd?
347, 658, 851, 3 741, 1 243, 3 965, 6 429, 86 725, 44 001, 73 464, 3 960, 6 576.
6. List all the even numbers and odd numbers between 1 340 and 1 370.

FACTORS

Consider the statement $3 \times 4 = 12$. The numbers 3 and 4 are called factors of 12 because they divide 12 without a remainder.

The table below shows factors of given numbers.

<i>Number</i>	<i>Factors</i>
12	1, 2, 3, 4, 6, 12
16	1, 2, 4, 8, 16
39	1, 3, 13, 39
7	1, 7

Every natural number is a factor of itself. Some natural numbers have only two factors. A natural number with only two factors, one and itself, is called a **prime number**. Examples are 2, 3, 5, 7, 11.

Note:

1 is not a prime number because it has only one factor. Natural numbers other than 1 which are not prime are called **composite numbers**. They can be expressed as a product of two or more prime numbers or prime factors. For example:

$$9 = 3 \times 3$$

$$12 = 2 \times 2 \times 3$$

$$105 = 3 \times 5 \times 7$$

In some situations, the same number is repeated several times. For example:

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

In short, $2 \times 2 \times 2 \times 2 \times 2$ is written as 2^5 and read as 'two raised to power five'.

$$\begin{aligned}\text{Similarly, } 72 &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^3 \times 3^2\end{aligned}$$

To express a number in terms of prime factors, it is useful to take the prime numbers in ascending order, i.e., 2, 3, 5 ... and divide by each of them as many times as possible before going to the next.

Example 1

Express the following numbers in terms of their prime factors:

- (a) 300 (b) 196

Solution

(a)	2	300
	2	150
	3	75
	5	25
	5	5
		1

$$\begin{aligned}
 300 &= 2 \times 2 \times 3 \times 5 \times 5 \\
 &= 2^2 \times 3 \times 5^2
 \end{aligned}$$

(b)	2	196
	2	98
	7	49
	7	7
		1

$$\begin{aligned}
 196 &= 2 \times 2 \times 7 \times 7 \\
 &= 2^2 \times 7^2
 \end{aligned}$$

Exercise 2.1

Express the following composite numbers as products of prime factors:

- (a) 30 (b) 40 (c) 64 (d) 81 (e) 169
- (a) 256 (b) 430 (c) 472 (d) 126 (e) 245
- (a) 900 (b) 105 (c) 231 (d) 385 (e) 189
- (a) 993 (b) 357 (c) 715 (d) 935 (e) 1 386
- (a) 1 078 (b) 5 929 (c) 1 573 (d) 1 859 (e) 2 057

Chapter Three

DIVISIBILITY TEST

It is important to use divisibility tests on numbers for easier computation.

3.1: Divisibility Test for 2

A number is divisible by 2 if its last digit is even or zero.

Example 1

Work out $350 \div 2$.

Solution

$$\begin{array}{r} 175 \\ 2 \overline{)350} \\ \underline{-2} \\ 15 \\ \underline{-14} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

$$350 \div 2 = 175$$

350 is divisible by 2 because its last digit is zero.

Example 2

What is $174 \div 2$?

Solution

$$\begin{array}{r} 87 \\ 2 \overline{)174} \\ \underline{-16} \\ 14 \\ \underline{-14} \\ 0 \end{array}$$

$$174 \div 2 = 87$$

174 is divisible by 2 because its last digit is an even number.

Exercise 3.1

1. Which of the following numbers are divisible by 2?

- (a) 4, 27, 521, 628, 738
- (b) 15, 70, 99, 102, 823, 998
- (c) 151, 300, 702, 1 001, 20 020

3.2: Divisibility Test for 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

Example 3

Work out: $1\,257 \div 3$

Solution

$$\begin{array}{r}
 419 \\
 3 \overline{) 1257} \\
 \underline{- 12} \\
 5 \\
 \underline{- 3} \\
 27 \\
 \underline{- 27} \\
 0
 \end{array}$$

$$1\,257 \div 3 = 419$$

1 257 is divisible by 3 because the sum of its digits is 15, which is divisible by 3, i.e., $1 + 2 + 5 + 7 = 15$.

Example 4

What is $1\,075 \div 3$?

Solution

$$\begin{array}{r}
 358 \\
 3 \overline{) 1075} \\
 \underline{- 9} \\
 17 \\
 \underline{- 15} \\
 25 \\
 \underline{- 24} \\
 1
 \end{array}$$

$$1\,075 \div 3 = 358 \text{ rem } 1$$

1 075 is not divisible by 3 because the sum of the digits, i.e., $1 + 0 + 7 + 5 = 13$, is not divisible by 3.

Exercise 3.2

- Test whether the following numbers are divisible by 3:
 - 20 121
 - 7 203
 - 891 037
 - 65 379
- Which of the following numbers are divisible by 3?
 - 574, 681, 24 381, 87 690
 - 659, 82 720, 78 426, 942 831
- Test whether 5 202 is divisible by:
 - 2
 - 3
- Which of the following numbers are divisible by both 2 and 3?
 - 132, 84, 970, 534, 722
 - 24, 712, 9 030, 6 754

3.3: Divisibility Test for 4

A number is divisible by 4 if its last two digits are both zero or form a number which is divisible by 4.

Example 5

Divide 1 132 by 4.

Solution

$$\begin{array}{r}
 283 \\
 4 \overline{) 1132} \\
 \underline{-8} \\
 33 \\
 \underline{-32} \\
 12 \\
 \underline{-12} \\
 0
 \end{array}$$

$$1\,132 \div 4 = 283$$

1 132 is divisible by 4 because the last two digits form 32, which is divisible by 4.

Example 6

Divide 34 700 by 4.

Solution

$$\begin{array}{r}
 8675 \\
 4 \overline{) 34700} \\
 \underline{-32} \\
 27 \\
 \underline{-24} \\
 30 \\
 \underline{-28} \\
 20 \\
 \underline{-20} \\
 0
 \end{array}$$

$$34\,700 \div 4 = 8\,675$$

34 700 is divisible by 4 because the last two digits are both zeros.

Exercise 3.3

- Which of the following numbers are divisible by 4?
 (a) 100, 324, 758, 640, 832
 (b) 917, 1 448, 72 005, 9 472, 173 462
- Test whether 61 500 is divisible by:
 (a) 2 (b) 3 (c) 4
- Which of the following numbers are divisible by both 2 and 4?
 (a) 10, 82, 132, 416, 600
 (b) 50, 256, 496, 882, 930
- Which of the following numbers are divisible by both 3 and 4?
 (a) 36, 86, 192, 1 504
 (b) 120, 744, 306, 9 564
- Which of the following numbers are divisible by all three numbers 2, 3 and 4?
 1 080, 1 842, 9 216, 65 432, 12 636

3.4: Divisibility Test for 5

A number is divisible by 5 if its last digit is zero or 5.

Example 7

$$2\,340 \div 5$$

Solution

$$\begin{array}{r}
 468 \\
 5 \overline{) 2340} \\
 \underline{-20} \\
 34 \\
 \underline{-30} \\
 40 \\
 \underline{-40} \\
 0
 \end{array}$$

$$2\,340 \div 5 = 468$$

2 340 is divisible by 5 because its last digit is zero.

Example 8

$$7\,835 \div 5$$

Solution

$$\begin{array}{r}
 1567 \\
 5 \overline{) 7835} \\
 \underline{-5} \\
 28 \\
 \underline{-25} \\
 33 \\
 \underline{-30} \\
 35 \\
 \underline{-35} \\
 0
 \end{array}$$

$$7\,835 \div 5 = 1\,567$$

7 835 is divisible by 5 because its last digit is 5.

Exercise 3.4

- Which of the following numbers are divisible by 5?
 - 78, 175, 804, 930, 1 050
 - 95, 374, 535, 800, 2 172, 4 325
- Which of the following numbers are divisible by both 2 and 5?
 - 572, 825, 720, 9 430, 10 505
 - 820, 41 325, 57 640, 684 320
- Which of the following numbers are divisible by both 4 and 5?
 - 825, 74 320, 32 432, 643 000
 - 300, 4 345, 67 420, 931 700
- Which of the following are divisible by both 3 and 5?
 - 30, 170, 900, 520, 635, 10 710
 - 3 250, 432 120, 832 710, 92 715
- Which of the following are divisible by all the four numbers 2, 3, 4 and 5?

720, 87 400, 10 320, 57 435, 637 410

3.5: Divisibility Test for 6

A number is divisible by 6 if it is divisible by both 2 and 3.

Example 9

Work out: $612 \div 6$

Solution

$$\begin{array}{r} 102 \\ 6 \overline{) 612} \\ \underline{-6} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

$$612 \div 6 = 102$$

612 is divisible by 6 because it is divisible by both 2 and 3.

$6 + 1 + 2 = 9$, which is divisible by 3 and the last digit of 612 is even.

Exercise 3.5

- Which of the following numbers are divisible by 6?
(a) 37 622, 4 320, 8 730, 93 744, 1 083 470
(b) 834, 7 368, 98 704, 3 672, 48 732
- Test whether 83 472 is divisible by:
(a) 2 (b) 3 (c) 4 (d) 6
- Which of the following numbers are both divisible by 2 and 3?
(a) 390, 441, 5 310, 6 732, 7 544
(b) 531, 6 822, 7 452, 8 732, 95 490
- Identify the numbers which are divisible by 6 in question 3.
- Which of the following numbers are divisible by both 5 and 6?
(a) 78, 920, 53 250, 634 710, 83 475
(b) 435, 572, 62 510, 78 300, 83 210
- Which of the following numbers are divisible by both 4 and 6?
(a) 660, 7 212, 84 243, 9 404, 50 520, 42 564
(b) 5 140, 6 336, 6 028, 7 260, 30 468, 850 152
- Which of the following numbers are both divisible by 5 and 6?
(a) 210, 435, 450, 510, 655, 895, 900
(b) 750, 805, 920, 630, 1 290, 1 385

3.6: Divisibility Test for 8

A number is divisible by 8 if the number formed by its last 3 digits is divisible by 8.

Example 10

$$3\,027\,144 \div 8$$

Solution

$$\begin{array}{r}
 378393 \\
 8 \overline{) 3027144} \\
 \underline{-24} \\
 62 \\
 \underline{-56} \\
 67 \\
 \underline{-64} \\
 31 \\
 \underline{-24} \\
 74 \\
 \underline{-72} \\
 24 \\
 \underline{-24} \\
 0
 \end{array}$$

$$3\,027\,144 \div 8 = 378\,393$$

3 027 144 is divisible by 8 because its last three digits form a number divisible by 8, i.e., $144 \div 8 = 18$.

Exercise 3.6

- Which of the following numbers are divisible by 8?
 (a) 78 104, 83 412, 634 112, 857 124, 932 160
 (b) 532 168, 432 120, 864 324, 934 152, 1 034 128
- Which of the following numbers are divisible by both 4 and 8?
 (a) 732, 8 112, 93 136, 85 724, 1 123 136, 2 732 160
 (b) 6 104, 7 325, 93 128, 483 194, 754 368
- Which of the following numbers are divisible by both 3 and 8?
 (a) 73 104, 48 144, 501 144, 634 129, 754 104
 (b) 38 272, 484 248, 532 438, 231 672, 2 098 944
- Which of the following numbers are divisible by both 5 and 8?
 (a) 420, 535, 1 120, 1 230, 8 640, 7 320
 (b) 640, 3 240, 4 360, 8 470, 5 160, 5 800
- Test whether 77 080 is divisible by:
 (a) 2 (b) 3 (c) 4 (d) 6 (e) 8

3.7: Divisibility Test for 9

A number is divisible by nine if the sum of its digits is divisible by 9.

Example 11

$$32\,157 \div 9$$

Solution

$$\begin{array}{r}
 3573 \\
 9 \overline{) 32157} \\
 \underline{-27} \\
 51 \\
 \underline{-45} \\
 65 \\
 \underline{-63} \\
 27 \\
 \underline{-27} \\
 0
 \end{array}$$

$$32\,157 \div 9 = 3\,573$$

32 157 is divisible by 9 since the sum of its digits, i.e.,
 $3 + 2 + 1 + 5 + 7 = 18$, is divisible by 9.

Exercise 3.7

- Which of the following numbers are divisible by 9?
 (a) 405, 5 346, 6 726, 8 432, 9 315
 (b) 854, 9 792, 7 245, 8 549, 41 202
- Test whether 1 108 809 is divisible by:
 (a) 2 (b) 3 (c) 4 (d) 5 (e) 6 (f) 8 (g) 9
- Which of the following numbers are divisible by both 3 and 9?
 (a) 3 572, 4 320, 8 955, 9 540, 7 436
 (b) 2 846, 38 475, 4 386, 57 483, 64 756
- Which of the following numbers are divisible by both 5 and 9?
 (a) 7 540, 875 610, 975 420, 84 735
 (b) 3 279, 2 745, 3 411, 54 540

3.8: Divisibility Test for 10

A number is divisible by 10 if its last digit is 0.

Example 12

What is $3\,470 \div 10$?

Solution

$$\begin{array}{r}
 347 \\
 10 \overline{) 3470} \\
 \underline{-30} \\
 47 \\
 \underline{-40} \\
 70 \\
 \underline{-70} \\
 0
 \end{array}$$

$$3\,470 \div 10 = 347$$

3 470 is divisible by 10 because its last digit is 0.

Exercise 3.8

- Which of the following numbers are divisible by 10?
 (a) 432, 655, 720, 430, 801, 910, 990, 550
 (b) 610, 540, 604, 708, 880, 909, 850
- Which of the following numbers are divisible by both 5 and 10?
 (a) 725, 640, 855, 980, 1 140, 1 235, 3 460
 (b) 870, 935, 4 350, 5 780, 6 325, 7 840
- Which of the following numbers are divisible by both 3 and 10?
 (a) 270, 430, 510, 624, 720
 (b) 3 250, 4 320, 8 230, 97 410, 112 340

3.9: Divisibility Test for 11

A number is divisible by 11 if the sum of its digits in the 1st, 3rd, 5th, 7th, etc, positions, and the sum of its digits in the 2nd, 4th, 6th, 8th, etc, positions are equal, or differ by 11, or by a multiple of 11.

Example 13

$$8\,260\,439 \div 11$$

Solution

$$\begin{array}{r}
 750949 \\
 11 \overline{) 8260439} \\
 \underline{-77} \\
 56 \\
 \underline{-55} \\
 104 \\
 \underline{-99} \\
 53 \\
 \underline{-44} \\
 99 \\
 \underline{-99} \\
 0
 \end{array}$$

$$8\,260\,439 \div 11 = 750\,949$$

8 260 439 is divisible by 11 because $8 + 6 + 4 + 9 = 27$; $2 + 0 + 3 = 5$; and $27 - 5 = 22$, which is a multiple of 11.

Exercise 3.9

- Which of the following numbers are divisible by 11?
 (a) 2 596, 4 397, 5 896, 8 151, 1 052, 4 132
 (b) 4 213, 5 753, 5 016, 53 152, 104 844
- Which of the following numbers are divisible by both 10 and 11?
 (a) 3 520, 4 670, 4 730, 6 930, 7 470
 (b) 2 530, 5 170, 60 750, 10 230, 10 780
- Test whether 712 008 is divisible by:
 (a) 2 (b) 3 (c) 8 (d) 9 (e) 11
- Copy and match:

<i>Number</i>	<i>Divisible by</i>
3 572	2
743 815	3
5 289	4
83 427	5
95 712	6
348 246	8
7 384 370	9
87 534 216	10
1 048 564	11

Chapter Four

GREATEST COMMON DIVISOR

The factors of 12 are 1, 2, 3, 4, 6 and 12.

The factors of 16 are 1, 2, 4, 8 and 16.

1, 2 and 4 are common factors of both 12 and 16.

The greatest among them is 4. Thus, 4 is referred to as:

- (a) the greatest common factor (G.C.F.) of 12 and 16, or
- (b) the greatest common divisor (G.C.D.) of 12 and 16, or
- (c) the highest common factor (H.C.F.) of 12 and 16.

To find the G.C.D. of two or more numbers, it is easier to first list the factors of the stated or given numbers, identify common factors and state the greatest among them.

Alternatively, the G.C.D. of 2 or more numbers can be obtained by first expressing each number as a product of its prime factors. The factors which are common are determined and their product obtained.

Example 1

1. Find the G.C.D. of 72, 96 and 300.

Solution

72: 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72

96: 2, 3, 4, 6, 8, 12, 16, 24, 32, 48 and 96

300: 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150 and 300

The common factors are 2, 3, 4, 6, 12.

The greatest among them is 12.

$\therefore$ G.C.D. of 72, 96 and 300 is 12.

The G.C.D. of a set of numbers can also be obtained using a table.

Example 2

Find the G.C.D. of 72, 96, 300.

	72	96	300
2	36	48	150
2	18	24	75
3	6	8	25

$$\begin{aligned}\therefore \text{G.C.D.} &= 2 \times 2 \times 3 \\ &= 4 \times 3 \\ &= 12\end{aligned}$$

In the above example, we look for the factors which divide the given numbers exactly, starting with the least and then find their product.

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$300 = 2 \times 2 \times 3 \times 5 \times 5$$

The common factors are 2^2 and 3

$\therefore$ the G.C.D. is $2^2 \times 3$, which is 12.

Exercise 4.1

- Find the G.C.D. of the following pair of numbers:
(a) 30, 45 (b) 36, 64 (c) 48, 60
(d) 50, 80 (e) 75, 90 (f) 60, 45
- Find the G.C.D. of:
(a) 42, 105, 63 (b) 210, 135, 330
(c) 70, 210, 154 (d) 240, 360, 600, 700
- Three tanks are capable of holding 36, 84 and 90 litres of milk. Determine the capacity of the greatest vessel which can be used to fill each one of them an exact number of times.
- What is the greatest mass that can be taken in an exact number of times from 144 g, 216 g and 126 g?
- Three similar steel bars of length 200 cm, 300 cm and 360 cm are cut into equal pieces. Find the largest possible area of a square which can be made from any of the three pieces.
- Three masses of sugar are grouped into 0.36 kg, 0.504 kg and 0.672 kg. Find the greatest mass of sugar that can be taken an exact number of times from the three masses. (Give your answer in kg)
- Find the greatest number which, when divided by 181 and 236 leave a remainder of 5 in each case. (Hint: Subtract 5 from each number).

Chapter Five

LEAST COMMON MULTIPLE

5.1: Multiples

Multiples are products of natural numbers. Consider the table below:

<i>Number</i>	<i>Multiples</i>				
2	2 x 1	2 x 2	2 x 3	2 x 4	2 x 5 ...
	2	4	6	8	10
3	3 x 1	3 x 2	3 x 3	3 x 4	3 x 5 ...
	3	6	9	12	15
4	4 x 1	4 x 2	4 x 3	4 x 4	4 x 5 ...
	4	8	12	16	20

The numbers on the right are multiples of the numbers on the left. In the table, the multiples of 3 and 4 are;

3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ...

4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

12, 24 and 36 are the common multiples. So, the least common multiple (L.C.M.) of 2, 3 and 4 is 12.

5.2: L.C.M of Two or more Numbers

Method 1

- (i) List the multiples of each of the numbers given, extending until common multiples appear.
- (ii) The least common multiple will be the first common multiple to appear.

Example 1

Find the L.C.M. of 6, 8 and 12.

Solution

6: 6, 12, 18, 24, 30, 36, 42, 48, ...

8: 8, 16, 24, 32, 40, 48, 56, ...

12: 12, 24, 36, 48, 60, 72, ...

The common multiples are 24 and 48.

The least is 24.

∴ The L.C.M. of 6, 8 and 12 is 24.

Method 2

The L.C.M. of a set of numbers can be found using tables.

Example 2

Find the L.C.M. of 8, 12, 18, and 20.

	8	12	18	20
2	4	6	9	10
2	2	3	9	5
2	1	3	9	5
3	1	1	3	5
3	1	1	1	5
5	1	1	1	1

The L.C.M. is the product of all the prime factors.

$$\begin{aligned}
 \text{Therefore, L.C.M. of 8, 12, 18 and 20} &= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \\
 &= 2^3 \times 3^2 \times 5 \\
 &= 360
 \end{aligned}$$

Note:

Unlike the G.C.D. tables, if the divisor (factor) does not divide a number exactly, then the number is retained, e.g., 2 does not divide 9 exactly, therefore 9 is retained. The last row must have all values 1.

Method 3: Use of Prime Factors

- Express each number as a product of its prime factors (numbers).
- By taking every prime factor that occurs in the products of the numbers, underline or encircle the one with the highest power.
- The L.C.M. is the product of the underlined or encircled numbers.

Example 3

Find the L.C.M. of:

- 24 and 36.
- 990, 525 and 490.

Solution

$$\begin{aligned} \text{(a)} \quad 24 &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3 \end{aligned}$$

$$\begin{aligned} 36 &= 2 \times 2 \times 3 \times 3 \\ &= 2^2 \times 3^2 \end{aligned}$$

$$\begin{aligned} \text{L.C.M.} &= 2^3 \times 3^2 \\ &= 8 \times 9 \\ &= 72 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 990 &= 2 \times 3 \times 3 \times 5 \times 11 \\ &= 2 \times 3^2 \times 5 \times 11 \end{aligned}$$

$$\begin{aligned} 525 &= 3 \times 5 \times 5 \times 7 \\ &= 3 \times 5^2 \times 7 \end{aligned}$$

$$\begin{aligned} 490 &= 2 \times 5 \times 7 \times 7 \\ &= 2 \times 5 \times 7^2 \end{aligned}$$

$$\begin{aligned} \text{L.C.M.} &= 2 \times 3^2 \times 5^2 \times 7^2 \times 11 \\ &= 242\,550 \end{aligned}$$

Exercise 5.1

- Find the first six multiples of the following numbers:
5, 6, 7 and 9
- Find the L.C.M. of each of the following pairs of numbers, leaving the answer in prime factors (use powers where possible):

(a) 48, 45	(b) 36, 64	(c) 75, 100
(d) 84, 182	(e) 60, 225	(f) 421, 4 631
- Find the L.C.M. of each of the following sets of numbers:

(a) 4, 6, 9	(b) 10, 30, 165	(c) 42, 84, 70
(d) 140, 105, 150	(e) 48, 100, 72	(f) 34, 68, 170
- Find the L.C.M. of each of the following sets of numbers:

(a) 8, 12, 20, 32	(b) 36, 24, 40, 16
(c) 14, 35, 30, 49	(d) 28, 24, 40, 16
(e) 2, 3, 5, 7	(f) 240, 360, 600, 720
- Three bells ring at intervals of 40 minutes, 45 minutes and 60 minutes. If they ring simultaneously at 6.30 a.m., at what time will they next ring together?
- What is the least length of a school working day in hours if it can be split into exact periods of 30 minutes, 40 minutes or 45 minutes?
- A number n is such that when it is divided by 27, 30 or 45, the remainder is 3. Find the smallest possible value of n .

8. Find the length of the shortest piece of pipe that can be cut into equal lengths, each 25 cm or 36 cm or 42 cm.
9. Find the least mass of meat that can be cut into equal amounts of 2 kg, 3 kg, 5 kg or 6 kg.
10. Find the least amount of cement that can be put into bags which all contain 20 kg or 25 kg or 40 kg or 50 kg.
11. Four light signals are programmed at intervals of 40 seconds, 50 seconds, 60 seconds and 75 seconds. What is the earliest time they will give out light signals simultaneously if the last time they did this was at 8.15 a.m?
12. The G.C.D. of two numbers is 12 and their L.C.M. is 240. If one of the numbers is 60, find the other number.
13. The G.C.D. of three numbers is 30 and their L.C.M. is 900. Two of the numbers are 60 and 150. What are the other possible numbers?

Chapter Six

INTEGERS

6.1: Introduction

Kenya attained independence on 12th December, 1963. Achola, Wanjiku and Wanyama were born 2, 4 and 8 days respectively before the Independence Day. Fatuma, Ruto and Mwanzia were born 1, 4 and 7 days respectively after the Independence Day. This information can be represented as shown in table 6.1 and figure 6.1

Table 6.1

<i>Before</i>		<i>After</i>	
Achola	2	Fatuma	1
Wanjiku	4	Ruto	4
Wanyama	8	Mwanzia	7

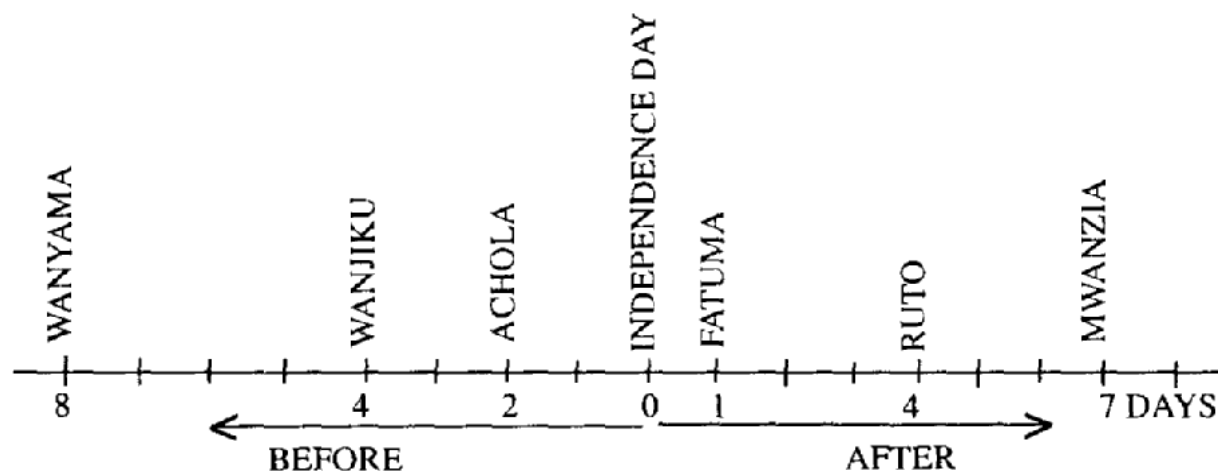


Fig. 6.1

Note that in table 6.1, we are giving the days each one of them was born before or after the Independence Day. In this case, the Independence Day is our reference point. We assign 'zero' (0) to the reference point. Notice that in figure although Wanjiku and Ruto were each born 4 days from the Independence they are on opposite directions (sides). We may use positive (+ve) and

negative (–ve) signs to refer to directions. In our example, we assign positive (+ve) to ‘after’ and negative (–ve) to ‘before’. This can be illustrated as in figure 6.2.

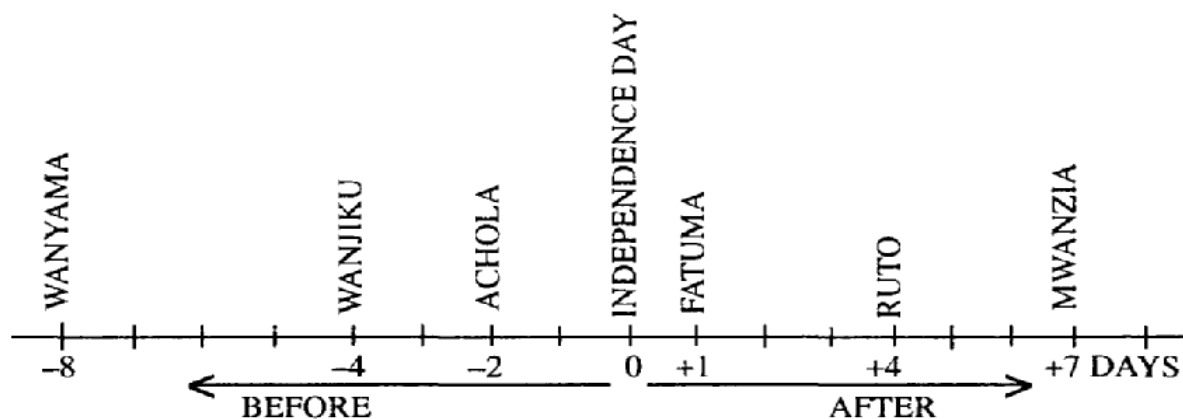


Fig. 6.2

Using figure 6.2, what number would you assign to each of the following dates of birth?

- (i) 11th Dec. 1963 (ii) 18th Dec. 1963 (iii) 9th Dec. 1963
 (iv) 30th Nov. 1963 (v) 1st Jan. 1964 (vi) 20th Nov. 1963

Project

With the aid of diagrams, give examples of other situations that require the use of reference points and directions.

6.2: The Number Line

Positive whole numbers, negative whole numbers and zero are called **integers**. Integers are usually represented on the **number line** at equal intervals, as shown in figure 6.3, where each interval is equal to one unit.

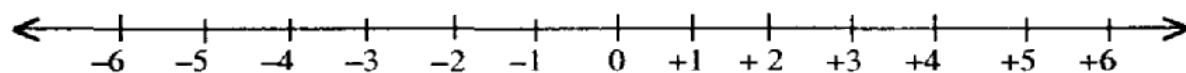


Fig. 6.3

Any integer is less than all other integers to the right of it and greater than all those to the left of it. Thus, -2 is less than -1 but greater than -3 .

The symbols $<$ and $>$ are used to denote 'less than and greater than' respectively.

Thus, $-2 < -1$, and $-2 > -3$.

Use $<$ or $>$ to compare the following pairs of numbers:

- | | | |
|---------------------|-----------------------|----------------------|
| (i) -5 and $+1$ | (ii) -3 and $+4$ | (iii) -5 and $+5$ |
| (iv) -10 and $+1$ | (v) -7 and -9 | (vi) -20 and -36 |
| (vii) 1 and -25 | (viii) 15 and -30 | (ix) -25 and -38 |

6.3: Operations on Integers

Addition of Integers

Addition of integers can be represented on a number line. For example, to add $+3$ to $+2$, we begin at $+2$ and move 3 units to the right, as shown in figure 6.4.

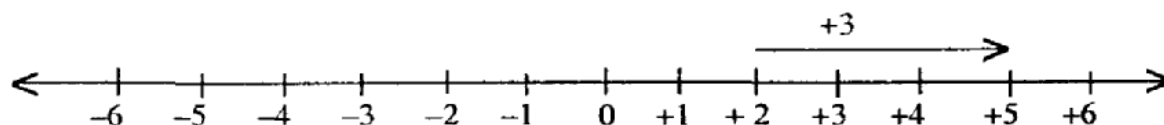


Fig. 6.4

If, instead, we begin at $+3$ and move 2 units to the right, what would be the result?

Similarly, to add -1 to $+5$, we begin at $+5$ and move 1 unit to the left, as shown in figure 6.5.

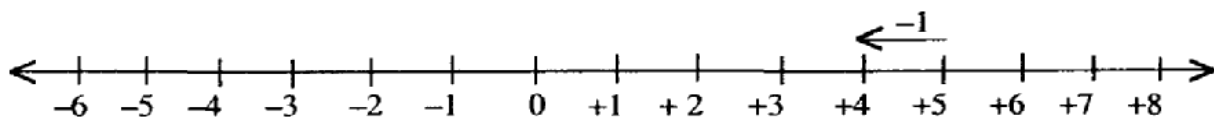


Fig. 6.5

If we begin at -1 , through how many steps and in what direction must we move to arrive at the same result?

Exercise 6.1

Show how the following additions can be done using a number line and give the results:

- | | |
|---------------------------------|---------------------------|
| 1. (a) $(+2) + (+3)$ | (b) $(+8) + (+7)$ |
| (c) $(+12) + (+9)$ | (d) $(+7) + (+10)$ |
| 2. (a) $(+7) + (-4)$ | (b) $(-8) + (+5)$ |
| (c) $(+15) + (-14)$ | (d) $(-9) + (+2)$ |
| 3. (a) $(+7) + (-4)$ | (b) $(-13) + (+13)$ |
| (c) $(+4) + (-13)$ | (d) $(-11) + (+5)$ |
| 4. (a) $(-3) + (-4)$ | (b) $(-7) + (+2)$ |
| (c) $(-15) + (+12)$ | (d) $(-6) + (-6)$ |
| 5. (a) $(+2) + (+3) + (+5)$ | (b) $(+4) + (-2) + (-3)$ |
| (c) $(+6) + (-2) + (+6)$ | (d) $(-7) + (-2) + (+6)$ |
| 6. (a) $(-4) + (-3) + (-2)$ | (b) $(-1) + (-7) + 0$ |
| (c) $(+6) + (+2) + (-5)$ | (d) $(+8) + (-3) + (+12)$ |
| 7. (a) $(+3) + (+3) + (+3)$ | (b) $(-5) + (-5) + (-5)$ |
| (c) $(-2) + (-2) + (-2) + (-2)$ | (d) $(-6) + (+4) + (-8)$ |

Fill in the boxes in numbers 8-11:

- | | |
|--------------------------------------|----------------------------------|
| 8. (a) $\square + (+3) = 8$ | (b) $(+7) + \square = 10$ |
| 9. (a) $(-3) + \square = +10$ | (b) $\square + (+7) = +7$ |
| (c) $\square + (+2) = -5$ | (d) $(+3) + \square = -6$ |
| 10. (a) $\square + (-7) = -11$ | (b) $(-9) + \square = -14$ |
| 11. (a) $(+3) + \square + (+4) = +5$ | (b) $(-4) + (-2) + \square = +3$ |
| (c) $\square + (+5) + (-2) = +7$ | (d) $(-3) + \square + (-4) = 0$ |

Subtraction of Integers

From the last section, we have seen that;

$$(+2) + (+3) = +5$$

This leads to the expression $(+5) - (+2) = +3$, and $(+5) - (+3) = (+2)$.

To subtract $+2$ from $+5$, we need to find a number n which, when added to $+2$, gives $+5$, see figure 6.6. In this case, $n = +3$.

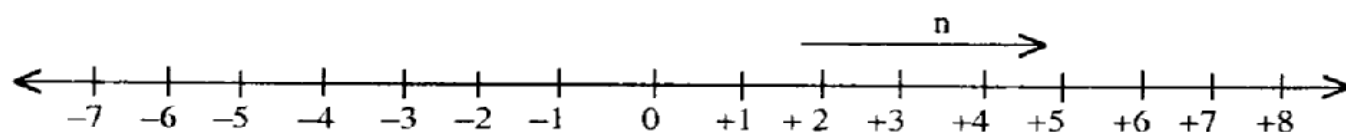


Fig. 6.6

Similarly, to subtract +3 from +5, we need to find a number n which, when added to +3, gives +5, see figure 6.7. In this case, $n = +2$.

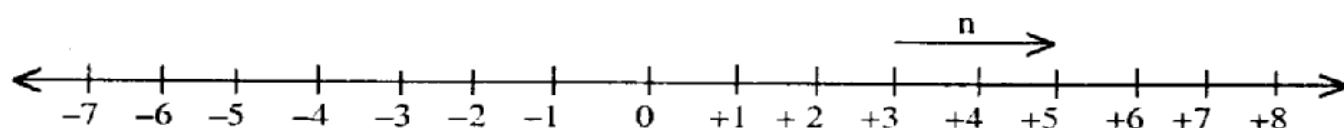


Fig. 6.7

Consider $(+2) - (+3)$. As in the examples above, we look for a number n which, when added to +3, gives +2, as shown in figure 6.8. The number in this case is -1 .

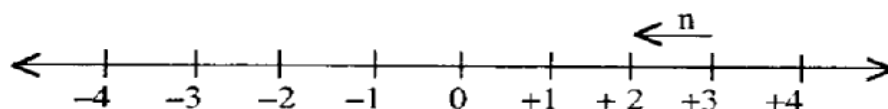


Fig. 6.8

Note:

- (i) The number n is given by the number of spaces between the two numbers.
- (ii) The number n is positive (+ve) if the arrow is towards the right, and negative (-ve) if the arrow is towards the left.

Example 1

Perform the following operations using a number line:

- (a) $(+5) - (-2)$ (b) $-3 - (+6)$ (c) $-7 - (-8)$

Solution

Start at -2 . Move to $+5$. The steps in between are 7 towards the right. The answer is $+7$.

(a)

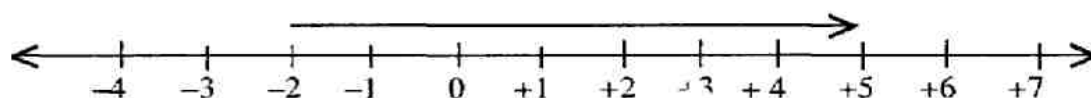


Fig. 6.9

(b)

Start at $+6$ and move to -3 . 9 steps will be made towards the left. The answer is -9 .

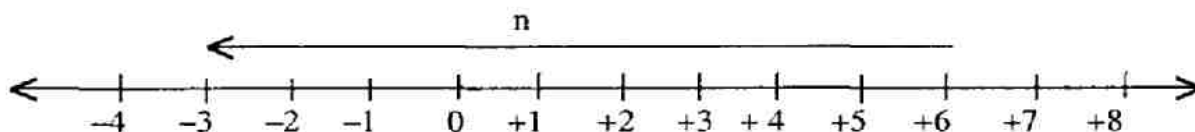


Fig. 6.10

(c)

Start at -8 and move to -7 . One step is made towards the right. The answer is $+1$.

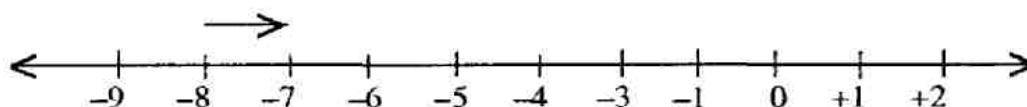


Fig. 6.11

Exercise 6.2

Show how the following subtractions can be performed using a number line and give the results:

1. (a) $(+3) - (+2)$

(c) $(+6) - (+3)$

2. (a) $(+4) - (-5)$

(c) $(+1) - (-8)$

3. (a) $(-2) - (+5)$

(c) $(-6) - (+6)$

4. (a) $(-3) - (-4)$

(c) $(-10) - (-3)$

(b) $(+4) - (+7)$

(d) $(+8) - (+8)$

(b) $(+13) - (-6)$

(d) $(+11) - (-4)$

(b) $(-4) - (+3)$

(d) $(-9) - (+12)$

(b) $(-6) - (-5)$

(d) $(-14) - (-5)$

Fill in the boxes in numbers 5-8:

5. (a) $\square - (+3) = +2$

(c) $\square - (+8) = +3$

6. (a) $\square - (-2) = +3$

(c) $\square - (-4) = -5$

7. (a) $(-7) - \square = -3$

(c) $\square - (+2) = -5$

8. (a) $(+1 - +3) - \square = -5$

(c) $\square - (-3) = -7$

(b) $\square - (-3) = +12$

(d) $(+7) - \square = +4$

(b) $\square - (-3) = -12$

(d) $(-4) - \square = 9$

(b) $(-8) - \square = +12$

(d) $\square - (+3) = +1$

(b) $\square - (-2) = -1$

(d) $(-6) - \square = -6$

Ordinarily, $+3$ is written as 3, but -2 can only be written as -2 .

Thus, $5 - (+3) = 5 - 3$

$= 2$

$2 - (+6) = 2 - 6$

$= -4$

Positive integers are also referred to as **natural numbers**. The result of subtracting the negative of a number is the same as adding that number.

Thus, $3 - (-4) = 3 + 4$

$= 7$

$(-5) - (-2) = -5 + 2$

$= -3$

In mathematics, it is assumed that a number with no sign before it has a positive sign. The number line can now be drawn as follows:

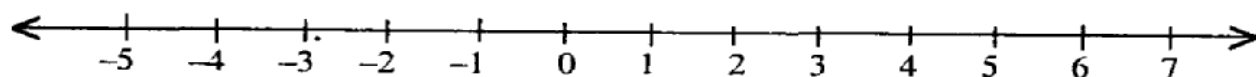


Fig. 6.12

Exercise 6.3

Evaluate the following:

- (a) $45 - 15$ (b) $35 - 16$ (c) $17 - 42$ (d) $19 - 70$
- (a) $12 - (-7)$ (b) $25 - (-36)$ (c) $30 - (-50)$ (d) $55 - (-28)$
- (a) $(-5) - (+16)$ (b) $(-11) - (+18)$ (c) $(-40) - (20)$ (d) $(-36) - (+52)$
- (a) $(-15) - (-22)$ (b) $(-33) - (-23)$ (c) $(-26) - (-19)$ (d) $76 - (-58)$

Multiplication of Integers

Repeated addition is similar multiplication, e.g., $(-2) + (-2) + (-2) + (-2)$ can be written as $(-2) \times 4 = -8$.

Copy and complete the tables below by continuing the pattern of the results.

Table 6.2 (a)

$4 \times 5 = 20$
$3 \times 5 = 15$
$2 \times 5 = 10$
$1 \times 5 = 5$
$0 \times 5 = 0$
$-1 \times 5 = -5$
$-2 \times 5 =$
$-3 \times 5 =$
$-4 \times 5 =$

Table 6.2 (b)

$3 \times 4 = 12$
$3 \times 3 = 9$
$3 \times 2 = 6$
$3 \times 1 = 3$
$3 \times 0 = 0$
$3 \times -1 = -3$
$3 \times -2 =$
$3 \times -3 =$
$3 \times -4 =$

Note:

- (a negative number) $\times$ (a positive number) = (a negative number)
- (a positive number) $\times$ (a negative number) = (a negative number)

Thus, $-50 \times 3 = -150$, and $24 \times -2 = -48$

Exercise 6.4

Evaluate:

- (a) $2 \times (-6)$ (b) $4 \times (-7)$
- (a) $12 \times (-5)$ (b) $11 \times (-10)$
- (a) $25 \times (-8)$ (b) $(-9) \times 4$
- (a) $(-10) \times 10$ (b) $(-15) \times 6$
- (a) $(-20) \times 4$ (b) $(-50) \times 6$

We have so far multiplied:

- positive integers by positive integers.
- positive integers by negative integers.
- negative integers by positive integers.

We now need to multiply a negative integer by a negative integer. Copy and complete the tables below by continuing the pattern of results:

Table 6.3 (a)

$5 \times -6 = -30$
$4 \times -6 = -24$
$3 \times -6 = -18$
$2 \times -6 = -12$
$1 \times -6 = -6$
$0 \times -6 = 0$
$-1 \times -6 = 6$
$-2 \times -6 = 12$
$-3 \times -6 =$
$-4 \times -6 =$
$-5 \times -6 =$

Table 6.3 (b)

$-2 \times 5 = -10$
$-2 \times 4 = -8$
$-2 \times 3 = -6$
$-2 \times 2 = -4$
$-2 \times 1 = -2$
$-2 \times 0 = 0$
$-2 \times -1 = 2$
$-2 \times -2 = 4$
$-2 \times -3 =$
$-2 \times -4 =$
$-2 \times -5 =$

What do you notice?

Note:

- The products are increasing from top to bottom.
- Whenever we multiply a negative integer by a negative integer, the result is a positive integer. For example;

$$-5 \times -6 = 30$$

$$-2 \times -4 = 8$$

$$-9 \times -3 = 27$$

$$\begin{aligned} -2 \times 3 \times -4 &= -6 \times -4 \\ &= 24 \end{aligned}$$

When we multiply a negative number by a negative number, the result is always a positive number.

Exercise 6.5

Evaluate:

- -3×-7
 - -8×-10
 - -13×-3
 - -16×-2
- -60×-4
 - -16×-8
 - -33×-3
 - -45×-20
- -56×-2
 - $-5 \times 8 \times -2$
 - $-7 \times -3 \times 10$
- $-4 \times -4 \times -4 \times -4$
 - $-10 \times 2 \times 10$

5. Fill in the box in each of the following:

$$(a) \ 5 \times \square = -20 \quad (b) \ -8 \times \square = -24 \quad (c) \ \square \times -24 = -48$$

Division of Integers

From the last section on multiplication, we saw that $5 \times 4 = 20$. This implies that $20 \div 4 = 5$ and $20 \div 5 = 4$. Division can thus be looked at as the inverse of multiplication.

To divide 20 by 5, i.e., $20 \div 5$, we need to find a number which when multiplied by 5 gives 20.

For example;

$$\begin{array}{lll} (i) \text{ If } 24 \div -6 = n, & (ii) \text{ If } -36 \div 4 = n, & (iii) \text{ If } -64 \div -8 = n, \\ \text{then, } n \times -6 = 24 & \text{then, } n \times 4 = -36 & \text{then, } n \times -8 = -64 \\ \therefore n = -4 & \therefore n = -9 & \therefore n = 8 \end{array}$$

Note:

- (i) (a positive number) $\div$ (a positive number) = (a positive number)
- (ii) (a positive number) $\div$ (a negative number) = (a negative number)
- (iii) (a negative number) $\div$ (a positive number) = (a negative number)
- (iv) (a negative number) $\div$ (a negative number) = (a positive number)

In general, for multiplication and division of integers:

- (i) two like signs give positive sign,
- (ii) two unlike signs give negative sign.

Note:

Multiplication by zero always gives zero, but division by zero is not defined.

Exercise 6.6

Evaluate:

1. (a) $10 \div 2$ (b) $50 \div -25$ (c) $98 \div -14$ (d) $126 \div 9$
2. (a) $288 \div -24$ (b) $42 \div 6$ (c) $-90 \div 10$ (d) $-125 \div 5$
3. (a) $-615 \div 15$ (b) $-1080 \div 90$ (c) $-140 \div -20$ (d) $-256 \div 16$
4. (a) $-289 \div 17$ (b) $-560 \div 16$ (c) $-912 \div 19$ (d) $-570 \div 19$

Find the value of the unknown in each of the following:

5. (a) $27 \div y = -3$ (b) $-144 \div y = 16$ (c) $84 \div y = 12$
6. (a) $-56 \div n = 7$ (b) $195 \div n = 13$ (c) $340 \div n = 17$
7. (a) $-2418 \div x = 39$ (b) $-2247 \div x = 321$ (c) $-4860 \div x = 81$

6.4: Order of Operations

The order in which operations are performed can be shown by the use of brackets.

(i) 'Subtract 8 from 18 and then subtract 5 from the result' can be written as
 $(18 - 8) - 5 = 10 - 5$
 $= 5.$

(ii) 'Subtract 5 from 8 and then subtract the result from 18' can be written as
 $18 - (8 - 5) = 18 - 3$
 $= 15.$

Note that $(18 - 8) - 5 \neq 18 - (8 - 5)$

If there are no brackets, the operations are usually done in the order in which they are read.

Evaluate the following pairs:

- | | |
|----------------------------------|-----------------------------------|
| (i) $5 + 3 + 9; 5 + (3 + 9)$ | (ii) $5 - 3 + 9; 5 - (3 + 9)$ |
| (iii) $5 - 3 - 9; 5 - (3 - 9)$ | (iv) $5 + 3 - 9; 5 + (3 - 9)$ |
| (v) $-5 + 3 + 9; -5 + (3 + 9)$ | (vi) $-5 - 3 + 9; -5 - (3 + 9)$ |
| (vii) $-5 - 3 - 9; -5 - (3 - 9)$ | (viii) $-5 + 3 - 9; -5 + (3 - 9)$ |

Compare your results for each pair. What do you notice?

In all the cases, where there is a positive sign in front of the brackets, the answers for each pair are the same and where there is a negative sign in front of the brackets, the answers for each pair are different.

Copy and complete the following by inserting the sign which will make the two sides equal. Part (i) is done for you.

- | | |
|--|---|
| (i) $5 - (3 + 9) = 5 - 3 - 9$ | (ii) $5 - (3 - 9) = 5 \quad 3 \quad 9$ |
| (iii) $-5 - (3 + 9) = 5 \quad 3 \quad 9$ | (iv) $-5 - (3 - 9) = 5 \quad 3 \quad 9$ |
| (v) $x - (a + b) = x \quad a \quad b$ | (vi) $x - (a - b) = x \quad a \quad b$ |

Multiplication and Division

Evaluate and compare the results for each of the following pairs:

- (i) $4 \times (3 \times 2); (4 \times 3) \times 2$
 (ii) $3 \times (6 \div 2); (3 \times 6) \div 2$
 (iii) $16 \div (4 \times 2); (16 \div 4) \times 2$
 (iv) $36 \div (6 \div 3); (36 \div 6) \div 3$

What do you notice?

Different positions of the brackets may lead to different results. In all the cases, where there is a multiplication sign in front of the bracket, the results for each pair are the same and when the division sign is in front of the brackets, the results for each pair are different.

At times, more than two operations may occur in one expression, e.g., $6 \times 3 - 4 \div 2 + 5$. In such a case, we begin by brackets, then division, followed by multiplication, addition and finally subtraction, in that order. This can be shown by the use of the brackets, as below:

$$(6 \times 3) - (4 \div 2) + 5 = 18 - 2 + 5 \\ = 21$$

Exercise 6.7

Evaluate each of the following:

1. (a) $(18 - 24) + 30$ (b) $32 + (17 + 30)$
 (c) $13 - (18 + 7)$ (d) $(62 - 94) + 20$
 (e) $84 - (100 + 2)$ (f) $(74 - 24) + 30$
 (g) $(77 + 54) - 110$ (h) $(100 - 150) + 180$
 (i) $222 - (158 + 90)$ (j) $1\ 120 - (1\ 450 + 120)$
2. (a) $72 - 30 + 25$ (b) $86 - 109 + 4$
 (c) $209 + 43 - 300$ (d) $348 + 60 - 510$
 (e) $890 - 100 + 23$ (f) $989 + 100 - 1\ 470$
 (g) $763 - 26 + 471$ (h) $1\ 190 + 340 + 670$
 (i) $666 - 892 + 238$ (j) $3\ 004 - 563 + 1\ 044$
3. (a) $2 \times (10 \div 5)$ (b) $(6 \times 18) \div 9$
 (c) $90 \div (10 \times 3)$ (d) $-84 \div (7 \times 4)$
 (e) $(-39 \div 13) - 8$ (f) $21 \times (14 \div 7)$
 (g) $1\ 320 \div (11 \times 5)$ (h) $(-420 \div 28) \times 5$
 (i) $20 \times (525 \div 21)$ (j) $(1\ 125 \div 15) \times 19$
 (k) $11 \times 12 \div 4$ (l) $19 \times 8 \div 2$
 (m) $64 \div 16 \times 9$ (n) $-256 \div 64 \times 10$
 (p) $3 \times 68 \div 17$ (q) $91 \div 13 \times 5$
 (r) $-11 \times 125 \div 5$ (s) $-369 \div 123 \times 8$
 (t) $235 \times 10 \div 5$ (u) $-1\ 156 \div 34 \times 7$
4. (a) $12 + 6 \div 2 - 34$ (b) $24 \div 3 + 4 \times 5 - 8 \div 4 \times 10 + 1$
 (c) $-6 \times 9 + 7 - 12 \div 3 - 5$ (d) $42 \div 2 - 8 \times 2 + 9$
 (e) $56 \div 14 \times 6 - 3 - 9 + 3$ (f) $12 \div 3 + 5 \times 6$
 (g) $4 \times 5 - 18 \div 3$ (h) $-16 \times 9 + 56 \div 7$
 (i) $-15 \div 3 - 72 \times 4$ (j) $26 \div 2 + 3 \times 7 - 4 \times 5$
 (k) $5 \times 6 - 76 \div 4 + 27 \div 3$ (i) $-7 \times 41 + 36 \div 9 + 12 \times 12$
 (m) $4 \times 4 - 2 \times 4 + 27 \div 3$ (n) $96 \div 6 + 7 \times 15 - 14 \times 5$
 (p) $121 \times 55 \div 11 - 55$

5. If $x = -2$, $y = -6$ and $z = 4$, find the values of each of the following:
- (a) $4z + 2y - x$ (b) $2y - 3x + z$
- (c) $\frac{4xy}{z}$ (d) $\frac{3yz}{x}$
6. On a certain day, a student measured the temperature inside a deep freezer and found that it was -3°C while the room temperature was 24°C . What was the temperature difference between the room and the deep freezer?
7. Rhoda walked four floors down from the tenth floor and then took a lift to the eighteenth floor. How many floors did she go through while in the lift?
8. Kericho is a town on Kisumu – Nakuru road. The distance between Kisumu and Kericho is 85 km, while that between Kericho and Nakuru is 105 km. What is the distance between Nakuru and Kisumu?
9. A man was born in 1966. His father was born in 1928 and the mother three years later. If the man's daughter was born in 1992 and the son 5 years earlier, find the difference between the age of the man's mother and that of his son.
10. The temperature of a patient admitted to a hospital with fever was 42°C . After treatment, his temperature settled at 36.8°C . Find the change in temperature.

FRACTIONS

7.1: Introduction

If an orange is cut into four equal parts, we call each part one fourth (one quarter) and write it as $\frac{1}{4}$. A fraction is written in the form $\frac{a}{b}$ where a and b are numbers and $b \neq 0$. The upper number is called the **numerator** and the one below the **denominator**.

If the numerator is smaller than the denominator, the fraction is called a **proper fraction**, e.g. $\frac{2}{3}$ and $\frac{4}{7}$.

If the numerator is bigger than or equal to the denominator, the fraction is called an **improper fraction**, e.g., $\frac{5}{3}$, $\frac{15}{4}$ and $\frac{8}{7}$.

An improper fraction can be written as the sum of an integer and a proper fraction. For example, $\frac{5}{3} = 1 + \frac{2}{3}$
 $= 1\frac{2}{3}$

When written in this form, the fraction is called a **mixed number**.

Example 1

Convert $5\frac{3}{7}$ into an improper fraction.

Solution

$$\begin{aligned} 5\frac{3}{7} &= \frac{(7 \times 5) + 3}{7} \\ &= \frac{35 + 3}{7} \\ &= \frac{38}{7} \end{aligned}$$

Exercise 7.1

1. Write each of the following in numerals:

- | | |
|------------------|----------------------|
| (a) Two-thirds | (b) One-seventh |
| (c) Three-fifths | (d) Seven-tenths |
| (e) Six-sevenths | (f) Three-hundredths |

2. Write each of the following fractions in words:

- | | | |
|---------------------|---------------------|----------------------|
| (a) $\frac{3}{4}$ | (b) $\frac{5}{17}$ | (c) $\frac{6}{23}$ |
| (d) $\frac{11}{30}$ | (e) $\frac{23}{50}$ | (f) $\frac{37}{124}$ |

3. Express the first quantity as a fraction of the second:

- (a) 22 m, 53 m (b) 60 cm^2 , 90 cm^2
 (c) 120 m^2 , 80 m^2 (d) 4 tonnes, 250 kg
 (e) 379 g, 1 kg (f) 1 m 10 cm, 23 cm

4. Express each of the following as a mixed number:

- (a) $\frac{8}{3}$ (b) $\frac{15}{7}$ (c) $\frac{38}{9}$ (d) $\frac{105}{4}$
 (e) $\frac{340}{13}$ (f) $\frac{361}{15}$ (g) $\frac{422}{11}$ (h) $\frac{523}{12}$

5. Express each of the following mixed numbers as an improper fraction:

- (a) $2\frac{6}{7}$ (b) $1\frac{11}{12}$ (c) $9\frac{1}{10}$
 (d) $5\frac{1}{2}$ (e) $6\frac{3}{4}$ (f) $7\frac{2}{13}$

7.2: Comparing Fractions

In order to compare fractions, it is important to convert them into their equivalent forms using the same denominator.

Equivalent Fractions

Figure 7.1 shows four equal rectangles which have been divided into 2, 4, 6 and 8 equal parts. The shaded parts are all equal:

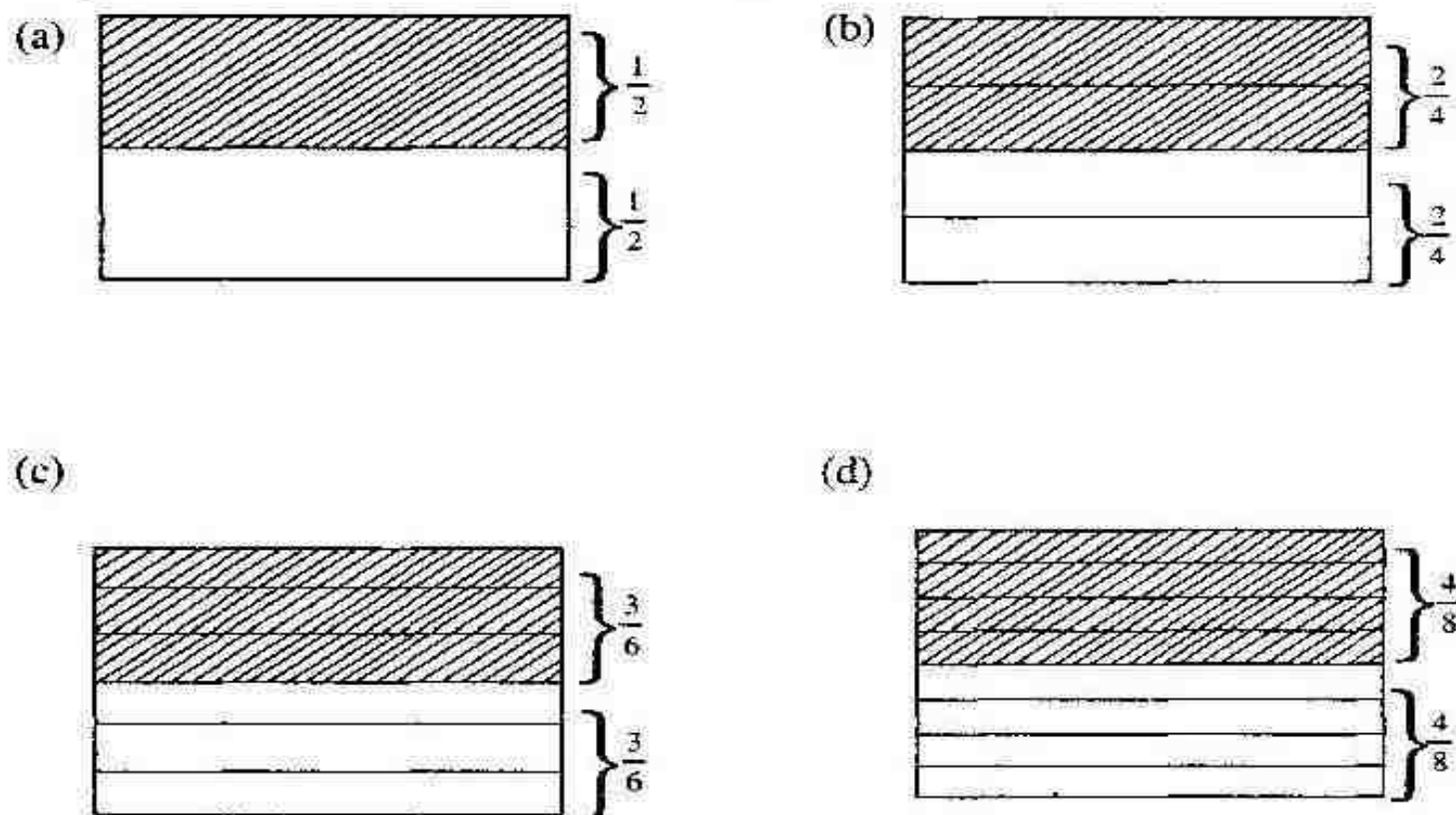


Fig. 7.1

In each rectangle, the same area is shaded. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are the fractions that represent the equal areas. Thus, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$. The fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ are called **equivalent fractions**. Write down two more fractions which are equivalent to $\frac{1}{2}$.

To get equivalent fractions, we multiply or divide the numerator and denominator of a given fraction by the same number. When the numerator and the denominator have no factor in common other than 1, the fraction is said to be in its simplest form.

Equivalent fractions may also be used to compare the sizes of fractions.

Example 2

Which of the fractions $\frac{2}{3}$ and $\frac{3}{4}$ is greater?

Solution

Using equivalent fractions, $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$

Clearly, $\frac{9}{12}$ is greater than $\frac{8}{12}$.

Therefore, $\frac{3}{4}$ is greater than $\frac{2}{3}$.

Note:

12 is the least common multiple (L.C.M.) of the denominators 3 and 4.

Example 3

Arrange the following in ascending order: $\frac{5}{12}$, $\frac{7}{3}$, $\frac{11}{5}$, $\frac{9}{4}$

Solution

$\frac{5}{12}$, $\frac{7}{3}$, $\frac{11}{5}$ and $\frac{9}{4}$ can, respectively, be written as $\frac{25}{60}$, $\frac{140}{60}$, $\frac{132}{60}$ and $\frac{135}{60}$.

$$\frac{25}{60} < \frac{132}{60} < \frac{135}{60} < \frac{140}{60}$$

$\therefore$ The fractions in ascending order: $\frac{5}{12}$, $\frac{11}{5}$, $\frac{9}{4}$, $\frac{7}{3}$

Example 4

Arrange the following in descending order: $\frac{2}{5}$, $\frac{3}{10}$, $\frac{6}{5}$, $\frac{5}{11}$

Solution

$$\frac{2}{5} = \frac{44}{110}, \quad \frac{3}{10} = \frac{33}{110}, \quad \frac{6}{5} = \frac{132}{110} \text{ and } \frac{5}{11} = \frac{50}{110}$$

$$\frac{132}{110} > \frac{50}{110} > \frac{44}{110} > \frac{33}{110}$$

∴ The fractions in descending order: $\frac{6}{5}$, $\frac{5}{11}$, $\frac{2}{5}$, $\frac{3}{10}$

Exercise 7.2

1. Copy and insert the missing numbers:

(a) $\frac{2}{3} = \frac{\quad}{12}$

(b) $\frac{\quad}{7} = \frac{28}{49}$

(c) $\frac{1}{16} = \frac{2}{\quad}$

(d) $\frac{3}{\quad} = \frac{6}{28}$

(e) $\frac{144}{180} = \frac{4}{\quad}$

(f) $\frac{35}{\quad} = \frac{7}{9}$

(g) $\frac{1}{7} = \frac{4}{\quad}$

(h) $\frac{40}{\quad} = \frac{\quad}{15} = \frac{4}{3}$

(i) $\frac{\quad}{15} = \frac{\quad}{10} = \frac{7}{5}$

(j) $\frac{18}{90} = \frac{\quad}{30} = \frac{3}{\quad} = \frac{\quad}{5}$

(k) $\frac{12}{\quad} = \frac{6}{\quad} = \frac{\quad}{6} = \frac{1}{2}$

(l) $\frac{104}{\quad} = \frac{\quad}{16} = \frac{26}{\quad} = \frac{13}{4}$

2. Express each of the following fractions in its lowest terms:

(a) $\frac{24}{36}$

(b) $\frac{25}{78}$

(c) $\frac{51}{78}$

(d) $\frac{125}{625}$

(e) $\frac{9576}{64}$

(f) $\frac{289}{51}$

(g) $\frac{2166}{396}$

(h) $\frac{6}{30}$

(i) $\frac{28}{21}$

(j) $\frac{9}{12}$

(k) $\frac{126}{18}$

(l) $\frac{169}{130}$

3. Find the value of the unknown:

(a) $\frac{3}{5} = \frac{x}{15}$

(b) $\frac{16}{y} = \frac{4}{7}$

(c) $\frac{3}{11} = \frac{x}{55}$

(d) $\frac{2x}{5} = \frac{18}{15}$

(e) $\frac{9}{21} = \frac{18}{28}$

(f) $\frac{3p}{5} = \frac{36}{15p}$

4. Which fraction is greater than the other in each of the following pairs?

(a) $\frac{1}{2}$, $\frac{1}{3}$

(b) $\frac{1}{5}$, $\frac{1}{6}$

(c) $\frac{4}{5}$, $\frac{5}{6}$

(d) $\frac{3}{4}$, $\frac{7}{8}$

(e) $\frac{8}{5}$, $\frac{24}{15}$

(f) $\frac{2}{3}$, $\frac{9}{12}$

(g) $\frac{5}{6}$, $\frac{3}{4}$

(h) $\frac{4}{9}$, $\frac{3}{4}$

(i) $\frac{2}{3}$, $\frac{4}{7}$

(j) $\frac{10}{18}$, $\frac{4}{6}$

(k) $\frac{24}{16}$, $\frac{30}{20}$

(l) $\frac{50}{40}$, $\frac{80}{60}$

5. Arrange the following fractions in ascending order:

(a) $\frac{2}{9}$, $\frac{1}{6}$, $\frac{5}{12}$

(b) $\frac{7}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$

(c) $\frac{5}{9}, \frac{3}{4}, \frac{7}{12}, \frac{1}{6}$

(d) $\frac{5}{12}, \frac{7}{8}, \frac{5}{6}, \frac{3}{4}$

(e) $\frac{2}{5}, \frac{4}{15}, \frac{1}{6}, \frac{1}{2}, \frac{2}{3}$

(f) $\frac{1}{21}, \frac{1}{10}, \frac{1}{15}$

(g) $\frac{11}{14}, \frac{7}{10}, \frac{5}{6}, \frac{11}{15}, \frac{19}{21}$

(h) $\frac{5}{6}, \frac{3}{4}, \frac{11}{12}, \frac{2}{3}$

6. Arrange the following fractions in descending order.

(a) $\frac{1}{2}, \frac{5}{7}, \frac{3}{4}$

(b) $\frac{7}{9}, \frac{11}{3}, \frac{4}{5}, \frac{2}{3}$

(c) $\frac{13}{15}, \frac{2}{3}, \frac{7}{8}, \frac{4}{5}$

(d) $\frac{3}{2}, \frac{7}{5}, \frac{9}{10}, \frac{11}{15}, \frac{3}{7}$

(e) $\frac{1}{2}, \frac{3}{5}, \frac{4}{7}, \frac{8}{9}, \frac{2}{3}$

(f) $\frac{4}{5}, \frac{9}{10}, \frac{11}{16}, \frac{12}{17}$

(g) $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{8}{9}$

(h) $\frac{4}{9}, \frac{7}{18}, \frac{1}{2}, \frac{2}{5}$

7.3: Operations on Fractions

Addition and Subtraction

The numerators of fractions whose denominators are equal can be added or subtracted directly.

Example 5

Evaluate: (a) $\frac{1}{5} + \frac{2}{5}$

(b) $\frac{5}{7} - \frac{3}{7}$

(c) $\frac{2}{9} - \frac{7}{9}$

Solution

(a) $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$

(b) $\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$

(c) $\frac{2}{9} - \frac{7}{9} = \frac{2-7}{9} = \frac{-5}{9}$

In case of different denominators, equivalent fractions with the same denominators may be used. Usually, this common denominator is the L.C.M.

Example 6

Evaluate: (a) $\frac{3}{4} + \frac{1}{7}$

(b) $\frac{5}{8} - \frac{3}{10}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{3}{4} + \frac{1}{7} &= \frac{21}{28} + \frac{4}{28} \\ &= \frac{21+4}{28} \end{aligned}$$

$$= \frac{13}{40}$$

Mixed numbers can be added or subtracted easily by first expressing them as improper fractions.

Example 7

Work out: (a) $5\frac{2}{3} + 1\frac{4}{5}$ (b) $3\frac{3}{5} + 5\frac{1}{2}$

Solution

$$(a) \quad 5\frac{2}{3} = 5 + \frac{2}{3}, \text{ which is } \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$$

$$1\frac{4}{5} = 1 + \frac{4}{5} \text{ which is } \frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

$$\begin{aligned} \therefore 5\frac{2}{3} + 1\frac{4}{5} &= \frac{17}{3} + \frac{9}{5} \\ &= \frac{85}{15} + \frac{27}{15} \\ &= \frac{112}{15} \\ &= 7\frac{7}{15} \end{aligned}$$

Alternatively,

$$\begin{aligned} 5\frac{2}{3} + 1\frac{4}{5} &= 5 + \frac{2}{3} + 1 + \frac{4}{5} = (5 + 1) + \frac{2}{3} + \frac{4}{5} \\ &= 6 + \frac{10+12}{15} \\ &= 6 + \frac{22}{15} \\ &= 6 + 1\frac{7}{15} \\ &= 7\frac{7}{15} \end{aligned}$$

$$(b) \quad 3\frac{3}{5} = 3 + \frac{3}{5}$$

$$= \frac{15+3}{5}$$

$$= \frac{18}{5}$$

$$5\frac{1}{2} = 5 + \frac{1}{2}$$

$$= \frac{10+1}{2}$$

$$= \frac{11}{2}$$

$$\therefore 3\frac{3}{5} - 5\frac{1}{2} = \frac{18}{5} - \frac{11}{2}$$

$$\begin{aligned}
 &= \frac{36-55}{10} \\
 &= \frac{-19}{10} \\
 &= -1\frac{9}{10}
 \end{aligned}$$

Alternatively,

$$\begin{aligned}
 3\frac{3}{5} - 5\frac{1}{2} &= 3 + \frac{3}{5} - \left(5 + \frac{1}{2}\right) \\
 &= 3 - 5 + \frac{3}{5} - \frac{1}{2} \\
 &= -2 + \frac{6-5}{10} \\
 &= -2 + \frac{1}{10} \\
 &= \frac{-20+1}{10} \\
 &= \frac{-19}{10} \\
 &= -1\frac{9}{10}
 \end{aligned}$$

Example 8

Evaluate: $\frac{2}{3} + \frac{-1}{8}$

Solution

$$\begin{aligned}
 \frac{2}{3} + \frac{-1}{8} &= \frac{-16+3}{24} \\
 &= \frac{-13}{24}
 \end{aligned}$$

Example 9

Express $5\frac{2}{3}$ and $1\frac{4}{5}$ as improper fractions.

Solution

$$5\frac{2}{3} = 5 + \frac{2}{3} \text{ which is } \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$$

$$1\frac{4}{5} = 1 + \frac{4}{5} \text{ which is } \frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Exercise 7.3

1. Convert each of the following into an improper fraction:

(a) $3\frac{1}{9}$

(b) $4\frac{5}{14}$

(c) $9\frac{6}{13}$

(d) $1\frac{9}{17}$

(e) $2\frac{4}{9}$

(f) $5\frac{7}{10}$

(g) $11\frac{3}{8}$

(h) $14\frac{5}{6}$

(i) $20\frac{2}{3}$

2. Evaluate each of the following:

(a) $\frac{1}{2} + \frac{1}{9}$

(b) $\frac{5}{6} + \frac{4}{11}$

(c) $\frac{4}{9} + \frac{3}{17}$

(d) $\frac{5}{6} + \frac{3}{10}$

(e) $\frac{2}{13} - \frac{1}{26}$

(f) $\frac{9}{4} + \frac{4}{9}$

(g) $\frac{19}{5} - \frac{32}{4}$

(h) $2\frac{1}{2} - 1\frac{1}{8}$

(i) $5\frac{4}{8} + 7\frac{1}{8}$

(j) $4\frac{15}{32} - \frac{21}{8}$

(k) $-2\frac{6}{13} + 1\frac{1}{2}$

(l) $3\frac{7}{8} - 4\frac{2}{3}$

3. Evaluate each of the following:

(a) $8\frac{1}{9} - 2\frac{3}{4} + \frac{9}{4}$

(b) $\frac{2}{9} + \frac{1}{7} - \frac{3}{4}$

(c) $1\frac{4}{5} - 3\frac{1}{2} + 5\frac{3}{8}$

(d) $2\frac{7}{12} + 1\frac{2}{3} - 9\frac{3}{5}$

(e) $\frac{12}{5} - 1\frac{3}{8} + 1\frac{1}{9}$

(f) $7\frac{2}{3} + 6\frac{3}{5} + 11\frac{5}{6}$

(g) $10\frac{1}{2} - 5\frac{1}{3} + \frac{1}{27}$

(h) $3\frac{1}{6} - 2\frac{1}{3} + \frac{7}{12}$

4. Evaluate:

(a) $-\frac{1}{4} - (-\frac{1}{2})$

(b) $-\frac{2}{3} + (-\frac{1}{8})$

(c) $(+\frac{1}{17}) + (-\frac{3}{5}) - (+\frac{1}{3})$

(d) $-\frac{2}{3} - (+\frac{1}{10}) + (+\frac{1}{7})$

Multiplication

Multiplication of a fraction by a whole number is a repeated addition of the fraction. For example,

$$\begin{aligned}
 5 \times \frac{1}{7} &= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} \\
 &= \frac{5 \times 1}{7} \\
 &= \frac{5}{7}
 \end{aligned}$$

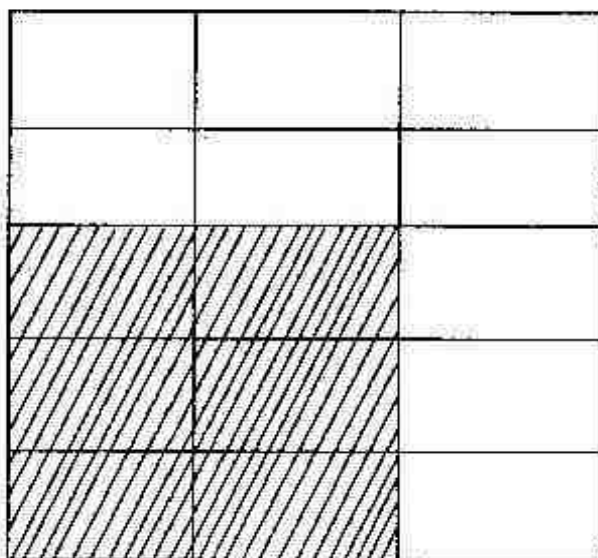


Fig. 7.2

Figure 7.2 shows a square of side 1 unit which has been divided into 15 equal regions.

The shaded part represents $\frac{8}{15}$ of the square. The area of the shaded part is

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$$

To multiply two fractions, we multiply the numerators to get the numerator of the product and multiply the denominators to get the denominator of the product.

Example 10

Evaluate: (a) $\frac{3}{8} \times \frac{2}{3}$ (b) $2\frac{1}{4} \times 4\frac{3}{5}$

Solution

$$\begin{aligned}
 \text{(a)} \quad \frac{3}{8} \times \frac{2}{3} &= \frac{3 \times 2}{8 \times 3} \\
 &= \frac{6}{24} \\
 &= \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 2\frac{1}{4} \times 4\frac{3}{5} &= \frac{9}{4} \times \frac{23}{5} \\
 &= \frac{207}{20} \\
 &= 10\frac{7}{20}
 \end{aligned}$$

Example 11

Evaluate: $4\frac{1}{5} \times 1\frac{1}{14}$

Solution

$$\begin{aligned} 4\frac{1}{5} \times 1\frac{1}{14} &= \frac{21}{5} \times \frac{15}{14} \\ &= \frac{3 \times 3}{1 \times 2} \\ &= \frac{9}{2} \\ &= 4\frac{1}{2} \end{aligned}$$

This method shows that when multiplying two fractions, it may be easier to first divide both the numerator and the denominator by common factors. This process is called **cancellation**.

What is $\frac{1}{2}$ of $\frac{3}{4}$?

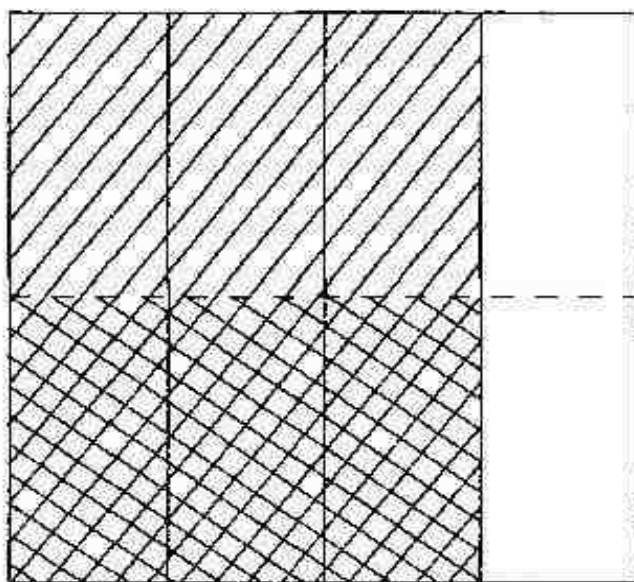
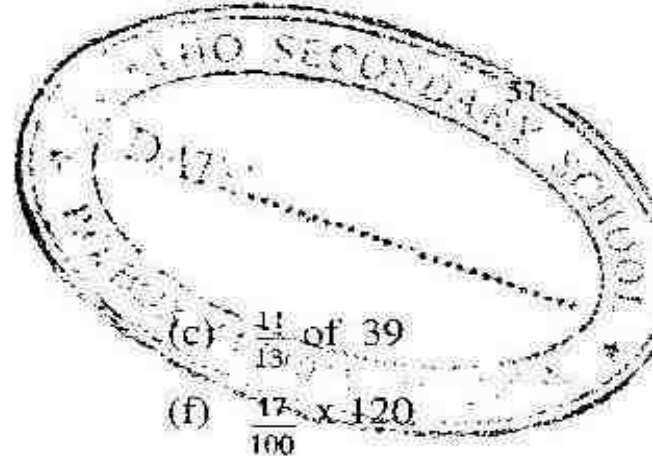


Fig. 7.3

Figure 7.3 shows a square which has been divided vertically into four equal regions. The shaded region is $\frac{3}{4}$ of the square. Half of this shaded region is shown by the dotted line and it is $\frac{3}{8}$ of the square. But, $\frac{1}{2} \times \frac{3}{4}$ is $\frac{3}{8}$. This example illustrates that if 'of' is used in fractions, it can be replaced by the multiplication sign.

**Exercise 7.4**

1. Evaluate each of the following:

(a) $\frac{3}{8} \times 48$

(b) $\frac{1}{6}$ of 72

(d) $42 \times \frac{1}{4}$

(e) $\frac{4}{11}$ of 121

(c) $\frac{11}{13}$ of 39

(f) $\frac{17}{100} \times 120$

2. Evaluate each of the following:

(a) $\frac{2}{5} \times \frac{6}{7}$

(b) $\frac{1}{3} \times \frac{1}{2} \times \frac{7}{8}$

(c) $\frac{3}{5} \times \frac{7}{11} \times \frac{22}{25}$

(d) $\frac{1}{6}$ of $\frac{3}{5} \times (-\frac{1}{2})$

(e) $\frac{4}{9} \times \frac{4}{10}$ of $\frac{3}{8}$

(f) $\frac{5}{7}$ of $(-\frac{6}{10}) \times \frac{2}{3}$

3. Simplify:

(a) $2\frac{1}{3} \times 7\frac{1}{4}$

(b) $-4\frac{3}{7} \times 12\frac{1}{2}$

(c) $3\frac{4}{7} \times 4\frac{2}{3} \times 5\frac{1}{2}$

(d) $-6\frac{4}{7} \times 7\frac{2}{5} \times (-\frac{7}{11})$

(e) $\frac{11}{4}$ of $5\frac{1}{2}$

(f) $\frac{10}{17}$ of $6\frac{4}{5}$

4. Simplify:

(a) $1\frac{1}{4}$ of $5\frac{1}{3} \times \frac{9}{10}$

(b) $3\frac{1}{2} \times 2\frac{1}{4}$ of $2\frac{2}{7}$

(c) $\frac{7}{3} \times 4\frac{1}{2}$ of $\frac{2}{9} \times \frac{3}{4}$

(d) $6\frac{2}{3} \times \frac{7}{20} \times 4\frac{2}{3}$ of $2\frac{6}{7}$

(e) $\frac{26}{5} \times \frac{3}{52}$ of $\frac{7}{8} \times 1\frac{5}{7}$

(f) $7\frac{1}{4} \times \frac{1}{8}$ of $\frac{8}{29} \times 1\frac{1}{3}$

5. A certain machine uses $1\frac{1}{2}$ litres of diesel in one hour. How much diesel does it use in $5\frac{1}{4}$ hours?
6. A car travels at $75\frac{1}{3}$ km/h. How far does it travel in $2\frac{2}{5}$ hours?
7. A tailor needs $1\frac{2}{3}$ m of materials to make a skirt. How much material will she need to make 15 skirts?
8. What is the cost of $8\frac{1}{2}$ kg of meat if one kilogram costs sh. 40?
9. A farmer plants cotton on $\frac{5}{8}$ of his $21\frac{1}{5}$ hectares piece of land. How many hectares are planted with cotton?
10. It takes $1\frac{1}{2}$ days to make a toy train. How many such toys can be made in two weeks?
11. A dressmaker buys 45 rolls of dress material, each $3\frac{1}{2}$ m long. How much does he pay if the price is sh. 220 per metre?

12. A cyclist delivered 3 cartons weighing $3\frac{1}{2}$ kg each, 8 parcels weighing $2\frac{1}{4}$ kg each and 125 sachets weighing $\frac{1}{4}$ kg each to a shop. What was the total load?
13. A bag of onions weighs $44\frac{1}{2}$ kg. What is the weight of $3\frac{2}{3}$ bags of the same type?
14. Two business partners received $\frac{3}{7}$ and $\frac{2}{7}$ of the business proceeds after a year. The businessman who received the larger share was required to spend $\frac{1}{8}$ of his share to pay all workers. If the business realised sh. 180 000, how much did the workers receive?

Division

If the product of two numbers is equal to 1, then we call them **reciprocals** of each other. For example $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals of each other because $\frac{2}{3} \times \frac{3}{2} = 1$. In the same way, the reciprocal of 5 is $\frac{1}{5}$ and that of $\frac{1}{5}$ is 5.

We have also seen that division by integers is the reverse of multiplication. For example $18 \div 3 = 6$, so that $6 \times 3 = 18$. Similarly, division by fractions is the reverse of multiplication. For example,

- (i) $3 \div \frac{1}{2} = n$ can be written as

$$3 = n \times \frac{1}{2}$$

$$\therefore n = 6$$

$$\text{But } 3 \times \frac{2}{1} = 6$$

$$\text{Thus, } 3 \div \frac{1}{2} = 3 \times \frac{2}{1}$$

- (ii) $\frac{1}{2} \div \frac{1}{4} = n$ can be written as;

$$n \times \frac{1}{4} = \frac{1}{2}$$

$$\therefore n = 2$$

$$\text{But, } \frac{1}{2} \times 4 = 2$$

$$\text{Thus, } \frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2$$

These examples show that dividing by a fraction gives the same results as multiplying by the reciprocal of the divisor. We can arrive at the same result in the following way; $\frac{3}{4} \div \frac{5}{7}$ can be written as $\frac{3}{4} \times \frac{7}{5}$. Recalling that a fraction is not

altered if the numerator and the denominator are multiplied by the same number,

$$\begin{aligned}\text{we have; } \frac{3}{4} \div \frac{5}{7} &= \frac{\frac{3}{4} \times \frac{7}{5}}{\frac{5}{7} \times \frac{7}{5}} \\ &= \frac{3}{4} \times \frac{7}{5} \\ &= \frac{21}{20}\end{aligned}$$

This again shows that $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$

Exercise 7.5

1. Find the reciprocal of:

(a) $6\frac{3}{8}$

(b) 10

(c) $\frac{4}{5}$

(d) $\frac{13}{6}$

(e) $\frac{7}{11}$

(f) $9\frac{5}{6}$

(g) $\frac{m}{n}$

(h) $\frac{2k}{r}$

2. Evaluate:

(a) $5 \div \frac{3}{7}$

(b) $6 \div \frac{2}{3}$

(c) $\frac{2}{11} \div 4$

(d) $\frac{3}{4} \div 12$

(e) $4\frac{1}{5} \div 7$

(f) $8 \div 2\frac{2}{3}$

(g) $\frac{18}{5} \div 9$

(h) $12 \div \frac{15}{13}$

(i) $9\frac{1}{3} \div \frac{1}{7}$

(j) $11\frac{3}{5} \div 29$

(k) $12\frac{4}{5} \div 16$

(l) $25 \div 4\frac{1}{3}$

3. Work out:

(a) $\frac{4}{5} \div \frac{2}{3}$

(b) $3 \div \frac{6}{5}$

(c) $1\frac{5}{7} \div \frac{12}{7}$

(d) $\frac{18}{11} \div 1\frac{4}{5}$

(e) $5\frac{3}{4} \div 6\frac{11}{23}$

(f) $2\frac{6}{11} \div \frac{3}{11}$

(g) $\frac{13}{14} \div 5\frac{1}{5}$

(h) $7\frac{3}{7} \div 3\frac{1}{4}$

7

(i) $3\frac{9}{17} \div 5\frac{1}{5}$

 $37\frac{1}{3} \div 3\frac{1}{4}$

(j) $\frac{\frac{1}{2} \div \frac{1}{4}}{\frac{1}{8} \div \frac{7}{16}}$

4. A pile of books is $\frac{3}{4}$ metres high and each book is $\frac{3}{4}$ cm thick. How many books are in the pile?

5. Two thirds of a loaf of bread is shared equally among four children. What fraction of the loaf does each get?

6. The product of two numbers is $\frac{2}{7}$. If one of them is $\frac{8}{21}$, find the other.
7. A man's stride is $\frac{7}{8}$ m long. How many strides does he take in walking a distance of 98 m?
8. How many pieces of paper, each $3\frac{1}{7}$ m long, can be cut out from a piece of paper $34\frac{4}{7}$ m long?
9. A car consumes $8\frac{5}{8}$ litres of fuel to cover $51\frac{3}{4}$ km. What average distance does it travel for every litre?

7.4: Order of Operations on Fractions

The same rules regarding the order in which operations are performed on integers apply to fractions. However, in dealing with fractions, a further operation 'of' may appear.

In case where there are no brackets, the operation 'of' is performed before all other operations. If there are brackets, the operation inside the brackets should always be performed first. For example,

$$\begin{aligned} 15 \div \frac{1}{4} \text{ of } 12 &= 15 \div \left(\frac{1}{4} \times 12 \right) \\ &= 15 \div 3 \\ &= 5 \end{aligned}$$

Notice that this is different from $15 \div \frac{1}{4} \times 12$, which is equal to 720.

In cases where we have brackets inside other brackets, the operations enclosed by the innermost brackets are performed first. For example,

$$\begin{aligned} \frac{1}{6} + \frac{1}{2} \times \left\{ \frac{3}{8} + \left(\frac{1}{3} - \frac{1}{4} \right) \right\} &= \frac{1}{6} + \frac{1}{2} \times \left\{ \frac{3}{8} + \frac{1}{12} \right\} \\ &= \frac{1}{6} + \frac{1}{2} \times \frac{11}{24} \\ &= \frac{1}{6} + \frac{11}{48} \\ &= \frac{19}{48} \end{aligned}$$

Example 12

Evaluate: $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6} \right)} + \frac{1}{2}$

Solution

We first work out: $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6}\right)}$

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\begin{aligned} \frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6}\right) &= \frac{1}{7} \times \frac{7}{30} \\ &= \frac{1}{30} \end{aligned}$$

$$\begin{aligned} \text{Therefore, } \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6}\right)} &= \frac{\frac{5}{6}}{\frac{1}{30}} \\ &= \frac{5}{6} \times 30 \\ &= 25 \end{aligned}$$

$$\begin{aligned} \text{Thus, } \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6}\right)} + \frac{1}{2} &= 25 \frac{1}{2} \\ &= 25 \frac{1}{2} \end{aligned}$$

Operations on fractions should therefore be performed in the following order:

- (i) Perform the operations enclosed within the brackets first.
- (ii) If 'of' appears, perform that operation before any other.
- (iii) Divide and/or multiply in the order the operations are written.
- (iv) Add and/or subtract in the order the operations are written.

In the above expression, the bar (—) is a special type of bracket. It indicates that the operation above and/or below it must be performed first.

Example 13

Evaluate: $\frac{1}{2} \left\{ \frac{3}{5} + \frac{1}{4} \left(\frac{7}{3} - \frac{3}{7} \right) \text{ of } 1 \frac{1}{2} \div 5 \right\}$

Solution

$$\begin{aligned}
\frac{1}{2} \left\{ \frac{3}{5} + \frac{1}{4} \left(\frac{7}{3} - \frac{3}{7} \right) \text{ of } 1\frac{1}{2} \div 5 \right\} &= \frac{1}{2} \left\{ \frac{3}{5} + \frac{1}{4} \left(\frac{40}{21} \right) \text{ of } 1\frac{1}{2} \div 5 \right\} \\
&= \frac{1}{2} \left(\frac{3}{5} + \frac{1}{4} \times \frac{40}{21} \times \frac{3}{2} \div 5 \right) \\
&= \frac{1}{2} \left(\frac{3}{5} + \frac{10}{21} \times \frac{3}{2} \div 5 \right) \\
&= \frac{1}{2} \left(\frac{3}{5} + \frac{5}{7} \div 5 \right) \\
&= \frac{1}{2} \left(\frac{3}{5} + \frac{5}{35} \right) \\
&= \frac{1}{2} \left(\frac{21+5}{35} \right) \\
&= \frac{1}{2} \times \frac{26}{35} \\
&= \frac{13}{35}
\end{aligned}$$

Exercise 7.6

1. (a) $\frac{4}{9} \div \frac{8}{63} + 3\frac{1}{2}$

(c) $4\frac{1}{3} - \left(5\frac{1}{5} + \frac{4}{5} \text{ of } \frac{25}{3} \right)$

(e) $\frac{6}{11} \text{ of } \frac{55}{21} \div \frac{33}{7} + \frac{1}{3}$

(g) $3\frac{1}{4} \times 2\frac{2}{3}$

(i) $\frac{4\frac{1}{3} \div \frac{1}{6}}{1\frac{1}{4} + 3\frac{3}{4}}$

2. (a) $1\frac{3}{4} \div \frac{3}{5} \times 2\frac{2}{3}$

(c) $1\frac{2}{3} \times \frac{3}{4} + \frac{14-5}{12}$

(b) $\frac{1}{8} \text{ of } \frac{1}{3} \div \frac{1}{6} \times \frac{5}{12}$

(d) $\frac{7}{4} \div \left(\frac{8}{3} + 3\frac{7}{12} \right)$

(f) $6\frac{1}{4} \div \frac{8}{13} \times \frac{8}{13} \text{ of } \frac{26}{52}$

(h) $\frac{8}{11} + \frac{3}{22} \div \frac{5}{66}$

(j) $\frac{2\frac{3}{4} \div \frac{3}{4}}{\frac{1}{4} + 1\frac{2}{5}}$

(b) $1\frac{3}{4} \div \frac{3}{5} \text{ of } 2\frac{2}{3}$

(d) $\frac{7\frac{1}{5} \times \frac{2}{3}}{9\frac{3}{5}} + 1\frac{2}{5}$

(e) $1\frac{3}{4} \times \frac{5}{28} - 2\frac{6}{3}$

(f) $\left(2\frac{1}{5} - 2\frac{1}{4}\right) - \frac{3\frac{1}{2} - 2\frac{1}{4}}{\frac{5}{8}}$

(g) $\frac{45}{60} \div \left(5\frac{1}{12} - 4\frac{1}{5}\right)$

(h) $\left\{\left(5\frac{1}{2} - 2\frac{1}{4}\right) \div 6\frac{1}{2}\right\} \div \frac{2}{5}$

(i) $\left\{\left(1\frac{1}{4} - \frac{3}{8}\right) \div 3\frac{1}{2} + 1\frac{5}{6} \div 1\frac{1}{4}\right\}$

(j) $\frac{3}{8}$ of $\left\{7\frac{3}{5} - \frac{1}{3}\left(1\frac{1}{4} + 3\frac{1}{3}\right) \times 2\frac{2}{5}\right\}$

(k) $\frac{3}{5} \div \frac{2}{3} - \frac{1}{2} \times \frac{1}{13}$ of $\left(\frac{1}{2} + \frac{4}{5}\right)$

(l) $\left(\frac{5}{7} \times \frac{2}{3}\right) + \left(\frac{5}{6} - \frac{8}{9}\right) \div \frac{7}{15}$ of $\frac{5}{6}$

3. What must be subtracted from the product of $5\frac{1}{2}$ and $3\frac{1}{3}$ to get $18\frac{1}{2}$?
4. By how much is the product of $\frac{9}{5}$ and $8\frac{1}{4}$ greater than 5?
5. How many exercise books, each costing sh. 9.50, can be bought with sh. 100.00? How much money is left over?
6. Ochieng' had sh. 250 as pocket money at the beginning of the term. In the middle of the term, he was left with $\frac{2}{5}$ of this amount. How much did he spend?
7. The distance between two schools, A and B, is 2 km. A market is situated between A and B, one third of the distance from A to B. How far is this market from B?
8. After spending $\frac{5}{6}$ of his January salary on paying school fees, Bokole was left with sh. 1 500. How much money did he earn that month?
9. Kiplang'at's family uses three tenths of its daily milk supply in making breakfast while one seventh of the remainder is given to children. This leaves the family with 6 litres. How much is the daily supply?
10. An integer p is two thirds of another and their difference is 10. Find the two integers.
11. A classroom floor is made of small square tiles of side $\frac{1}{20}$ m. If the floor measures 6 m by 5 m, how many such square tiles are needed to cover the floor?
12. Three boys shared some money. The youngest got $\frac{1}{12}$ of it, the next got $\frac{1}{9}$ and the eldest got the remainder. What fraction of the money did the eldest receive? If the eldest boy got sh. 330, what was the original sum of the money?
13. Charles spent $\frac{1}{4}$ of his net January salary on school fees. He spent $\frac{1}{4}$ of the remainder on electricity and water bills. He then spent $\frac{1}{9}$ of what was left on transport. If he finally had sh. 3 400, what was his net January salary?

DECIMALS

8.1: Introduction

A fraction whose denominator can be written as a power of 10, e.g., $\frac{1}{10}$, $\frac{3}{100}$ and $\frac{50}{100}$ is called a **decimal fraction** or simply a **decimal**.

Decimal fractions are usually written in a special way, e.g., $\frac{3}{10}$ is written as 0.3 while $\frac{15}{100}$ is written as 0.15. The dot in this notation is called a **decimal point**. A number such as 356 can be written as $3 \times 100 + 5 \times 10 + 6 \times 1$, i.e.,

3 hundreds plus 5 tens plus 6 ones. Similarly, 0.247 means $2 \times \frac{1}{10} + 4 \times \frac{1}{100} + 7 \times \frac{1}{1000}$, i.e., 2 tenths plus 4 hundredths plus 7 thousandths. This decimal is read as **zero point two four seven**.

The chart below illustrates how decimals extend our system of notation to the right of the decimal point.

Ten thousands	Ten thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths
2	7	8	0	5	•	6	3	9	1	4

A decimal fraction such 8.3 means $8 + \frac{3}{10}$. Such a decimal fraction which represents the sum of a whole number and a proper fraction is called a **mixed decimal**.

8.2: Conversion of Fractions into Decimals

A fraction whose denominator can be changed to power of 10 can easily be expressed as a decimal.

Example 1

Express each of the following fractions as decimal:

(a) $\frac{3}{4}$ (b) $\frac{1}{8}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{3}{4} &= \frac{3 \times 25}{4 \times 25} \\ &= \frac{75}{100} \\ &= 0.75 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{1}{8} &= \frac{1 \times 125}{8 \times 125} \\ &= \frac{125}{1000} \\ &= 0.125 \end{aligned}$$

Alternatively, the conversions can be done directly by dividing the numerator by the denominator.

$$\begin{array}{rcl} \text{Thus, } \frac{3}{4} & = & 4 \overline{) 3.0} \quad \text{and} \quad \frac{1}{8} = 8 \overline{) 1.000} \\ & & \underline{-2.8} \qquad \qquad \underline{-.8} \\ & & .20 \qquad \qquad .20 \\ & & \underline{-.20} \qquad \qquad \underline{-.16} \\ & & 0 \qquad \qquad 40 \\ & & \underline{-.40} \\ & & 0 \end{array}$$

To convert a decimal into a fraction, place values to the right of the decimal point are used.

Example 2

Convert each of the following decimals into fractions:

(a) 0.75 (b) 0.314

Solution

$$\begin{aligned} \text{(a)} \quad 0.75 &= \frac{7}{10} + \frac{5}{100} \\ &= \frac{70}{100} + \frac{5}{100} \\ &= \frac{75}{100} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 0.314 &= \frac{3}{10} + \frac{1}{100} + \frac{4}{1000} \\ &= \frac{300}{1000} + \frac{10}{1000} + \frac{4}{1000} \\ &= \frac{314}{1000} \end{aligned}$$

Note:

- (i) The digits in the numerator are the same as those in the decimal, but without the decimal point.
- (ii) The number of zeros in the denominator are equal to the number of digits to the right of the decimal point.

Exercise 8.1

1. Express each of the following in decimal notation:

(a) $\frac{8}{10}$

(b) $\frac{24}{100}$

(c) $\frac{3}{10}$

(d) $\frac{27}{10}$

(e) $\frac{7}{100}$

(f) $\frac{21}{10000}$

(g) $\frac{102}{10000}$

(h) $\frac{273}{10000}$

(i) $\frac{1}{1000}$

(j) $\frac{15}{1000}$

(k) $\frac{6.7}{100}$

(l) $\frac{3.5}{100}$

(m) $\frac{5.4}{1000}$

(n) $\frac{10.11}{1000}$

(p) $\frac{3.7}{100}$

(q) $\frac{81}{1000}$

(r) $\frac{1.45}{1000}$

(s) $\frac{92}{1000}$

(t) $\frac{36}{1000}$

(u) $\frac{2.21}{100}$

2. Write each of the following as a single decimal:

(a) $\frac{1}{10} + \frac{2}{100}$

(b) $\frac{1}{100} + \frac{2}{100}$

(c) $1 + \frac{1}{100} + \frac{2}{1000} + \frac{3}{10000}$

(d) $26 + \frac{1}{100} + \frac{1}{10000}$

(e) $1 + \frac{1}{100} + \frac{1}{10000} + \frac{1}{1000000}$

3. Write each of the following in decimal notation:

(a) Four-tenths.

(b) Thirty-four hundredths.

(c) Two hundreds and thirty-four hundredths.

(d) Ninety-six thousandths.

(e) Six hundred and twenty-five ten thousandths.

(f) Seven hundred and thirty-three millionths.

(g) Two hundred and sixteen thousandths.

(h) Twenty-five ten thousandths.

(i) Seventy-eight tenths.

(j) Forty-five ten millionths.

4. Arrange the following decimal fractions in ascending order:

(a) 0.25, 0.75, 2.05

(b) 0.45, 0.045, 0.55

(c) 0.35, 0.25, 0.5, 0.05

5. Arrange the following decimals in descending order:
- (a) 0.53, 0.75, 0.56, 0.45
 (b) 0.024, 0.24, 0.0024, 2.4
 (c) 5.6, 5.68, 0.59, 0.591
6. Express each of the following as decimal fraction by first changing the denominator to a power of ten:
- (a) $\frac{7}{20}$ (b) $\frac{17}{50}$ (c) $\frac{15}{40}$
 (d) $\frac{33}{125}$ (e) $\frac{4}{5}$ (f) $\frac{73}{200}$
 (g) $\frac{23}{25}$ (h) $\frac{45}{80}$ (i) $\frac{59}{40}$
 (j) $\frac{345}{2000}$ (k) $\frac{50}{3000}$ (l) $\frac{45}{900}$
7. Use division to convert each of the following into a decimal:
- (a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{3}{8}$
 (d) $\frac{5}{16}$ (e) $\frac{7}{16}$ (f) $\frac{25}{15}$
 (g) $\frac{9}{20}$ (h) $\frac{45}{80}$ (i) $\frac{1.08}{9}$
 (j) $\frac{1.96}{14}$ (k) $\frac{6.25}{25}$ (l) $\frac{3.75}{75}$
8. Convert each of the following mixed numbers into a decimal:
- (a) $2\frac{1}{2}$ (b) $3\frac{4}{5}$ (c) $40\frac{3}{8}$ (d) $58\frac{7}{10}$
9. Express each of the following as a fraction in its simplest form:
- (a) 0.125 (b) 0.34 (c) 0.677
 (d) 49.75 (e) 6.675 (f) 0.15
 (g) 0.235 (h) 4.375 (i) 2.125
 (j) 3.825 (k) 4.753 (l) 8.450

8.3: Recurring Decimals

In division, we sometimes obtain a decimal fraction in which a digit or a group of digits repeat continuously without ending. Such a decimal fraction is called a **recurring decimal**. For example;

$$(i) \frac{1}{3} = 0.33333... \quad (ii) \frac{5}{11} = 0.454545... \quad (iii) \frac{12}{37} = 0.324324324...$$

In short, we place a dot above a digit that recurs. If more than one digit recurs in a pattern, we place a dot above the first and the last digit in the pattern, e.g.,

0.3333..... is written as $0.\dot{3}$

0.4545 is written as $0.4\dot{5}$ and

0.324324... is written as $0.\dot{3}2\dot{4}$.

Any division whose divisor has prime factors other than 2 or 5 forms a recurring decimal or non-terminating decimal, otherwise the division is terminating. The following examples show how to convert a recurring decimal to a fraction.

Example 3

Express each as a fraction: (a) $0.\dot{6}$ (b) $0.7\dot{3}$ (c) $0.\dot{1}\dot{5}$

Solution

(a) Let $r = 0.6666 \dots$ (i)

$$10r = 6.666 \dots \text{ (ii)}$$

Subtracting (i) from (ii);

$$9r = 6$$

$$r = \frac{6}{9}$$

$$= \frac{2}{3}$$

$$\therefore 0.\dot{6} = \frac{2}{3}$$

(b) Let $r = 0.733333 \dots$ (i)

$$10r = 7.33333 \dots \text{ (ii)}$$

$$100r = 73.3333 \dots \text{ (iii)}$$

Subtracting (ii) from (iii);

$$90r = 66$$

$$r = \frac{66}{90}$$

$$= \frac{11}{15}$$

$$\therefore 0.7\dot{3} = \frac{11}{15}$$

(c) Let $r = 0.151515 \dots$ (i)

$$100r = 15.1515 \dots \text{ (ii)}$$

$$99r = 15$$

$$r = \frac{15}{99}$$

$$= \frac{5}{33}$$

$$\therefore 0.\dot{1}\dot{5} = \frac{5}{33}$$

Exercise 8.2

1. Express each of the following as a single decimal fraction:

(a) $\frac{7}{11}$ (b) $\frac{5}{6}$ (c) $\frac{5}{27}$ (d) $6\frac{3}{11}$

(e) $\frac{7}{9}$ (f) $1\frac{2}{3}$ (g) $1\frac{1}{7}$ (h) $8\frac{27}{99}$

2. Express each of the following as a fraction:

(a) 0.3 (b) $0.\dot{7}$ (c) 0.15 (d) 0.04
 (e) $0.6\dot{7}$ (f) $2.\dot{8}\dot{3}$ (g) 4.37 (h) 28.13
 (i) $0.\dot{2}1\dot{5}$ (j) $1.\dot{5}2\dot{3}$ (k) $3.2\dot{5}\dot{6}$ (l) 2.750

8.4: Decimal Places

In carrying out division, the process may go on and on without ending. In such a case, we may round off the answer to any number of required digits to the right of the decimal point. These are called **decimal places**. For example;

$$1.5 \div 1.3 = 1.15384615\dots$$

This answer may be rounded off to:

- (i) 1.2 to the nearest tenth (1 decimal place).
- (ii) 1.15 to the nearest hundredth (2 decimal places).
- (iii) 1.154 to the nearest thousandth (3 decimal places).
- (iv) 1.1538 to the nearest ten thousandth (4 decimal places), and so on.

The steps to follow in rounding off a number to the required decimal places are:

- (i) find the answer to one place more than the required decimal.
- (ii) if the extra digit is less than 5, omit it.
- (iii) if the extra digit is 5 or more, add one to the preceding digit.

Note:

If we need to omit more than one digit in the rounding off process, then only the first of the digits to be omitted should be considered.

For example, 1.1538... to 2 decimal places becomes 1.15 and 1.1568.... to 2 decimal places becomes 1.16.

Exercise 8.3

1. Round off each of the following to 1, 2, 3 and 4 decimal places:

- | | | |
|--------------|--------------|---------------|
| (a) 0.139784 | (b) 0.203485 | (c) 0.0431285 |
| (d) 0.029372 | (e) 3.00569 | (f) 5.10946 |
| (g) 7.280291 | (h) 9.56777 | (i) 11.645811 |

8.5: Standard Form

A number is said to be in standard form if it is expressed in form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Example 4

Write each of the following numbers in standard form:

- (a) 36 (b) 576 (c) 0.052 (d) 0.0065

Solution

$$\begin{aligned}
 \text{(a)} \quad \frac{36}{10} \times 10 &= 3.6 \times 10^1 & \text{(b)} \quad \frac{576}{100} \times 100 &= 5.76 \times 10^2 \\
 \text{(c)} \quad 0.052 &= 0.052 \times \frac{100}{100} & \text{(d)} \quad \frac{0.0065 \times 1000}{1000} &= \frac{6.5}{1000} \\
 &= 5.2 \times \frac{1}{100} & &= 6.5 \times 10^{-3} \\
 &= 5.2 \times \left(\frac{1}{10}\right)^2 & & \\
 &= 5.2 \times 10^{-2} & &
 \end{aligned}$$

Exercise 8.4

1. Express each of the following numbers in standard form:

- | | | |
|------------|------------|------------|
| (a) 0.4 | (b) 369.4 | (c) 48.2 |
| (d) 0.0289 | (e) 509.78 | (f) 63.247 |

2. Write the number whose standard form is:

- | | | |
|---------------------------|---------------------------|---------------------------|
| (a) 3.12×10^2 | (b) 9.01×10^{-1} | (c) 7.85×10^{-3} |
| (d) 4.93×10^{-1} | (e) 3.7×10^1 | (f) 8.88×10^{-2} |

3. The cost of a minibus is sh. 3.6 million. Parents of a school that wanted to buy vehicle bus raised sh. 625 000. The school then realised sh. 4.05 million in a harambee. How much was the school left with after paying for the minibus? Express your answer in millions.

8.6: Operations on Decimals

Addition and Subtraction

Decimals are added or subtracted in the same way as whole numbers. However, it is important to have the decimal points one beneath the other. This ensures that tenths are added to tenths, hundredths to hundredths, e.t.c. The same applies to subtraction.

Example 5

Evaluate: (a) $15.25 + 1.2 + 0.067$ (b) $4.038 - 1.8$

Solution

(a) $15.25 + 1.2 + 0.067$ may be arranged as;

$$\begin{array}{r} 15.25 \\ 1.2 \\ + 0.067 \\ \hline 16.517 \end{array}$$

(b) $4.038 - 1.9$ may be arranged as;

$$\begin{array}{r} 4.038 \\ - 1.9 \\ \hline 2.138 \end{array}$$

Exercise 8.5

1. Evaluate:

- | | | |
|----------------------|-------------------------|-----------------------|
| (a) $1.372 + 2.008$ | (b) $3.45 + 6.03$ | (c) $16.84 + 0.1684$ |
| (d) $25.701 + 3.001$ | (e) $0.08 + 2.001$ | (f) $0.978 + 9.7801$ |
| (g) $5.739 + 16.482$ | (h) $2.983 + 0.307$ | (i) $0.145 + 0.0092$ |
| (j) $0.006 + 0.099$ | (k) $0.002481 + 92.381$ | (l) $0.0148 + 93.131$ |

2. Evaluate:

- | | |
|------------------------------|------------------------------|
| (a) $1.2 + 4.35 + 0.0764$ | (b) $8.06 + 0.009 + 4.923$ |
| (c) $11.233 + 250.16 + 2.96$ | (d) $42.56 + 0.291 + 7.2300$ |
| (e) $400.01 + 0.005 + 92.3$ | (f) $14.25 + 0.02 + 1.35$ |
| (g) $452.4 + 16.294 + 7.12$ | (h) $16.4 + 15.3 + 192.7$ |
| (i) $0.0012 + 0.012 + 0.12$ | (j) $2.034 + 20.34 + 203.4$ |

3. Evaluate:

- | | |
|---------------------|------------------------|
| (a) $2.55 - 1.201$ | (b) $39.231 - 3.874$ |
| (c) $0.002 - 0.034$ | (d) $425.631 - 0.004$ |
| (e) $0.2378 - 1.46$ | (f) $56.4895 - 43.652$ |

(g) $96.1 - 100.8$

(i) $212.25 - 152.79$

(k) $15.12 - 9.325$

(m) $0.025 - 0.0075$

(h) $495.001 - 548.8$

(j) $15.147 - 9.349$

(l) $16.132 - 5.74$

(n) $7 - 5.387$

4. Evaluate:

(a) $12.35 - 7.61 + 0.328$

(c) $0.348 - 0.089 + 2.0003$

(e) $45.68 - 13.984 + 4.019$

(g) $16.804 - 17.569 + 0.708$

(i) $101.045 - 120.034 + 11.468$

(b) $8.7 + 4.008 - 9.479$

(d) $8.941 + 11.031 - 22.003$

(f) $2.783 - 28.317 + 30.255$

(h) $55.31 + 100.184 - 140.912$

(j) $222 + 22.022 - 25.222$

5. Evaluate:

(a) $2.543 + 0.02 - 1.83$

(b) $9.27 - 10 + 42.9 - 15.37$

(c) $419.02 + 0.11 - 600 - 38.5$

(d) $3007.1 - 500.375 - 29.8625$

(e) $23.6 + 48.7 - 0.94$

(f) $(2.35 + 7.2) - (2.56 + 1.67)$

(g) $(17.2 - 13.12) - 2.56 + 1.67$

(h) $12.12 - [(6.34 + 6.45) - 2.78]$

(i) $15.21 + (2.7 - 3.1) - 4.3$

(j) $45.13 - [(6.34 - 2.17) - 9.1]$

6. A tailor used 2.15 m of material to make a dress, 1.8 m to make a skirt and 0.75 m to make a blouse. How much cloth did he use?
7. A botanist measured the growth of a seedling every week. The increase in growth in four weeks were 2.1 mm, 4.9 mm, 8.15 mm and 10.35 mm. What was the total increase in length of the seedling?
8. Four children bought 2.5 litres, 4.0 litres, 5.5 litres and 3.5 litres of milk from a dairy farm. If one litre of milk costs sh. 42, how much money did they pay altogether?
9. The length of a rectangular farm is 1.42 km and its breadth is 0.48 km. What is the perimeter of the farm?
10. A man spent 0.25 of his income on bills, 0.45 of the remainder on food and saved the rest. What fraction (in decimal notation) of his income did he save?
11. A wood-cutter had logs measuring 0.34 m, 0.038 m, 0.036 m and 0.04 m long, all from a single plank. How long was the plank?
12. The total mass of two pigs and 12 goats is 241.64. The mass of the two pigs is 145.36 kg. Find the average mass of each goat to the nearest 1 kg.

13. The distance between towns M and N is 58.6 km. Town Q, which is between the two towns, is 39.78 km from N. How far is Q from M?
14. Three jugs of capacity 0.75 litres, 0.68 litres and 0.91 litres are filled using a container holding 2.5 litres of liquid. How much liquid is left in the container?

Multiplication

Multiplication of decimals is the same as multiplication of fractions.

Example 6

Evaluate each of the following:

- (a) 0.5×0.7 (b) 0.2×0.45
 (b) 3.25×0.03 (d) $0.3 \times 1.05 \times 2.35$

Solution

$$\begin{aligned} \text{(a) } 0.5 \times 0.7 &= \frac{5}{10} \times \frac{7}{10} \\ &= \frac{35}{100} \\ &= 0.35 \end{aligned}$$

$$\begin{aligned} \text{(b) } 0.2 \times 0.45 &= \frac{2}{10} \times \frac{45}{100} \\ &= \frac{90}{1000} \\ &= 0.090 \\ &= 0.09 \end{aligned}$$

$$\begin{aligned} \text{(c) } 3.25 \times 0.03 &= \frac{325}{100} \times \frac{3}{100} \\ &= \frac{975}{10000} \\ &= 0.0975 \end{aligned}$$

$$\begin{aligned} \text{(d) } 0.3 \times 1.05 \times 2.35 &= \frac{3}{10} \times \frac{105}{100} \times \frac{235}{100} \\ &= \frac{315 \times 235}{100000} \\ &= \frac{74025}{100000} \end{aligned}$$

This process simplifies to:

- (i) Multiply the decimals, disregarding the decimal points.
- (ii) Add the number of digits to the right of the decimal points in the numbers to be multiplied.
- (iii) The sum gives the number of digits to the right of the decimal point in the product.
- (iv) If the number of digits obtained in the product is less, insert zero between the decimal point and the digits to bring up the number of required digits.

Example 7

Evaluate: 4.15×0.021

Solution

4.15×0.021 may be arranged as:

$$\begin{array}{r}
 415 \\
 \times 21 \\
 \hline
 415 \\
 +830 \\
 \hline
 8715
 \end{array}$$

The number of digits after the decimal point in the product is five.

Therefore, $4.15 \times 0.021 = 0.08715$

Multiply each of the following by 10, 100, 1 000 and 10 000:

- (i) 12.037 (ii) 0.2589

Note:

To multiply by powers of ten, the decimal point moves to the right as many places as the number of zeros in the multiplier.

Exercise 8.6

1. Evaluate:

(a) 2.3×5

(b) 7.8×3

(c) 1.28×4

(d) 43×0.205

(e) 295×26.04

(f) 29.6×6.21

2. Multiply each of the following by 10, 100, 1 000, 10 000, 100 000 and 1 000 000:

(a) 16.3

(b) 5.16

(c) 0.0532

(d) 0.60241

(e) 0.090256

(f) 0.7381

3. Evaluate:

- | | | |
|----------------------------|--------------------------|----------------------------|
| (a) 0.14×0.05 | (b) 0.603×0.04 | (c) 0.071×0.035 |
| (d) 0.63×0.7 | (e) 0.89×0.56 | (f) 0.532×0.487 |
| (g) 0.0016×0.34 | (h) 0.987×0.125 | (i) 0.505×0.0704 |
| (j) 0.0351×0.0402 | (k) 0.678×0.075 | (l) 0.00784×0.231 |

4. Evaluate:

- | | | |
|----------------------------|---------------------------|---------------------------|
| (a) 4.25×3.8 | (b) 17.24×3.05 | (c) 41.325×6.4 |
| (d) 58.16×7.85 | (e) 5.38×95.6 | (f) 36.72×8.86 |
| (g) 70.3×0.26 | (h) 17.12×1.375 | (i) 1040.85×0.62 |
| (j) 15.1432×1.726 | (k) 13.362×0.081 | (l) 22.041×0.62 |

5. A machine consumes 4.5 litres of petrol every hour. How much petrol does it consume in 6.4 hours?
6. A tailor uses 2.8 m of material to make one uniform. How much cloth does he use to make 45 such uniforms?
7. A woman paid sh. 150.00 for every kilogram of meat. How much money did she pay for 5.50 kilograms?
8. A family consumes 5.5 litres of milk everyday. How much milk does the family consume in the month of April?
9. A field measures 100.34 m by 54.63 m. Find its area.
10. A doctor prescribed a particular tablet for a patient who had malaria. The patient had to take 4 tablets on the first day and 2 tablets on each day for the subsequent 5 days. How much medicine in grams had this patient taken by the end of the dosage if each of the tablets weighed 0.025 g?
11. Clara bought 6 kg of sugar and 2 kg of rice at sh. 44.50 and sh. 35.35 per kilogram respectively. How much money did she spend?
12. If a metre of curtain material costs sh. 245.65, what is the price of 12.5 m of the material?
13. The distance between two ports is 215 nautical miles. What is this distance in kilometres if one nautical mile is approximately equal to 1.85 km?
14. Abiero made three trips from town P to town Q by bus. On two occasions, he returned to P by minibus and once by bus. If the fare to Q from P is sh. 180 by bus and sh. 220 by minibus, how much did the trips cost him?

Division

A decimal can be divided by a whole number in the usual way.

Example 8

Work out: (a) $1.532 \div 4$ (b) $0.0021 \div 14$

Solution

(a) $1.532 \div 4$

$$\begin{array}{r}
 0.383 \\
 4 \overline{)1.532} \\
 \underline{-1.2} \\
 33 \\
 \underline{-32} \\
 12 \\
 \underline{-12} \\
 0
 \end{array}$$

(b) $0.0021 \div 14$

$$\begin{array}{r}
 0.00015 \\
 14 \overline{)0.0021} \\
 \underline{-14} \\
 70 \\
 \underline{-70} \\
 0
 \end{array}$$

Divide each of the following by 10, 100, 1 000 and 10 000:

(i) 0.256

(ii) 123.4

Note:

Division by a power of 10 moves the decimal point to the left as many places as the number of zeros in the divisor.

To divide a decimal by a decimal, we first change the divisor to a whole number. This is done by multiplying both the divisor and the dividend by a power of ten which makes the divisor a whole number.

Example 9

Evaluate:

(a) $0.036 \div 0.5$

(b) $1.078 \div 0.07$

Solution

$$\begin{aligned}
 \text{(a)} \quad \frac{0.036}{0.5} &= \frac{0.036}{0.5} \times \frac{10}{10} \\
 &= \frac{0.36}{5} \\
 &= 0.072
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{1.078}{0.07} &= \frac{1.078}{0.07} \times \frac{100}{100} \\
 &= \frac{107.8}{7} \\
 &= 15.4
 \end{aligned}$$

Exercise 8.7

1. Evaluate:

(a) $1.7 \div 2$

(b) $15.32 \div 4$

(c) $6.75 \div 2$

(d) $0.0015 \div 4$

(e) $2.34519 \div 3$

(f) $1.33 \div 7$

(g) $12.632 \div 5$

(h) $26.92 \div 8$

(i) $28.068 \div 8$

(j) $0.0236 \div 2$

(k) $22.0407 \div 11$

(l) $0.228 \div 12$

(m) $4.95 \div 11$

(n) $0.0286 \div 11$

(p) $109.8006 \div 12$

(q) $437.5 \div 25$

(r) $112.0812 \div 14$

(s) $244.008 \div 8$

(t) $0.372 \div 30$

(u) $787.792 \div 3716$

(v) $828.45 \div 15$

2. Divide each of the following numbers by 10, 100, 1 000, 10 000, 100 000 and 1 000 000:
 (a) 3.7 (b) 0.28 (c) 10.3 (d) 125.39 (e) 1.026
3. Evaluate:
 (a) $2.435 \div 2.5$ (b) $0.36 \div 0.03$ (c) $10.5 \div 4.2$
 (d) $0.0106 \div 0.02$ (e) $0.378 \div 0.18$ (f) $2.88 \div 0.36$
 (g) $5.78 \div 1.7$ (h) $0.00256 \div 0.16$ (i) $1.26 \div 1.8$
 (j) $0.252 \div 0.42$ (k) $4.016 \div 5.02$ (l) $4.0446 \div 1.26$
 (m) $28.6 \div 0.013$ (n) $7.008 \div 0.003$ (p) $0.34 \div 3.835$
 (q) $126.7 \div 1.267$ (r) $1009.992 \div 1.25$ (s) $334.842 \div 0.12$
4. The product of two numbers is 11.9. If one of the numbers is 4.25, what is the other number?
5. Ratemo's car travels 12.5 km on one litre of petrol. If he travelled 312.5 km in one day, how many litres of petrol did he use?
6. A certain district realised 778.8 mm of rainfall in seven days. What was the average rainfall per day?
7. The cost of twenty four bananas is sh. 16.80. Find the cost of each.
8. The area of a rectangular garden is $1384\frac{74}{100} \text{ m}^2$. If its length is 44.1 m, find its width.
9. Maria's car consumes 86.8 litres of petrol per week. Find its average consumption per day.
10. A pile of 18 identical books, each of 125 equal leaves, is 2.16 m high. Find the thickness of each leaf in centimetres.

8.7: Order of Operations

The same rules regarding the order in which operations are performed on integers apply to decimals.

Example 10

Evaluate:

$$0.02 + 3.5 \times 2.6 - 0.1(6.2 - 3.4)$$

Solution

$$\begin{aligned} 0.02 + 3.5 \times 2.6 - 0.1 \times 2.8 &= 0.02 + 9.1 - 0.28 \\ &= 8.84 \end{aligned}$$

Exercise 8.8

1. Evaluate:

$$(a) \frac{3.17 - 2.45}{1.8}$$

$$(b) \frac{0.21 + 0.7}{4.2 - 2.9}$$

$$(c) \quad \frac{1.4 + 2.7 - 1.7}{0.61 + 0.09}$$

$$(e) \quad \frac{13.5 \times 1.7}{1.53 \times 0.09}$$

$$(g) \quad 4.1 + 2.3 + 11.2 \div 2.8$$

$$(i) \quad \frac{9.93 \times 5.94}{0.0331 \times 1.1}$$

$$(k) \quad \frac{7.16 + 0.64}{0.24}$$

$$(m) \quad \frac{0.0734 - 0.008}{0.5 + 10.4}$$

$$(p) \quad (13.42 + 0.321) - (0.31 \times 2.56)$$

$$(q) \quad 0.04 \times (4.375 - 1.205 + 0.008)$$

$$(r) \quad \frac{0.0195 \times 4.55}{13.0 \times 0.35}$$

$$(t) \quad \frac{11.7 \times 0.036 \times 5.8}{130 \times 1.45 \times 7.2}$$

$$(d) \quad 3.64 \times 7.5 \div 3.9$$

$$(f) \quad 3.6 - 4.6 \times 9.0 + 7.5$$

$$(h) \quad \frac{10.8 \times 7.5}{3.6}$$

$$(j) \quad \frac{2.5 \times 3.6}{1.5}$$

$$(l) \quad \frac{0.232 \times 0.04}{2.5 \times 6.4}$$

$$(n) \quad \frac{0.35 \times 1.28 \times 7.3}{0.25}$$

$$(s) \quad \frac{4.2 \times 6.029 \times 7.8}{60 \times 0.7}$$

$$(u) \quad \frac{15.2 \times 0.051 \times 6.5}{120 \times 2.5 \times 6.0}$$

2. Evaluate $\frac{29.6 - 72.3}{0.28 \times 0.09}$, correct to 3 decimal places.

3. Evaluate $\frac{0.389 - 0.15}{20}$, correct to 4 decimal places.

4. Evaluate $\frac{0.36 \times 4.2 \times 8.3}{0.008 \times 25.5}$, correct to 2 decimal places.

5. Evaluate $\frac{0.17 \times 1.05 \times 0.32}{4.5 \times 0.08 \times 0.089}$, correct to 5 decimal places.

6. Evaluate $\frac{2.27 \times 0.41}{0.03}$, correct to 1 decimal place.

7. Evaluate $\frac{(0.73 \times 0.02) + 0.03}{1.8}$ correct to 4 decimal places

SQUARES AND SQUARE ROOTS

9.1: Squares

Example 1

Find the square of each of the following:

- (a) 15 (b) $1\frac{1}{2}$ (c) 2.5

Solution

$$\begin{aligned}\text{(a)} \quad 15^2 &= 15 \times 15 \\ &= 225\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad \left(1\frac{1}{2}\right)^2 &= \frac{3}{2} \times \frac{3}{2} \\ &= \frac{9}{4}\end{aligned}$$

$$\begin{aligned}\text{(c)} \quad 2.5^2 &= 2.5 \times 2.5 \\ &= 6.25\end{aligned}$$

The square of a number, e.g., x , can be written as x^2 .

Exercise 9.1

1. Find the square of each of the following numbers:

- | | | | | |
|-------------------|--------------------|---------|----------|-------------------|
| (a) 6 | (b) 9 | (c) 11 | (d) 23 | (e) $\frac{4}{7}$ |
| (f) $\frac{8}{3}$ | (g) $1\frac{2}{3}$ | (h) 1.8 | (i) 2.25 | (j) 0.17 |
| (k) 0.19 | (l) 3.15 | | | |

9.2: Squares from Tables

Squares of numbers are tabulated and can be read from a table of squares. These tables, however, give only approximate values of the squares to 4 figures. The squares of numbers from 1.000 to 9.999 can be read directly from the tables (see table 9.1). The position of the decimal point for other numbers has to be determined by inspection.

The use of table is illustrated by the following examples.

Example 2

Find the square of:

- (a) 4.25 (b) 42.5 (c) 425 (d) 0.425

Solution

x^2 SQUARES											Add								
X	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
3.0	9.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425	9.486	9.548	6	12	18	24	30	37	43	49	55
3.1	9.610	9.672	9.734	9.797	9.860	9.923	9.986	10.05	10.11	10.18	6	13	19	2	1	38	44	50	57
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	1	1	2	2	3	4	4	5	5
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	1	1	2	3	3	4	5	6	6
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	1	1	2	3	3	4	5	6	6
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	1	1	2	3	4	4	5	6	6
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	1	1	2	3	4	4	5	6	6
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	1	2	2	3	4	5	6	6	7
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	1	2	2	3	4	5	6	6	7
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	1	2	2	3	4	5	6	6	7
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	1	2	2	3	4	5	6	6	7
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	1	2	2	3	4	5	6	6	7
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	1	2	2	3	4	5	6	6	7
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	1	2	3	4	4	5	6	7	8
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	1	2	3	4	4	5	6	7	8
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	1	2	3	4	5	5	6	7	8
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	1	2	3	4	5	5	6	7	8
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	1	2	3	4	5	6	7	8	9
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	1	2	3	4	5	6	7	8	9
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	1	2	3	4	5	6	7	8	9
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	1	2	3	4	5	6	7	8	9
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	1	2	3	4	5	6	7	8	9
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	1	2	3	4	5	6	7	8	9
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05	1	2	3	4	5	7	8	9	10
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	1	2	3	4	5	7	8	9	11
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	1	2	3	4	6	7	8	9	10
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	1	2	3	4	6	7	8	9	10
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	1	2	4	5	6	7	8	10	10
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	1	2	4	5	6	7	8	10	11
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	1	2	4	5	6	7	8	10	11

Table 9.1

- (a) To read the square of 4.25, look for 4.2 down the column headed x . Move to the right along this row, up to where it intersects with the column headed 5. The number in this position is the square of 4.25.

So, $4.25^2 = 18.06$ (to 4 figures)

Note:

The actual value of 4.25^2 by long multiplication is 18.0625.

The value obtained from tables is rounded off to 4 figures only.

- (b) The square of 42.5 lies between 40^2 and 50^2 , i.e., between 1 600 and 2 500.

$$\begin{aligned}
 42.5^2 &= (4.25 \times 10^1)^2 \\
 &= 4.25^2 \times 10^2 \\
 &= 18.06 \times 100 \\
 &= 1\,806
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad 425^2 &= (4.25 \times 10^2)^2 \\
 &= 4.25^2 \times 10^4 \\
 &= 18.06 \times 10\,000 \\
 &= 180\,600
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad 0.425^2 &= (4.25 \times \frac{1}{10})^2 \\
 &= 4.25^2 \times (\frac{1}{10})^2 \\
 &= 18.06 \times \frac{1}{100} \\
 &= 0.1806
 \end{aligned}$$

Example 3

Find the square of: (a) 3.162 (b) 56.129

Solution

- (a) Note from table 9.1 that square tables have extra columns labelled 1 to 9 to the right of the thick line. The numbers under these columns are called '**mean differences**.' To find $(3.162)^2$, read 3.16^2 to get 9.986. Then read the number in the position where the row containing 9.986 intersects with the 'differences' column headed 2. The difference is 13 and this should be added to the last digit(s) of 9.986.

$$\begin{array}{r}
 9.986 \\
 + \quad 13 \\
 \hline
 9.999
 \end{array}$$

Thus, $3.162^2 = 9.999$

- (b) 56.129 has 5 significant figures and in order to use 4 figure tables, we must first round it off to four figures.

$$56.129 = 56.13 \text{ to 4 figures}$$

$$\begin{aligned}
 56.13^2 &= (5.613 \times 10^1)^2 \\
 &= 31.50 \times 10^2 \\
 &= 3\,150
 \end{aligned}$$

Exercise 9.2

1. Use tables to find the square of each of the following numbers:

- | | | | |
|-----------|-----------|-----------|-----------|
| (a) 2.3 | (b) 4.11 | (c) 1.35 | (d) 9.73 |
| (e) 2.78 | (f) 7.39 | (g) 9.32 | (h) 3.97 |
| (i) 6.02 | (j) 5.39 | (k) 8.02 | (l) 7.21 |
| (m) 6.28 | (n) 4.723 | (p) 3.221 | (q) 6.438 |
| (r) 2.007 | (s) 5.672 | (t) 2.011 | (u) 8.291 |

2. Use tables to find square of each of the following numbers:

- | | | | |
|--------------|---------------|---------------|---------------|
| (a) 0.25 | (b) 0.32 | (c) 3.08 | (d) 0.04 |
| (e) 0.793 | (f) 0.0094 | (g) 0.231 | (h) 0.6128 |
| (i) 0.0914 | (j) 0.3072 | (k) 0.00566 | (l) 0.00475 |
| (m) 0.007829 | (n) 0.000642 | (p) 0.05487 | (q) 0.004136 |
| (r) 0.00824 | (s) 0.0031928 | (t) 0.0001497 | (u) 0.0023998 |

3. By expressing each of the following numbers in the form $A \times 10^n$, where A is a number between 1 and 10 and n an integer, find its square:

- | | | | |
|------------|-------------|------------|-------------|
| (a) 21.36 | (b) 62.04 | (c) 41 | (d) 57 |
| (e) 97.3 | (f) 68.04 | (g) 48.95 | (h) 70.12 |
| (i) 51.37 | (j) 31.72 | (k) 89.19 | (l) 22.346 |
| (m) 82.365 | (n) 43.023 | (p) 88.56 | (q) 17.136 |
| (r) 39.754 | (s) 28.0375 | (t) 99.999 | (u) 893.689 |

9.3: Square Roots

Since $5 \times 5 = 25$, we say that 5 is a square root of 25. Remember that $-5 \times -5 = 25$ and so -5 is also a square root of 25.

In general, any positive number has two square roots, one positive and the other negative. The symbol for the square root of a number is $\sqrt{\quad}$.

For example, $\sqrt{121} = \pm 11$, $\sqrt{2.25} = \pm 1.5$, $\sqrt{0.0256} = \pm 0.16$.

In this chapter, we will confine ourselves to positive square roots only. A number whose square root is an integer (or a terminating decimal) is called a **perfect square**. For example, 1, 4, 9, 25 and 36 are perfect squares.

Exercise 9.3

1. Find the positive square root of each of the following numbers:

- | | | | |
|---------|---------|---------|---------|
| (a) 81 | (b) 169 | (c) 196 | (d) 625 |
| (e) 400 | (f) 144 | (g) 256 | (h) 361 |
| (i) 324 | (j) 900 | (k) 676 | (l) 841 |

9.4: Square Root by Factorisation

The square root of a number can also be obtained using factorisation method.

Example 4

Find the square root of 81 by factorisation method.

Solution

$$\begin{aligned}
 \sqrt{81} &= \sqrt{3 \times 3 \times 3 \times 3} \\
 &= 3 \times 3 \\
 &= 9
 \end{aligned}$$

- Find the prime factors of 81.
- Group the prime factors into two identical numbers.
- Out of the two identical prime factors, choose one and find their product.

Note:

In general, pair the prime factors into two identical numbers. For every pair, pick only one number then obtain the product.

Example 5

Find $\sqrt{1764}$ by factorisation

Solution

$$\begin{aligned}\sqrt{1764} &= \sqrt{\overbrace{2 \times 2} \times \overbrace{3 \times 3} \times \overbrace{7 \times 7}} \\ &= 2 \times 3 \times 7 \\ &= 42\end{aligned}$$

Alternatively, to find the square root of a number by factorisation method, express the factors in power form, then divide the powers by two, i.e., multiply the power by $\frac{1}{2}$.

Thus, in example 5, the square root can also be found as follows:

$$\begin{aligned}\sqrt{1764} &= \sqrt{\overbrace{2 \times 2} \times \overbrace{3 \times 3} \times \overbrace{7 \times 7}} \\ &= \sqrt{(2^2 \times 3^2 \times 7^2)} \\ &= (2^{2 \times \frac{1}{2}} \times 3^{2 \times \frac{1}{2}} \times 7^{2 \times \frac{1}{2}}) \\ &= 2^1 \times 3^1 \times 7^1 \\ &= 42\end{aligned}$$

Example 6

Find $\sqrt{441}$ by factorisation.

Solution

$$\begin{aligned}\sqrt{441} &= \sqrt{\overbrace{3 \times 3} \times \overbrace{7 \times 7}} \\ &= \sqrt{3^2 \times 7^2} \\ &= 3^{2 \times \frac{1}{2}} \times 7^{2 \times \frac{1}{2}} \\ &= 3^1 \times 7^1 \\ &= 21\end{aligned}$$

Exercise 9.4

1. Find the square root of each of the following using the factor method:

- | | | | |
|-----------|------------|-----------|------------|
| (a) 625 | (b) 576 | (c) 7396 | (d) 15 625 |
| (e) 3 249 | (f) 76 176 | (g) 2401 | (h) 4 356 |
| (i) 4 225 | (j) 3 136 | (k) 5 184 | (l) 6 561 |

9.5: Square Root from Tables

Tables of square roots are used in a similar way to tables of squares. Square roots of numbers from 1.0 to 99.99 are given in the tables and can be read directly (see tables of square roots in your mathematical tables).

Example 7

Use tables to find the square root of:

- | | | | |
|----------|-----------|---------|------------|
| (a) 1.86 | (b) 42.57 | (c) 359 | (d) 0.8236 |
|----------|-----------|---------|------------|

Solution

- (a) To read the square root of 1.86, look for 1.8 in the column headed x . Move to the right along this row to where it intersects with the column headed 6. The number in this position is the square root of 1.86. Thus,
- $$\sqrt{1.86} = 1.364 \text{ (four figures)}$$

- (b) $\sqrt{42.57}$
 Look for 42 in the column headed x and move along the row containing 42 to where it intersects with the column headed 5. Read the number in this position, which is 6.519. The difference for 7 from the difference column along this row is 6. The difference is added to 6.519, as below;

$$\begin{array}{r} 6.519 \\ + 0.006 \\ \hline 6.525 \end{array}$$

Thus, $\sqrt{42.57} = 6.525$ (for figures)

For any number outside this range, it is necessary to first express it in the form $A \times 10^n$ where $1 \leq A < 100$ and in n is an even integer.

- (c) $359 = 3.59 \times 10^2$

$$\begin{aligned} \sqrt{359} &= \sqrt{(3.59 \times 100)} \\ &= 1.895 \times 10 \\ &= 18.95 \text{ (four figures)} \end{aligned}$$

$$(d) \quad 0.8236 = 82.36 \times \left(\frac{1}{10}\right)^2$$

$$\begin{aligned}\sqrt{0.8236} &= \sqrt{82.36 \times \frac{1}{100}} \\ &= (9.072 + 0.004) \times \frac{1}{10} \\ &= 0.9076 \text{ (four figures)}\end{aligned}$$

Exercise 9.5

In this exercise, use square root tables

1. Find the square root of each of the following numbers:

- | | | | |
|------------|-------------|-------------|--------------|
| (a) 40 | (b) 5.38 | (c) 0.146 | (d) 6.142 |
| (e) 0.0529 | (f) 952.695 | (g) 8.52 | (h) 0.009823 |
| (i) 7.326 | (j) 689 341 | (k) 641.978 | (l) 0.001952 |

2. Evaluate:

(a) $\frac{\sqrt{90 \times 6}}{\sqrt{50 \times 4}}$	(b) $\frac{\sqrt{50} + \sqrt{20}}{\sqrt{20}}$	(c) $\frac{\sqrt{150} + \sqrt{180}}{\sqrt{60}}$
---	---	---

3. If $a = 3$, $b = 4.7$ and $c = 6.4$, find the value of $\sqrt{\left(\frac{a^2 b^2}{c}\right)}$

(Give the answer in four figures)

4. If $x = 3$, $y = 2$ and $z = 7$, calculate $\sqrt{\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)}$ giving your answer correct to 3 figures.

5. Given that $a^3 + \frac{1}{b^2} - c^2 = d^2$, find the value of d if $a = 7$, $b = \frac{1}{2}$ and $c = \frac{1}{4}$.
(Give your answer to four figures)

6. If the area of a square is 38.44 cm^2 , calculate its perimeter.

7. The area of a triangle whose height is equal to the length of its base is 40.5 cm^2 . Calculate the length of the base.

8. The periodic time for the swing of a pendulum in seconds is given by the formula $T = 2\pi \sqrt{\frac{l}{g}}$. Calculate the value of T when $l = 23.7 \text{ cm}$ and $g = 1000 \text{ cms}^{-2}$

9. The surface area of a sphere of radius r is given by the formula $A = 4\pi r^2$. What is the radius of a sphere whose surface area is 120 cm^2 (correct to 3 decimal places)?

ALGEBRAIC EXPRESSIONS

10.1: Symbolic Representation

Consider the following:

- (i) Wakio had 5 goats and Baraza 19 goats. How many goats did they have altogether?

If instead Wakio had P goats and Baraza Q goats, how many goats would they have altogether?

- (ii) Miriam buys a number of oranges from Mponda's kiosk. She then moves to Mary's kiosk and buys twice as many oranges as she had from the first kiosk. Using a letter of your choice, write down an expression for the total number of oranges bought.

If we choose n to represent the number of oranges bought at Mponda's kiosk, then the number bought from the Mary's kiosk would be $2n$. Therefore, the expression for total number of oranges would be $(n + 2n)$.

In algebra, (i) letters are used to represent numbers.

(ii) $2n$ means $2 \times n$.

Exercise 10.1

- Use letters to represent the following statements:
 - The sum of two consecutive integers.
 - A number is multiplied by 3, then six is added to the result.
 - The area of a rectangle whose length is $2\frac{1}{2}$ times its width.
 - The distance covered in a given time at a given speed.
 - The number of days in m weeks.
 - The number of hours in $\frac{n}{4}$ days.
- John is twice as old as his brother Kogo, and their sister Jane is 7 years younger than Kogo. Write down an expression for the sum of their ages.
- Three children are given a certain number of bananas by their mother. If they share the bananas equally, how many does each get?
- Wafula earns twice as much as his wife Grace. Write down an expression for the difference between their earnings.

5. A number is added to $\frac{2}{3}$ of itself. If the sum is doubled, write down the final expression.
6. For the figure below, express in terms of a , b , c and d :
- the area of each of the rectangles, A, B, C and D.
 - the perimeter of each of the rectangles A, B, C and D.
 - the total area of the four rectangles A, B, C and D.

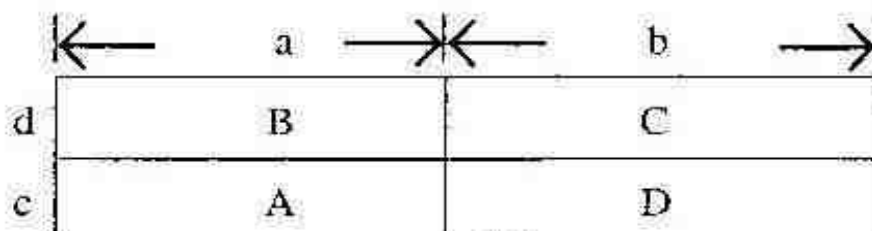


Fig. 10.1

7. George is ten years older than his brother Sam. Find an expression for:
- the sum of their ages.
 - the sum of their ages in 5 years time.
 - the product of their ages three years ago.
8. In a mathematics test, six pupils obtained the following scores; 40, c , 20, d , e and f . Find their average score.
9. A man spends half of his monthly salary on food and one eighth on water and electricity. Write down an expression for the remainder of his salary.
10. In the figure below, ABCD is a square. DEFC, BCGH and AEFB are all rectangles. Find:
- CF in terms of a and b .
 - the area of rectangle CDEF.

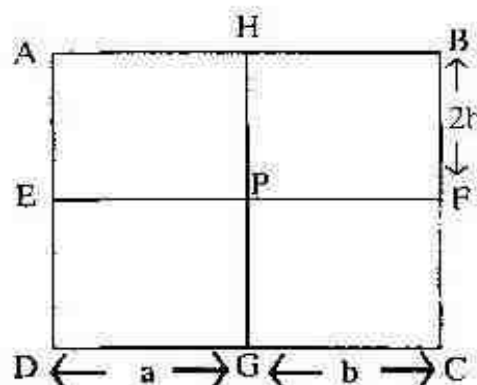


Fig. 10.2

11. An expression is obtained from three numbers by subtracting the difference between the first two from the third and adding twice the result to the first number. Write down the expression.
12. A man whose stride is $\frac{3}{4}$ m long walks all the way round a rectangular plot measuring 'a' m by 'b' m. Find an expression for the total number of steps.

10.2: Simplification of Algebraic Expressions

Like and Unlike Terms

Whereas we may add 3 cows to 4 cows to get 7 cows, the result of adding 3 cows to 4 houses cannot be expressed in any simpler form.

Similarly, $3a + 4a = 7a$, but $3a + 4b$ cannot be simplified further. $3a$ and $4a$ are like terms, while $3a$ and $4b$ are unlike terms.

Example 1

Simplify: $3x + 4y - x + z + 3y$

Solution

$$\begin{aligned} 3x + 4y - x + z + 3y &= (3x - x) + (4y + 3y) + z \\ &= 2x + 7y + z \end{aligned}$$

Example 2

Simplify: $2x - 6y - 4x + 5z - y$

Solution

$$\begin{aligned} 2x - 6y - 4x + 5z - y &= 2x - 4x - 6y - y + 5z \\ &= (2x - 4x) - (6y + y) + 5z \\ &= -2x - 7y + 5z \end{aligned}$$

Note:

$$-6y - y = -(6y + y)$$

Example 3

Simplify: $\frac{1}{2}a - \frac{1}{3}b + \frac{1}{4}a$

Solution

The L.C.M. of 2, 3 and 4 is 12.

$$\begin{aligned}
 \text{Therefore } \frac{1}{2}a - \frac{1}{3}b + \frac{1}{4}a &= \frac{6a - 4b + 3a}{12} \\
 &= \frac{6a + 3a - 4b}{12} \\
 &= \frac{9a - 4b}{12}
 \end{aligned}$$

Example 4

Simplify: $\frac{a+b}{2} - \frac{2a-b}{3}$

Solution

$$\begin{aligned}
 \frac{a+b}{2} - \frac{2a-b}{3} &= \frac{3(a+b) - 2(2a-b)}{6} \\
 &= \frac{3a + 3b - 4a + 2b}{6} \\
 &= \frac{3a - 4a + 3b + 2b}{6} \\
 &= \frac{-a + 5b}{6}
 \end{aligned}$$

Exercise 10.2

Simplify where possible:

1. (a) $2a + 3b + 4a - b$

(b) $-10k + 2m - 3k - 5m$

2. (a) $7t + 2p + 3t + 5p$

(b) $6x - 9x - 2y + 9y$

3. (a) $2a + 2b + 2c$

(b) $0.5r + 0.85 - 0.1r$

4. (a) $2a + 3b + 4a - b$

(b) $0.8r + 0.26 - 0.6r + 7c - 0.2r$

5. (a) $-4z - 7d + 2z$

(b) $8x - 2y - 2\frac{1}{2}x - 3y$

6. (a) $8w - p - 2w + 11p$

(b) $\frac{1}{3}r - \frac{1}{4}s + \frac{1}{6}s$

7. (a) $\frac{2a}{3} - \frac{1}{8}b + \frac{1}{6}a$

(b) $-\frac{1}{9} + \frac{5k}{6} - \frac{1}{3}$

8. (a) $\frac{2x}{3} + \frac{2y}{5} - \frac{4x}{3} - \frac{y}{5}$

(b) $\frac{3d}{4} - \frac{p}{8} - \frac{d}{2} + \frac{p}{4}$

9. (a) $\frac{1}{10} + \frac{d}{10} - d + 7a$

(b) $\frac{3x}{2} - \frac{5y}{6} + \frac{y}{4}$

10. In each of the following, pick the term which is not like the others:

- (a) $3a, 5a, 7a, 2a^2, 10a$
- (b) $m^5, 5m^3, 6m^5, 3m^5, 11m^5$
- (c) $a^2b, ab, ba^2, 5a^2b, 6ba^2$
- (d) $2cd, dc, -cd, c^2d, -\frac{3}{4}cd$

In general, terms are called 'like terms' if they have the same letters raised to the same power. Otherwise they are unlike terms. For example, $3a$ and $3a^2$ are unlike terms, while a^2 and $3a^2$ are like terms.

Example 5

Simplify: $5a + 2a^2 + 10a$

Solution

$$\begin{aligned} 5a + 2a^2 + 10a &= 5a + 10a + 2a^2 \\ &= 15a + 2a^2 \end{aligned}$$

Example 6

Simplify: $a^2b - 2b^3c + 3a^2b + b^3c$

Solution

$$\begin{aligned} a^2b - 2b^3c + 3a^2b + b^3c &= a^2b + 3a^2b - 2b^3c + b^3c \\ &= 4a^2b - b^3c \end{aligned}$$

Note:

W and w are unlike terms.

Exercise 10.3

Simplify where possible:

- | | |
|---|---|
| 1. (a) $p^2 - 3p^2 + 7p^2 + 8p^2$ | (b) $y^3 + 3y + 2y^3 + y$ |
| 2. (a) $x^2 + x - 2x^2 + 5x$ | (b) $3W^5 + 3w^5 + 6w^5 - 4w$ |
| 3. (a) $a^2b + 2ba + 3ba^2 - ab$ | (b) $a^2b + b^2a + 3ab^2$ |
| 4. (a) $pqr + rpq + qrp + 2pq$ | (b) $u^3vw + uv^3w + uvw^3$ |
| 5. (a) $4R^2 - 2r^2 + 5R^4 + r^2$ | (b) $\frac{1}{4}p^2q + \frac{1}{2}qp^2 - \frac{1}{2}q^2p$ |
| 6. (a) $\frac{1}{3}a^3 + \frac{2}{5}a^3 - \frac{1}{6}a^3$ | (b) $0.5st + \frac{3}{4}t^2s + \frac{1}{10}ts - 5$ |

10.3: Brackets

In algebra, brackets serve the same purpose as they do in arithmetic.

Example 7

Remove the brackets and simplify:

$$(a) \quad 3(a + b) - 2(a - b)$$

$$(b) \quad 5 - (6a - 3b - 8)$$

$$(c) \quad \frac{1}{3}a + 3(5a + b - c)$$

$$(d) \quad \frac{2}{y} \left(\frac{1}{2}y + y^2 + y^3 \right)$$

$$(e) \quad 2b + 3\{3 - 2(a - 5)\}$$

Solution

$$\begin{aligned} (a) \quad 3(a + b) - 2(a - b) &= 3a + 3b - 2a + 2b \\ &= 3a - 2a + 3b + 2b \\ &= a + 5b \end{aligned}$$

$$\begin{aligned} (b) \quad 5 - 6(a - 3b - 8) &= 5 - 6a + 18b + 48 \\ &= 53 - 6a + 18b \end{aligned}$$

$$\begin{aligned} (c) \quad \frac{1}{3}a + 3(5a + b - c) &= \frac{1}{3}a + 15a + 3b - 3c \\ &= 15\frac{1}{3}a + 3b - 3c \end{aligned}$$

$$\begin{aligned} (d) \quad \frac{2}{y} \left(\frac{1}{2}y + y^2 + y^3 \right) &= \frac{2}{y} \times \frac{1}{2}y + \frac{2}{y} \times y^2 + \frac{2}{y} \times y^3 \\ &= 1 + 2y + 2y^2 \end{aligned}$$

$$\begin{aligned} (e) \quad 2b + a\{3 - 2(a - 5)\} &= 2b + a\{3 - 2a + 10\} \\ &= 2b + 3a - 2a^2 + 10a \\ &= 2b + 3a + 10a - 2a^2 \\ &= 2b + 13a - 2a^2 \end{aligned}$$

Exercise 10.4

Remove the brackets and simplify:

$$1. \quad (a) \quad 3(r + s) + 4(r + s)$$

$$(b) \quad 4(2a + 3) - 3(5a - 6)$$

$$2. \quad (a) \quad (3 - y) - 2(x - y + 2)$$

$$(b) \quad (3 - y) - 3(y - x - 2)$$

$$3. \quad (a) \quad 2p(q + r) - r(p + q)$$

$$(b) \quad \frac{4(x+1) - 3(x-1)}{6}$$

4. (a) $\frac{3}{4}y(5y + 3) - 3y(y + 7)$

(b) $\frac{1}{2}w(8w - 2m) + \frac{1}{3}(m - w^2)$

5. (a) $-\frac{1}{2}xy(x - xy) - x(xy - x^2)$

(b) $\frac{(x-2x+3)}{x}$

6. (a) $\frac{1}{y}(2y^2 + 3ay)$

(b) $\frac{4}{am}(a^2m^3 + a) + \frac{1}{2}(a^2 + m^3)$

7. (a) $2\{x + 3(x + 2y)\}$

(b) $a\{3(b + c) + 4(c + a)\}$

8. (a) $\frac{1}{2}\{x - 4(2y - 3x)\}$

(b) $\frac{1}{4}\{x - (2y - 3x)\}$

9. (a) $3\{2p - \frac{1}{3}(p - q + r)\}$

(b) $3y - 4\{3 - (y + 2)\}$

10. (a) $-[(m - 2p - 5) - 3(2m + 4p - 3)]$

(b) $\frac{1}{9}\{3(3T - ST) - (4ST - 6T)\}$

You have learnt to remove brackets in some expressions like $3(m+n) = 3m + 3n$. This process is called **expansion**.

Sometimes the reverse process is required, e.g., in $3m + 3n$, since 3 is a common factor.

Therefore, $3m + 3n = 3(m + n)$.

Copy and complete the following:

(i) $2a + 2b = 2(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

(ii) $4c - 4d = \underline{\hspace{1cm}}(c - d)$

(iii) $3a + 6b = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

(iv) $3a + 6ab - 9a^2b = \underline{\hspace{1cm}}(1 + 2b - 3ab)$

What you have done in (i) – (iv) above is called **factorisation**.

Example 8

Factorise each of the following:

(a) $2a + 4b + 3a + 6b$

(b) $ar^3 + ar^4 + ar^5$

(c) $a^2p^3 - ap^2 + a^3p$

(d) $4x^2y + 20x^4y^2 - 36x^3y$

Solution

$$\begin{aligned} \text{(a)} \quad 2a + 4b + 3a + 6b &= 2a + 3a + 4b + 6b \\ &= 5a + 10b \end{aligned}$$

5 is common to both

$$\therefore 5a + 10b = 5(a + 2b)$$

(b) $ar^3 + ar^4 + ar^5$

ar^3 is common

$$\therefore r^3 + ar^4 + ar^5 = ar^3(1 + r + r^2)$$

(c) $a^3p^3 - ap^2 + a^3p$

 ap is common

$$\therefore a^3p^3 - ap^2 + a^3p = ap(ap^2 - p + a^2)$$

(d) $4x^2y + 20x^4y^2 - 36x^3y$

 $4x^2y$ is common,

$$\therefore 4x^2y + 20x^4y^2 - 36x^3y = 4x^2y(1 + 5x^2y - 9x)$$

Exercise 10.5

Factorise each of the following:

1. (a) $3p + 3r + 3q + 3r$

(b) $4k + 6r + 2s + 2r$

2. (a) $5a - 10b + 5$

(b) $28 - 21w + 14t$

3. (a) $6a + 18b + 27c - 12d$

(b) $8x + 16y - 32k - 64p$

4. (a) $cd^2 - c^2d$

(b) $a^3b + a^2b^2 - ab^3$

5. (a) $6a + 18a^2b - 27ac + 12a^3d$

(b) $4a^2b + 24a^2c - 14a^3d$

6. (a) $4a^3b + 6a^2b - 9ab^2$

(b) $4pqr^2 + 6p^2qr^2 - 2pq^2r^2$

7. (a) $-27p^2q^2 + 6p^3q^5 - 3p^4q^3$

(b) $x^4y^6 + x^3y^5 + 3x^3y^4$

8. (a) $p^2q^2 + p^3q^3$

(b) $28x^3y + 70x^2y^2 - 42xy^3$

9. (a) $\frac{1}{2}a^2 - \frac{1}{2}ab + \frac{1}{8}a$

(b) $\frac{c^2}{2} - \frac{a}{4} + \frac{3c^3}{4}$

10. (a) $\frac{a^2}{3} - \frac{ab^2}{9} + \frac{a}{27}$

11. Express each of the following using symbols and brackets:

(a) The product of two integers which differ by 3.

(b) Multiply the sum of b and c by a .(c) Multiply $h + 3$ by $h - 3$.(d) Subtract $a + b$ from $1 - b$.

12. Find the area of the rectangle given below. The dimensions are in centimetres:

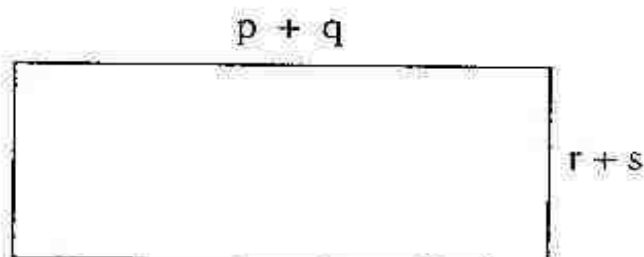


Fig. 10.3

13. Find the perimeter and the area of the figure 10.4:

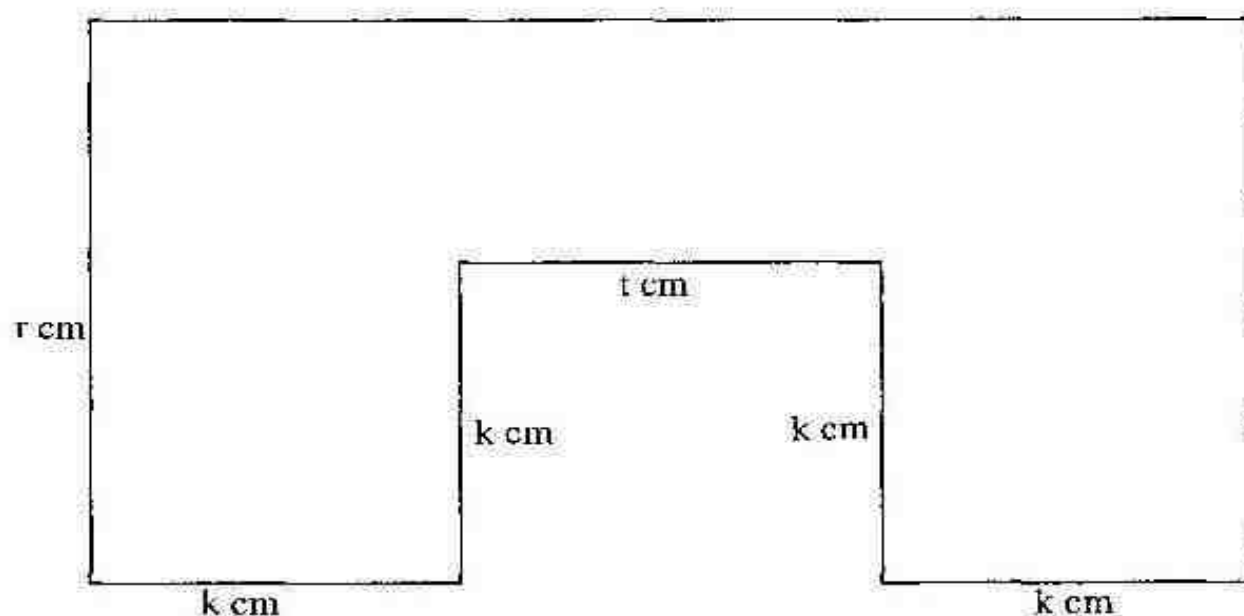


Fig. 10.4

14. An empty basket weighs w kg. Thirty eggs, each weighing p grams, are put in the basket. Find the mass of q such baskets.
15. Charo has sh. p . She buys a book for sh. q and a pen for sh. r . How much is she left with?
16. A man earns p shillings and his wife earns q shillings. They spend $\frac{1}{4}$ of their total earnings on food. How much are they left with?
17. A rectangular piece of cloth is $(x + 5)$ cm by $(x - 1)$ cm. A strip 2 cm wide is cut off all around it. Write an expression for the area of the strip.
18. A square lawn of side y m is surrounded by a path of width 1 m. Write an expression for the area of the path.
19. A car travels at a speed of $(x + 6)$ km/h. What distance does it cover in $(2y - 3)$ hours?
20. George has x books while Kilonzo has three more books than him. Kezia has t books and Edna one less than Kezia. How many books do they have altogether?
21. A father is three times as old as his son. Find an expression for the product of their ages five years ago, if the son is x years old now.
22. In figure 10.5, the outer radius is R cm while the inner radius is r cm. Find an expression for the area of the shaded region. (Leave your answer in terms of π)

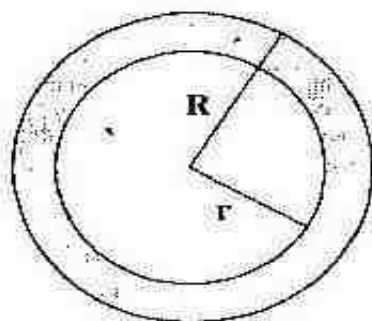


Fig. 10.5

10.4: Factorisation by Grouping

In the previous section, we dealt with factorisation of such expressions as $bx + b = b(x + 1)$. Factorise the following expressions:

- (i) $2x + 6x^2$
- (ii) $7y^2 + 14y + 21$
- (iii) $3mn + 9n + 12n^2$
- (iv) $ax + b + a + bx$

Note:

In (i), (ii) and (iii), there is a common factor while in (iv) there is no common factor. If the terms of the expression $ax + b + a + bx$ are taken pairwise, i.e., $ax + a$ and $bx + b$, then each pair has a common factor.

We can thus factorise the expression $ax + a + bx + b$ as; $a(x + 1) + b(x + 1)$. Now, $x + 1$ is a common factor. Therefore;

$$\begin{aligned} ax + b + a + bx &= a(x + 1) + b(x + 1) \\ &= (x + 1)(a + b) \end{aligned}$$

This method is known as **factorisation** by grouping.

Note that the terms of this expression could be paired differently to obtain the same result. Find out this by factorising the expression anew.

Example 9

Factorise: (a) $3ab + 2b + 3ca + 2c$ (b) $ab + bc - a - c$

Solution

$$\begin{aligned} \text{(a)} \quad 3ab + 2b + 3ca + 2c &= b(3a + 2) + c(3a + 2) \\ &= (3a + 2)(b + c) \\ \text{(b)} \quad ab + bc - a - c &= b(a + c) - 1(a + c) \\ &= (a + c)(b - 1) \end{aligned}$$

Exercise 10.6

Factorise each of the following expressions:

1. (a) $nx - 2n + 3mx - 6m$ (b) $x^2 + xy + 2x + 2y$

2. (a) $3n - 3w + mw - mn$ (b) $3ab - 3bc - 2c + 2a$
 3. (a) $x^2 - xy + 4x - 4y$ (b) $2ab + abk - 2m - mk$
 4. (a) $x^2 + xc + bx + bc$ (b) $xr - ym + yr - xm$
 5. (a) $ay + 3 + y + 3a$ (b) $ef^2 + gf + ef + g$
 6. $ar^2 + ap - 2r^2 - 2p$

10.5: Algebraic Fractions

In algebra, as in arithmetic, fractions can be added and subtracted by finding the L.C.M. of the denominators.

Example 10

Express each of the following as a single fraction:

- (a) $\frac{x-1}{2} + \frac{x+2}{4} + \frac{x}{5}$ (b) $\frac{a+b}{b} - \frac{b-a}{a}$
 (c) $\frac{1}{3(a+b)} + \frac{3}{8(a+b)} + \frac{5}{12a}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{x-1}{2} + \frac{x+2}{4} + \frac{x}{5} &= \frac{10(x-1) + 5(x+2) + 4x}{20} \\ &= \frac{19x}{20} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{a+b}{a} - \frac{b-a}{b} &= \frac{b(a+b) - a(b-a)}{ab} \\ &= \frac{a^2 + b^2}{ab} \end{aligned}$$

$$\text{(c)} \quad \frac{1}{3(a+b)} + \frac{3}{8(a+b)} + \frac{5}{12a}$$

The L.C.M. of 3, 8 and 12 is 24

The L.C.M. of a and $(a+b)$ is $a(a+b)$

$\therefore$ The L.C.M. of $3(a+b)$, $8(a+b)$, and $12a$ is $24a(a+b)$

$$\begin{aligned} \frac{1}{3(a+b)} + \frac{3}{8(a+b)} + \frac{5}{12a} &= \frac{8a + 9a + 10a + 10b}{24a(a+b)} \\ &= \frac{27a + 10b}{24a(a+b)} \end{aligned}$$

Exercise 10.7

1. Find the L.C.M. of:

- (a) $2t, 3t, 5t$ (b) r, s, t
 (c) $2 \times 3, 2^2 \times 7, 3^2 \times 5$ (d) ab, a^2b, b^2a
 (e) $4ab^2, 8a^2b$ (f) $a(2b+c), b(2c+a), c(2a+b), 2ap, 2bp$

Express each of the following as a single fraction in its lowest form:

2. (a) $\frac{x+1}{2} + \frac{x-1}{3}$

(b) $\frac{2a^2+ab}{ab} - \frac{3a^2-ab}{6ab}$

3. (a) $\frac{m}{3} + \frac{x-1}{2} + \frac{x}{6}$

(b) $\frac{3a^2+ab}{6(a+b)} + \frac{a}{3}$

4. (a) $\frac{p+q}{3} + \frac{p-2q}{5}$

(b) $\frac{1}{2} + \frac{3a+b}{2a+2b}$

5. (a) $\frac{2r-3}{4} + \frac{1-r}{3}$

(b) $\frac{2}{a^2} - \frac{1}{c^2d}$

6. (a) $\frac{3x-6}{4} - \frac{2x-2}{3}$

(b) $\frac{1}{cd^2} - \frac{1}{c^2d}$

7. (a) $\frac{1+v}{u} + \frac{1-u}{v}$

(b) $\frac{1}{a^2bc} + \frac{a+b^2}{ab^2c} + \frac{1}{abc^2}$

8. (a) $\frac{r+s}{r} + \frac{r+s}{s}$

(b) $\frac{1}{ab} + \frac{a+b}{a^2b+ab^2}$

9. (a) $\frac{p+q}{t} - \frac{p-q}{s}$

(b) $\frac{6ab-2ab^2}{2a+2ab} + \frac{8b}{2a+2b}$

10. (a) $\frac{p+q}{p} + \frac{p-q}{q}$

(b) $\frac{a^2b}{4ab} + \frac{b^2a}{4ab} + \frac{3}{4}$

11. (a) $\frac{2r+t}{t} + \frac{r+1}{r}$

(b) $\frac{4p^3r}{2pr-3r^2p} - \frac{2r^2p}{2pr-3r^2p}$

10.6: Simplification by Factorisation

One of the uses of factorisation is simplification of given expressions.

Example 11

Simplify:

(a) $\frac{ra+rb}{ma+mb}$

(b) $\frac{ax-ay+bx-by}{a+b}$

(c) $\frac{ay-ax}{bx-by}$

Solution

$$\begin{aligned} \text{(a)} \quad \frac{ra+rb}{ma+mb} &= \frac{r(a+b)}{m(a+b)} \\ &= \frac{r}{m} \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{ax-ay+bx-by}{a+b} &= \frac{a(x-y)+b(x-y)}{a+b} \\
 &= \frac{(a+b)(x-y)}{a+b} \\
 &= x-y
 \end{aligned}$$

$$\text{(c)} \quad \frac{ay-ax}{bx-by} = \frac{a(y-x)}{b(x-y)} = \frac{a(y-x)}{-b(y-x)} = \frac{a}{-b} = -\frac{a}{b}$$

Note:

$$x - y = -(y - x)$$

Exercise 10.8

Simplify by use of common factors:

$$1. \quad \text{(a)} \quad \frac{x^2 - 4x}{x - 4}$$

$$\text{(b)} \quad \frac{2a^2 + a^3}{2a + a^2}$$

$$2. \quad \text{(a)} \quad \frac{3bx - 3by + 4ax - 4ay}{4a + 3b}$$

$$\text{(b)} \quad \frac{3bx - 3by + 4ax - 4ay}{x - y}$$

$$3. \quad \text{(a)} \quad \frac{18ar - 18am}{9am - 9ar}$$

$$\text{(b)} \quad \frac{4xy - 3x + 8y^2 - 6y}{8y - 6}$$

$$4. \quad \text{(a)} \quad \frac{2m - am - 2y + ay}{2m + 2y - am - ay}$$

$$\text{(b)} \quad \frac{4xy - 3x + 8y^2 - 6y}{x + 2y}$$

$$5. \quad \text{(a)} \quad \frac{x^2 - 4ax - 4a + x}{(x + 1)(4a^2 - ax)}$$

$$\text{(b)} \quad \frac{(x + 1)\{a(x - 1) + b(x - 1)\}}{(1 - x)(ax + bx - a - b)}$$

10.7: Substitution

The process of giving variables specific values in an expression is known as **substitution**.

Consider the rectangle below. Its area is $(a + 3)(b + 2) \text{ cm}^2$:

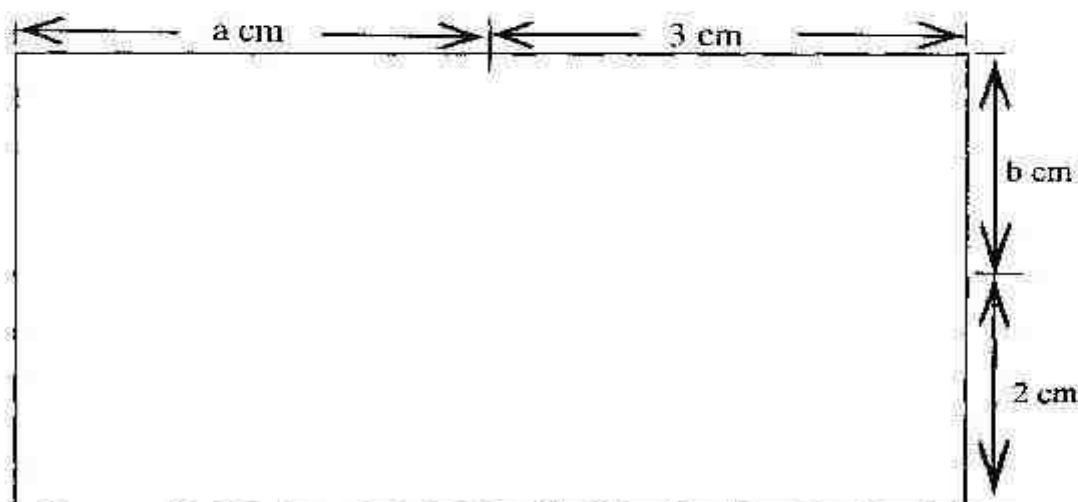


Fig. 10.6

If $a = 4$ and $b = 1$, then the area of the rectangle becomes $(4 + 3)(1 + 2) = 7 \times 3 = 21 \text{ cm}^2$

If $a = 10$ and $b = 7$ then the new area is $(10 + 3)(7 + 2) = 13 \times 9 = 117 \text{ cm}^2$

Example 12

Evaluate the expression $\frac{x^2 + y^2}{y + 2}$ if $x = 2$ and $y = 1$.

Solution

$$\begin{aligned}\frac{x^2 + y^2}{y + 2} &= \frac{2^2 + 1^2}{1 + 2} \\ &= \frac{4 + 1}{3} \\ &= \frac{5}{3} \\ &= 1\frac{2}{3}\end{aligned}$$

Exercise 10.9

1. Evaluate:

- (a) $a^2 - b^3$ when: (i) $a = 1, b = 2$ (ii) $a = 3, b = 1$
 (iii) $a = 0.2, b = 0.3$
 (b) $a^2 - ab$ when: (i) $a = 2, b = 1$ (ii) $a = 3, b = 0$
 (iii) $a = 3, b = 3$ (iv) $a = \frac{1}{16}, b = \frac{1}{4}$

2. If $r = 5$, $s = 2$, and $t = 3$, find the value of:

- (a) $r^2 + s^3 - t$ (b) $\frac{r}{s} + \frac{s}{t} + \frac{t}{r}$
 (c) $\frac{r}{s+t} - \frac{s}{t+r} - \frac{t}{r+s}$ (d) $\frac{s+t}{r}$
 (e) $\frac{r+s}{t} + 2s$ (f) $\frac{2s-r}{s} - \frac{3}{5}t$

3. If $A = (R^2 - r^2)$, find A when:

- (a) $R = 19$ and $r = 11$.
 (b) $R = 0.6$ and $r = 0.2$.

4. If $E = \frac{1}{2}mv^2$, find m when $E = 30$ and $v = 2$.

5. If $V = \frac{1}{3}l \times b \times h$, find:

- (a) h if $lb = 8$ and $V = 24$.
 (b) b if $lh = 12$ and $V = 96$.

6. The surface area of an open box of side a , b and c centimetres is given by $A = 2b(a + c) + ac$.

(a) Find A if $a = 40$, $b = 30$ and $c = 20$.

(b) Find c if $A = 512$, $a = 6$ and $b = 10$.

Mixed Exercise 1

1. Simplify each of the following expressions:

(a) $\frac{1}{a} - \frac{a}{b} + \frac{3}{a} + \frac{4b}{a}$

(b) $a^2b + b^2a + 3ba^2 - 3b^2a + b^3a + a^3b$

(c) $\frac{\frac{1}{c} + \frac{1}{d}}{c+d}$

(d) $\frac{3a^3r}{4r^2a^2}$

(e) $\frac{a-3}{(3+a)(3-a)}$

(f) $\frac{3a}{2b} + \frac{4a}{3b} + \frac{5c}{4b}$

(g) $3q - 3 - (1 + q) + 1 - q - \frac{1}{q}$

(h) $\frac{x-2y}{12p} - \frac{x+3y}{60p}$

2. Three girls share an amount of money. The eldest gets $\frac{b}{3c}$ of the total amount while the youngest gets $\frac{b}{6c}$ of the total. What fraction does the third girl get?

3. Find the sum of a third of $(a + b)$ and a fifth of $(a - b)$.

4. After the tenth month of the year, what fraction of the year still remains?

5. A rectangular field is 0.4 m longer than it is wide. If its length is 6m find its perimeter. When the breadth of the rectangle is reduced by 0.5 m, the length is increased such that the perimeter is increased by $\frac{1}{4}$ of its original. What is the change in the length of the rectangle?

6. (a) Convert each of the following into a decimal:

(i) $\frac{7}{8}$

(ii) $1\frac{1}{8}$

(iii) $5\frac{3}{7}$

(iv) $\frac{11}{9}$

(b) Convert each of the following into a fraction:

(i) 0.375

(ii) 0.84

(iii) 2.4

(iv) 0.275

(c) Copy and complete each of the following:

(i) $\frac{3}{8} = \frac{\quad}{2} = \frac{-15}{-40} = \frac{\quad}{40}$

(ii) $\frac{13}{4} = \frac{5.2}{1.6} = \frac{0.65}{\quad}$

7. The internal height of a rectangular box is 10.5 cm. The thickness of the bottom is $\frac{3}{5}$ cm and the thickness of the top is 1 cm. What is the external height of the box?

8. Three cisterns in a public lavatory are designed to flush at intervals of 8, 13 and 15 seconds. After how many minutes will they flush together?
9. Three-fifths of work is done on the first day. On the second day, $\frac{3}{4}$ of the remainder is completed. If third day $\frac{7}{8}$ of what remained is done, what fraction of work still remains to be done?
10. Juma spent half of his July salary on school fees, one-eighth on farming and two-thirds of the remainder on food. Calculate his July salary if he spent sh. 3200 on food.
11. A farmer has 3 containers of capacity 48 litres, 36 litres and 27 litres. Find the capacity of:
 - (a) the smallest container that can be filled by each one of them an exact number of times.
 - (b) the largest container that can be used to fill each one of them an exact number of times.
12. Onyango buys p oranges and discover that 1% are bad. How many oranges are fit for consumption?
13. Evaluate:

$$3\frac{7}{8} + \sqrt{\left(\frac{3\frac{7}{8} \div 7\frac{3}{4}}{5\frac{1}{10} + 2\frac{9}{10}} \right)}$$

14. Use tables of squares to evaluate:

$$(a) \quad 6250^2 \div 0.1750^2$$

$$(b) \quad 0.0225^2 \times 12800^2$$

$$(c) \quad \frac{(0.706 \times 20.5)^2}{32.2}$$

$$(d) \quad \frac{23.5^2 \times 0.701^2}{3.4}$$

15. Find the length of a square whose area is 0.0084 m^2 .
16. A foreign government donated sh. 67.9 billion while the Kenya Government contributed sh. 200 million towards a project. Of the total amount sh. 10.8 million was used to remunerate experts, sh. 670 000 for the purchase of stationery and sh. 12.8 million for the acquisition of machinery. How much money remained unused? (Express your answer in words)

RATE, RATIO, PROPORTION AND PERCENTAGE

11.1: Rates

A rate is a way of comparing one quantity with another of a different kind. If a car takes two hours to travel a distance of 160 km, then we will say that it is travelling at an average rate of 80 km per hour. If two kilograms of maize meal is sold for sh. 38.00, then we say that maize meal is selling at the rate of sh. 19.00 per kilogram.

Example 1

A labourer's wage is sh. 240 per eight-hour working day. What is the rate of payment per hour?

Solution

$$\begin{aligned}\text{Rate} &= \frac{\text{amount of money paid}}{\text{number of hours}} \\ &= \frac{240}{8} \\ &= \text{sh. 30 per hour}\end{aligned}$$

Example 2

What is the rate of consumption per day if twelve bags of beans are consumed in 120 days?

Solution

$$\begin{aligned}\text{Rate of consumption} &= \frac{\text{number of bags}}{\text{number of days}} \\ &= \frac{12}{120} \\ &= \frac{1}{10} \text{ bag per day}\end{aligned}$$

Exercise 11.1

1. Simplify each of the following rates as indicated:
 - (a) A 100 m dash in 10 s as metres per second.
 - (b) 168 km covered in 1 h 36 min as kilometres per hour.
 - (c) A tax of sh. 225 on an income of £ 75 as shillings per pound.

- (d) A commission of sh. 400 for £ 250 of sales as shillings per pound.
 - (e) A loss of 3 kg of body mass in 12 months as kg per month.
 - (f) A fee of sh. 600 to fill four teeth as charge per tooth.
 - (g) A fee of sh. 12 000 for three school terms as fees per term.
 - (h) A wage of sh. 920 for eight hours work as earning per hour.
2. A factory produced 4 200 rolls of barbed wire in a 5-day working week. What was the rate of production of rolls of wire per day?
 3. A train took two hours to travel a distance of 69 km. What was its average speed?
 4. A typist can type 4 800 words in one hour. What is her rate of typing per minute?
 5. A tenant paid sh. 36 000 to his landlord in one year. What was his rate of payment per month?
 6. A car travels 160 km on 20 litres of petrol. How far can it go on 12 litres of petrol?
 7. Convert a speed of 60 km/h to m/s.
 8. A farmer harvested 200 bags of wheat from 2 ha of his farm. How many bags of wheat would he harvest from 16 ha if he maintained the rate?

11.2: Ratio

A **ratio** is a way of comparing two similar quantities. For example, if Ali is 10 years old and his brother Bashir 14 years old, then Ali's age is $\frac{10}{14}$ of Bashir's age and their ages are said to be in the ratio of 10 to 14, written 10 : 14.

Ali's age : Bashir's age = 10 : 14

Bashir's age : Ali's age = 14 : 10

In stating a ratio, the units must be the same. If on a map 2 cm represents 5 km on the actual ground, then the ratio of map distance to ground distance is 2 cm : 5 × 100 000 cm, which is 2 : 500 000.

A ratio is expressed in its simplest form in the same way as a fraction,

e.g. $\frac{10}{14} = \frac{5}{7}$, hence 10 : 14 = 5 : 7.

Similarly, 2 : 500 000 = 1 : 250 000.

A **proportion** is a comparison of two or more ratios. If, for example, a, b and c are three numbers such that a : b : c = 2 : 3 : 5, then a, b, c are said to be proportional to 2, 3, 5 and the relationship should be interpreted to mean

$\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$. Similarly, we can say that a : b = 2 : 3, b : c = 3 : 5 and a : c = 2 : 5.

Example 3

If $a : b = 3 : 4$ and $b : c = 5 : 7$, find $a : c$.

Solution

$$a : b = 3 : 4 \quad \dots\dots\dots (i)$$

$$b : c = 5 : 7 \quad \dots\dots\dots (ii)$$

Consider the right hand side;

Multiply (i) by 5 and (ii) by 4 to get, $a : b = 15 : 20$ and $b : c = 20 : 28$

Thus, $a : b : c = 15 : 20 : 28$ and $a : c = 15 : 28$

Exercise 11.2

1. Write each of the following ratios in its simplest forms:

(a) $\frac{1}{2} : \frac{3}{5}$

(b) $\frac{1}{3} : \frac{4}{9}$

(c) $0.2 : 0.8$

(d) 12 cm to 48 cm

(e) 25 cm to 1.5 m

(f) 36 min to 1 h

(g) 35 cents to sh. 1

(h) $2\frac{1}{2} : 7\frac{1}{2}$

(i) $8 \text{ cm}^2 : 1 \text{ m}^2$

(j) sh. 48 : K£3

(k) $2 \text{ cm}^3 : 8 \text{ cm}^3$

(l) 2.5 : 10 000

(m) 0.03 : 0.009

(n) 2 km : 300 m

(p) 25 g : 1 kg

(q) 2 ha : 400 m^2

(r) $3 \text{ cm}^2 : 100 \text{ mm}^2$

(s) 3.8 l : 19 l

(t) 45 km/h : 75 km/h

(u) 4 days : 1 week

2. Find the ratio $a : c$ if:

(a) $a : b = 2 : 5$, $b : c = 5 : 3$

(b) $a : b = 1 : 4$, $b : c = 1 : 5$

(c) $a : b = 3 : 1$, $b : c = 3 : 1$

(d) $a : b = 3 : 5$, $b : c = 6 : 5$

(e) $a : b = 1 : 2$, $b : d = 4 : 5$, $d : c = 3 : 1$

(f) $a : k = 1 : 5$, $k : c = 1 : 9$

(g) $a : x = 3 : 1$, $x : 2 = 4 : 1$, $2 : c = 2 : 1$

(h) $a : b = 7 : 1$, $b : d = 1 : 2$, $d : c = 2 : e$

3. Find the value of x which makes the following ratios equal

(a) $2 : 3 = x : 9$

(b) $1\frac{1}{2} : 3 = 3\frac{1}{2} : x$

(c) $7 : 5 = 35 : x$

(d) $0.3 : 0.1 = x : 11$

(e) $150 : x = 25 : 3$

(f) $600 : x = 20 : 7$

4. The ratio of boys to girls in a mixed school is 2 : 3. If there are 160 boys, how many girls are there?

5. An alloy is to be made by combining copper and aluminium in the ratio 3 : 8. If there is 39 kg of copper, how much of aluminium is required to make the alloy?

11.3: Increase and Decrease in a given Ratio

To increase or decrease a quantity in a given ratio, we express the ratio as a fraction and multiply it by the quantity.

Example 4

Increase 20 in the ratio 5 : 4.

Solution

$$\begin{aligned}\text{New value} &= \frac{5}{4} \times 20 \\ &= 5 \times 5 \\ &= 25\end{aligned}$$

Example 5

Decrease 45 in the ratio 7 : 9.

Solution

$$\begin{aligned}\text{New value} &= \frac{7}{9} \times 45 \\ &= 7 \times 5 \\ &= 35\end{aligned}$$

Example 6

The price of a pen is adjusted in the ratio 6 : 5. If the original price was sh. 50, what is the new price?

Solution

New price : old price = 6 : 5

$$\begin{aligned}\frac{\text{New price}}{\text{Old Price}} &= \frac{6}{5} \\ \therefore \text{New price} &= \frac{6}{5} \times 50 \\ &= \text{sh. } 60\end{aligned}$$

Note:

When a ratio expresses a change in a quantity an (increase or decrease), it is usually put in the form; new value : old value

Exercise 11.3

- Increase the given number in the ratio in brackets:

(a) 20 (3 : 2)	(b) 15 (6 : 5)
(c) 125 (5 : 1)	(d) 120 (4 : 3)

2. Decrease the given number in ratio in brackets.
(a) 100 (2 : 5) (b) 450 (5 : 9)
(c) 160 (5 : 8) (d) 49 (3 : 7)
3. A trader plans to increase prices in the ratio 7 : 6. What will be the new price of an iron box which is marked at sh. 1 800?
4. When meat is fried, its mass reduces in the ratio 5 : 12. A piece of uncooked meat has a mass of 4.8 kg. What mass is lost when it is fried?
5. A photograph is reduced in the ratio 3 : 5 for a newspaper, and further reduced in the ratio 4 : 5 for a textbook. Find the ratio of the newspaper size to the textbook size.

11.2: Comparing Ratios

In order to compare ratios, they have to be expressed as fractions first, i.e., $a : b = \frac{a}{b}$. The resultant fractions can then be compared.

Example 7

Which ratio is greater, 2 : 3 or 4 : 5?

Solution

$$2 : 3 = \frac{2}{3}, 4 : 5 = \frac{4}{5}$$

$$\therefore \frac{2}{3} = \frac{10}{15}, \frac{4}{5} = \frac{12}{15} \Rightarrow \frac{4}{5} > \frac{2}{3}$$

Thus, $4 : 5 > 2 : 3$

Alternatively, ratios can be compared by writing them in the form $n : 1$

Example 8

Which of the fraction is greater, 6 : 5 or 11 : 10?

Solution

6 : 5 in the form $n : 1 = 1.2 : 1$ (divide through by 5)

11 : 10 in the form $n : 1$ is $1.1 : 1$ (divide through by 10)

Thus, 6 : 5 is greater than 11 : 10.

Exercise 11.4

1. Which of the following fractions is greater:
(a) 2 : 3 or 1 : 4? (b) 4 : 7 or 2 : 5?
(c) 3 : 5 or 6 : 11? (d) 8 : 9 or 10 : 11?
(e) 5 : 2 or 20 : 17? (f) 3 : 2 or 5 : 4?
2. Determine which of the following ratios is greater than the other by expressing them in the form $n : 1$.
(a) 3 : 4 and 5 : 6 (b) 5 : 8 and 4 : 7
(c) 9 : 4 and 8 : 3 (d) 16 : 6 and 14 : 4

11.5: Distributing a Quantity in a given Ratio

If a quantity is to be divided in the ratio $a : b : c$, the fraction of the quantity represented by:

(i) a will be $\frac{a}{a+b+c}$

(ii) b will be $\frac{b}{a+b+c}$

(iii) c will be $\frac{c}{a+b+c}$

Example 9

A 72-hectare farm is to be shared among three sons in the ratio $2 : 3 : 4$. What will be the sizes in hectares of the three shares?

Solution

The total number of parts is $2 + 3 + 4 = 9$

The shares are: $\frac{2}{9} \times 72 \text{ ha} = 16 \text{ ha}$

$$\frac{3}{9} \times 72 \text{ ha} = 24 \text{ ha}$$

$$\frac{4}{9} \times 72 \text{ ha} = 32 \text{ ha}$$

Exercise 11.5

1. Divide sh. 150 in the ratio:
(a) $1 : 2$ (b) $1 : 4$ (c) $2 : 3$
(d) $3 : 5$ (e) $1 : 1$ (f) $7 : 8$
2. Divide 2 320 ha in the ratio:
(a) $2 : 5$ (b) $1 : 3$
(c) $1 : 5$ (e) $1 : 6$
3. The angles of a quadrilateral are in the ratio $6 : 4 : 3 : 2$. Calculate the sizes of the angles.
4. The profit of a firm divided between new plant, reserves and dividends are in the ratio $3 : 7 : 9$. If the profit is sh. 380 000, what is the amount put to reserve?

11.6: Direct and Inverse Proportion

Direct Proportion

The table below shows the cost of various numbers of cups at sh. 20 per cup:

No. of cups	1	2	3	4	5
Cost (sh.)	20	40	60	80	100

The ratio of the number of cups in the fourth column to the number of cups in the second column is $4 : 2 = 2 : 1$. The ratio of the corresponding costs is $80 : 40 = 2 : 1$. By considering the ratio of costs in any two columns and the corresponding ratio of the number of cups, you should notice that they are always the same.

If two quantities are such that when one increases (decreases) in a particular ratio, the other one also increases (decreases) in the same ratio, they are said to be **directly proportional**.

Inverse Proportion

The table below shows the amount of money each pupil gets when sh. 120 is shared out among varying numbers of pupils:

No. of pupils	1	2	3	4	5
Amount of money	120	60	40	30	24

The ratio of the number of pupils in the fourth column to that in the second column is $4 : 2 = 2 : 1$. The ratio of the corresponding amounts is $30 : 60 = 1 : 2$. By considering ratios of corresponding pairs in any two columns, you should notice that the ratio of the number of pupils is the inverse of the ratio of the amount per pupil.

If two quantities are such that when one increases (decreases) in the ratio $a : b$ the other one decreases (increases) in the ratio $b : a$, they are said to be **inversely proportional**.

Example 10

A car travels 40 km on 5 litres of petrol. How far does it travel on 12 litres of petrol?

Solution

Petrol is increased in the ratio 12 : 5.

$$\begin{aligned}\text{Distance} &= 40 \times \frac{12}{5} \text{ km} \\ &= 96 \text{ km}\end{aligned}$$

Example 11

A train takes 3 hours to travel between two stations at an average speed of 40 km per hour. At what average speed would it need to travel to cover the same distance in 2 hours?

Solution

Time is decreased in the ratio 2 : 3.

Speed must be increased in the ratio 3 : 2.

$$\text{Average speed is } 40 \times \frac{3}{2} \text{ km} = 60 \text{ km/h}$$

Example 12

Ten men working six hours a day take 12 days to complete a job. How long will it take eight men working 12 hours a day to complete the same job?

Solution

Number of men decreases in the ratio 8 : 10.

Therefore, the number of days taken increases in the ratio 10 : 8.

Number of hours have increased in the ratio 12 : 6.

Therefore, number of days decreases in the ratio 6 : 12.

$$\begin{aligned}\text{Number of days taken} &= 12 \times \frac{10}{8} \times \frac{6}{12} \\ &= 7\frac{1}{2} \text{ days}\end{aligned}$$

Exercise 11.6

1. When a number of oranges is divided among six girls, each gets four oranges. How many oranges would each girl get if there were eight girls?
2. As a result of a transfer from one school to another, a boy has to walk one and a half times the distance he was walking from home to school. If he walks at the same pace, how long will he take to walk to school if he was taking 43 minutes previously?
3. A farmer takes 8 days to cultivate a plot of land $1\frac{1}{2}$ ha in area. What area of land will he cultivate in 20 days working at the same rate?
4. The price of milk is raised from sh. 40.50 to sh. 45.00 per litre. Calculate the new monthly bill of milk for a man who was spending sh. 1 215 per month if his milk consumption remains the same.

5. After selling 200 magazines, a vendor gets a commission of sh. 840. The vendor gets the same commission for selling 280 newspapers. How much would he get for selling 300 magazines and 350 newspapers if his commission is directly proportional to the number of items?
6. Curtain material 15 m long costs sh. 2 850. What is the cost of a similar material which is 2.4 m longer?
7. John paid for his gas cooker for 18 months at the rate of sh. 800 per month. How many months would he have taken to pay for the gas cooker had he increased his monthly payment by sh. 400?
8. Thirty six men can cultivate a piece of land in 20 days. How many more men, working at the same rate, would be needed to cultivate the same piece of land in 15 days?
9. Twelve tailors can make uniforms for a firm in three days. How long would it take eight tailors to make the same uniforms if they worked at the same rate?
10. A mother had enough money to buy 24 bananas at sh. 4.50 a banana. How many would she buy with the same money if the price of a banana was 50 cents less?
11. It takes 30 minutes for a man to travel distance at 12 km/h. How long will he take to cover the same distance at 18 km/h?
12. A farmer has enough feed to last his 15 pigs for 20 days. How long would the feed last if he had 10 pigs?
13. Four men can till a piece of land in six days. How long would it take two men to till the same piece of land?
14. Okuku's uses 60 bottles of milk per month. As a result of the price of milk being raised from sh. 20 to sh. 22 per bottle, he decides to reduce his monthly consumption to 55 bottles. In what ratio does his monthly bill for milk change?
15. Three girls, Rose, Doris and Pauline together scored 207 marks in a test. The ratios of their marks were: Rose to Doris, 3 to 2 and Doris to Pauline, 6 to 8. How many marks did each girl score?
16. Three businessmen, Peter, John and Thomas share a profit of sh. 620 000. Peter gets $3\frac{1}{2}$ times as much as John and John gets $1\frac{1}{2}$ times as much as Thomas. Find the amount of money each gets.
17. Fifteen tractors, each working eight hours a day, take eight days to plough a piece of land. How long would it take 24 tractors, each working 10 hours a day, to plough the same piece of land?

18. Four men can build a stone wall 32 m long in 12 days. What length of wall can eight men, working at the same rate, build in eight days?
19. Three lorries, each making five trips per day, transport 2 500 crates from a factory to a distributor in two days. How many lorries, each making six trips a day, are needed to transport 10 000 such crates in one day?
20. One hundred and fifty examiners, each marking 80 scripts per day, are needed to mark an examination in two weeks. How many days would 200 examiners, each marking 40 scripts a day, take to mark the same examination?
21. Three tractors, each working eight hours a day, can plough a field in five days. How many days would two such tractors, working 10 hours a day, take to plough the same field?

11.7: Percentages

A percentage (written %) is a fraction whose denominator is 100. For example, 27% means $\frac{27}{100}$.

Converting Fractions and Decimals into Percentages

Example 13

Change $\frac{2}{5}$ into a percentage.

Solution

$$\frac{2}{5} = \frac{x}{100}$$

$$\begin{aligned} x &= \frac{2}{5} \times 100 \\ &= 40\% \end{aligned}$$

Note:

To convert a fraction to a percentage, we multiply it by 100. A decimal is a fraction whose denominator is a multiple of 10.

Example 14

Convert 0.67 into a percentage.

Solution

$$0.67 = \frac{67}{100}$$

$$\begin{aligned} \text{As a percentage, } 0.67 &= \frac{67}{100} \times 100 \\ &= 67\% \end{aligned}$$

Note:

The same can be achieved by multiplying 0.67 by 100.

Exercise 11.7

- Convert each of the following decimals into a percentage:

(a) 0.32	(b) 0.02	(c) 0.167
(d) 0.88	(e) 3.2	(f) 0.275
(g) 0.428	(h) 1.25	(i) 2.73
(j) 25.23	(k) 33.45	(l) 98.92
- Convert each of the following fractions into percentage:

(a) $\frac{1}{2}$	(b) $\frac{1}{5}$	(c) $\frac{3}{7}$
(d) $1\frac{5}{6}$	(e) $\frac{6}{8}$	(f) $\frac{1}{3}$
(g) $1\frac{5}{8}$	(h) $2\frac{2}{3}$	(i) $3\frac{3}{4}$
- Convert each of the following percentages into a fraction:

(a) 120%	(b) 200%	(c) 40%
(d) 25%	(e) 60%	(f) 74%
- Convert each of the following percentages into a decimal:

(a) 75%	(b) 23%	(c) 147%
(d) 8%	(e) 0.4%	(f) 0.13%
(g) 0.03%	(h) 0.072%	(i) 0.805%

11.8: Percentage Increase and Decrease

A quantity can be expressed as a percentage of another by first writing it as a fraction of the given quantity.

Example 15

A farmer harvested 250 bags of maize in one season. If he sold 200 bags, what percentage of his crop does this represent?

Solution

Let x be the percentage sold.

$$\text{Then, } \frac{x}{100} = \frac{200}{250}$$

$$\begin{aligned}\text{So, } x &= \frac{200}{250} \times 100 \\ &= 80\%\end{aligned}$$

Example 16

A man earning sh. 4 800 per month was given a 25% pay rise. What was his new salary?

Solution

$$\begin{aligned}\text{New salary} &= \frac{25}{100} \times 4\,800 + 4\,800 \\ &= 1\,200 + 4\,800 \\ &= \text{sh. } 6\,000\end{aligned}$$

How else could you arrive at the same answer?

Example 17

A dress which was costing sh. 1 200 now goes for sh. 960. What is the percentage decrease?

Solution

Decrease in cost is $1\,200 - 960 = \text{sh. } 240$

$$\begin{aligned}\text{Percentage decrease} &= \frac{240}{1200} \times 100 \\ &= 20\%\end{aligned}$$

Example 15

The ratio of John's earnings to Musa's earnings is 5 : 3. If John's earnings increase by 12%, his new figure becomes sh. 5 600. Find the corresponding percentage change in Musa's earnings if the sum of their new earnings is sh. 9 600.

Solution

John's earnings before the increase is $\frac{100}{112} \times 5\,600 = \text{sh. } 5\,000$

$$\frac{\text{John's earnings}}{\text{Musa's earnings}} = \frac{5}{3}$$

$$\begin{aligned}\text{Musa's earnings before the increase} &= \frac{3}{5} \times 5\,000 \\ &= \text{sh. } 3\,000\end{aligned}$$

$$\begin{aligned}\text{Musa's new earnings} &= 9\,600 - 5\,600 \\ &= \text{sh. } 4\,000\end{aligned}$$

$$\begin{aligned}\text{Musa's change in earnings} &= 4\,000 - 1\,000 \\ &= \text{sh. } 3\,000\end{aligned}$$

$$\begin{aligned}\therefore \text{Percentage change in Musa's earnings} &= \frac{1000}{3000} \times 100 \\ &= 33\frac{1}{3}\%\end{aligned}$$

Exercise 11.8

1. A student pays 20% more for his bus fare from home to school than he used to pay two years ago. If he pays sh. 30, how much was he paying then?
2. A hawker bought a glass for sh. 24 and sold it later for sh. 36. What was his percentage gain?
3. A businesswoman bought cabbages at sh. 8 each and sold them at sh. 12 each. What was her percentage gain per cabbage?
4. Korir's shamba is 12 hectares in area. Five hectares is used for grazing and he plants tea in the rest. What percentage of his shamba is under tea?
5. The price of milk rose from sh. 30 to sh. 36 per bottle. What was the percentage increase in price?
6. In a quality control analysis, 3.5% of all parts of a machine were declared sub-standard. If there were 72 sub-standard parts, how many parts were analysed?
7. What percentage is 0.002 cm of 4 cm?
8. A mason was paid sh. 24 000 to build a house. What was the total cost of building the house if this amount constituted 20% of the cost?
9. It costs Moses 150% more to rent his house than it used to 10 years ago when the rent was sh. 800 per month. How much rent does he pay now?
10. An insurance firm reduced its annual premiums for a policy by $12\frac{1}{2}\%$ to sh. 850. What was the previous amount of the premium?
11. A rectangle measures 18 cm by 12 cm.
 - (a) If each dimension is reduced by 2 cm, by what percentage is:
 - (i) the perimeter, and
 - (ii) the area of the rectangle reduced?
 - (b) If each dimension is reduced by 2% by what percentage is:
 - (i) the perimeter and
 - (ii) the area of the rectangle reduced?
12. The price of an article was raised by 20% and week later the new price was lowered by 20%. What was the new price if the original price was sh. 50?
13. The ratio of girls to boys in a school is $3\frac{1}{2} : 4\frac{3}{4}$. What percent of the school do the boys represent?
14. Doris, Pauline and Joan share a number of sweets in the ratio $3\frac{1}{2} : 1\frac{5}{6} : 1\frac{1}{3}$. Calculate the percentage of the sweets each one gets.
15. A man invested sh. 36 000 in two companies P and Q. P pays a dividend of

- $11\frac{1}{4}\%$ while Q pays a dividend of $10\frac{1}{2}\%$. If from his total investment, he obtained a return of $10\frac{3}{4}\%$, how much money did he invest in each company?
6. The enrolment in Wengi Primary School was 1 283 pupils last year. The enrolment this year is 109% of last year's. What is this year's enrolment?

Chapter Twelve

LENGTH

12.1: Introduction

Length is the distance between two points. The SI unit of length is the metre.

Conversion of Units of Length

The table below shows the conversions of units of length:

1 kilometre (km)	=	1 000 metres
1 hectometre (Hm)	=	100 metres
1 decametre (Dm)	=	10 metres
1 decimetre (dm)	=	$\frac{1}{10}$ metre
1 centimetre (cm)	=	$\frac{1}{100}$ metre
1 millimetre (mm)	=	$\frac{1}{1000}$ metre

The following prefixes are often used when referring to length:

Mega	–	1 000 000
Kilo	–	1 000
Hecto	–	100
Deca	–	10
Deci	–	$\frac{1}{10}$
Centi	–	$\frac{1}{100}$
Milli	–	$\frac{1}{1 000}$
Micro	–	$\frac{1}{1 000 000}$

12.2: Significant Figures

The accuracy with which we state or write a measurement may depend on its relative size. It would be unrealistic to state the distance between towns A and B as 158.27 km. A more reasonable figure is 158 km.

158.27 km is the distance expressed to 5 significant figures and 158 km to 3 significant figures.

Example 1

Express each of the following numbers to 5, 4, 3, 2 and 1 significant figures:

- (a) 906 315 (b) 0.085641
(c) 40.0089 (d) 156 000

Solution

	Number	5 s.f.	4 s.f.	3 s.f.	2 s.f.	1 s.f.
(a)	906 315	906 320	906 300	906 000	910 000	900 000
(b)	0.085641	0.085641	0.08564	0.0856	0.086	0.09
(c)	40.0089	40.009	40.01	40.0	40	40
(d)	156 000	156 000	156 000	156 000	160 000	200 000

The above examples show how we would round off a measurement to a given number of **significant figures**.

Zero may or may not be significant. For example;

- (i) 0.085 – Zero is not significant. Therefore, 0.085 is a two-significant figure.
(ii) 2.30 – Zero is significant. Therefore 2.30 is a three-significant figure
(iii) 5000 – Zero may or may not be a significant figure. Therefore, 5 000 to three significant figure is 5 000 (zero after 5 is significant). 5 000 to one significant figure is 5 000. Zero after 5 is not significant.
(v) 31.805 or 305 – Zero is significant. Therefore, 31.805 is a five significant figure, 305 is a three significant figure.

Exercise 12.1

1. Copy and complete the table below:

	<i>km</i>	<i>Hm</i>	<i>Dm</i>	<i>m</i>	<i>dm</i>	<i>cm</i>	<i>mm</i>
a				250			
b	15						
c						135	
d							4 400
e				0.03			
f							1 000 000
g	0.47						
h							0.72
i			275				
j					805	.	

2. Add the following and express your answer in metres:

- (a) 2 km, 4 m
 (b) 8 Hm 8 m, 16 m
 (c) 17 m 12 cm, 78 mm
 (d) 178 cm 8 mm, 76 cm
 (e) 40 dm, 1 m, 13 dm
 (f) 25 Dm, 2.5 m, 96 dm
 (g) 0.5 Dm, 30.6 Hm, 120 mm
 (h) 4 m 27 dm, 3 Dm, 2 dm, 4 m 42 cm
 (i) 12 m 35 cm, 7 m 20 dm
 (j) 15 Dm, 12 cm, 2 m 15 cm, 8 m 18 dm 16 mm
 (k) 1.3 km, 3.8 Hm, 3.65 dm
 (l) 4 km, 4 cm 4 mm

3. Subtract the first quantity from the second, giving your answer in metres

- (a) 355 m 16 mm, 1 km 2 Dm
 (b) 407 dm 38 cm, 6 Hm 22 mm
 (c) 42.6 cm, 0.35 km
 (d) 5 431 cm, 5160 m
 (e) 56 cm, 1 m 14 cm
 (f) 73 cm, 59 m
 (g) 95 mm, 320 cm
 (h) 150 mm, 24 cm 3 mm
 (i) 1.4 dm, 0.08 m 40 cm
 (j) 0.001 m, 68 cm
 (k) 6.8 mm, 34 m
 (l) 0.8 m, 4 m 7 mm

4. Express each of the following to 4, 3, 2, and 1 significant figures:

- (a) 125 689
 (b) 3.647
 (c) 36.0320

- | | | |
|------------|--------------|-------------|
| (d) 89.545 | (e) 3.845 | (f) 906.24 |
| (g) 53.2 | (h) 0.030024 | (i) 736 000 |

5. Express each of the following measurements to:

- | | | |
|-------------|--------------|--------------|
| (i) 3 d.p. | (ii) 3 s.f. | (iii) 2 s.f. |
| (a) 4.013 | (b) 0.3651 | (c) 36.7892 |
| (d) 0.03475 | (e) 200.01 | (f) 0.1072 |
| (g) 0.09854 | (h) 341.0032 | (i) 21.03 |

12.3: Perimeter

The perimeter of a plane figure is the total length of its boundaries. Perimeter is a length and is therefore expressed in the same units as length.

Rectangular Shapes

Figure 12.1 is a rectangle of length 5 cm and breadth 3 cm

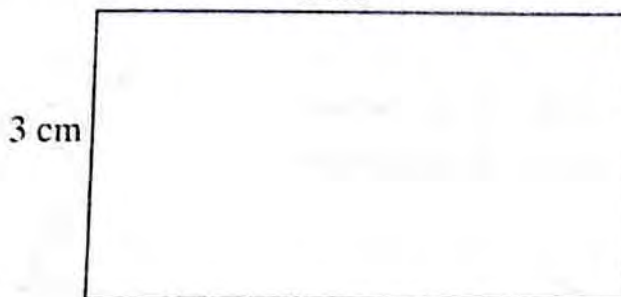


Fig. 12.1

$$\begin{aligned}
 \text{Its perimeter is } 5 + 3 + 5 + 3 &= 2(5 + 3) \text{ cm} \\
 &= 2 \times 8 \\
 &= 16 \text{ cm}
 \end{aligned}$$

In general, if a rectangle has a length of l units and breadth of b units, its perimeter $p = 2(l + b)$

Square Shapes

Figure 12.2 shows a square of length 6 cm.

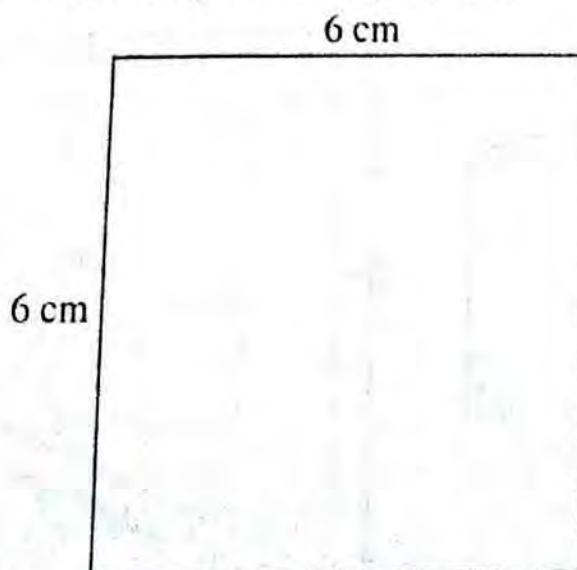


Fig. 12.2

$$\begin{aligned}\text{Its perimeter} &= (6 + 6 + 6 + 6) \text{ units} \\ &= 24 \text{ units}\end{aligned}$$

$$\text{But, } 6 \times 4 = 24$$

Therefore, perimeter of a square can also be expressed as $(l \times 4)$ units, where l = length.

Triangular shapes

To find the perimeter of a triangle, add all the three sides.

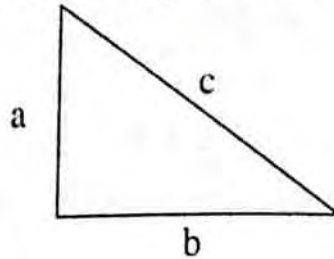


Fig.12.3

Perimeter = $(a + b + c)$ units, where a , b and c are the lengths of the sides of the triangle.

Exercise 12.2

In numbers 1–4, the dimensions are in centimetres

- Find the perimeter of each of the following:

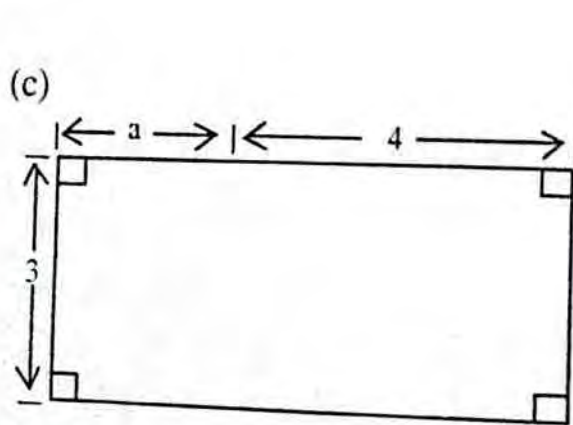
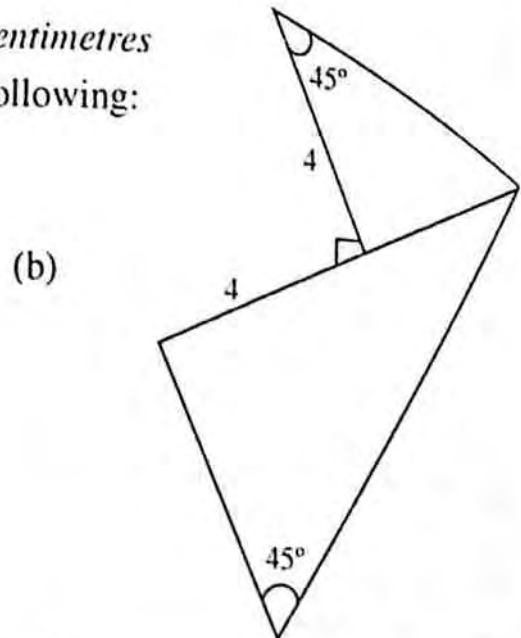
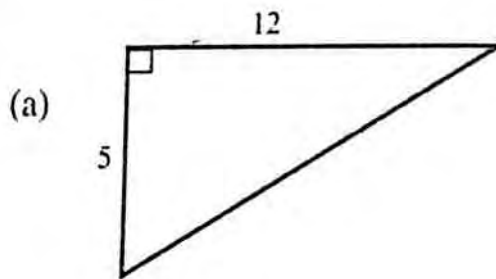
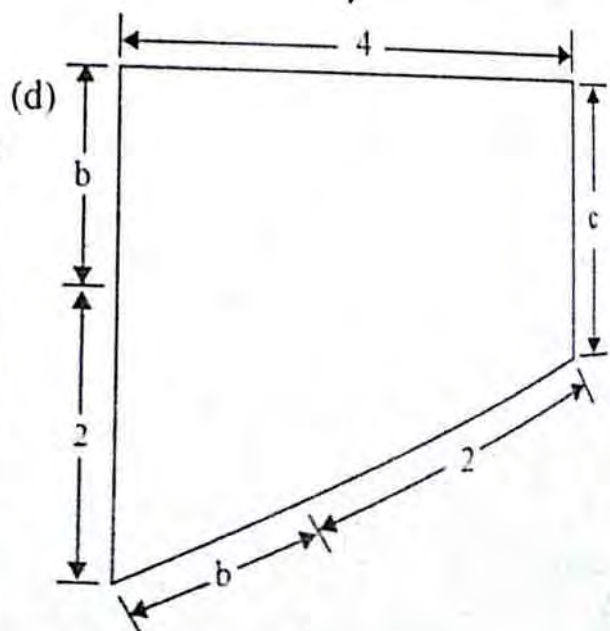


Fig. 12.4



2. Copy and complete the following table for rectangles:

<i>Length</i>	<i>Breadth</i>	<i>Perimeter</i>
15	8	—
—	4	24
—	7.5	40
9	—	63
12.5	—	39.6

3. Find the perimeter of each of the following:

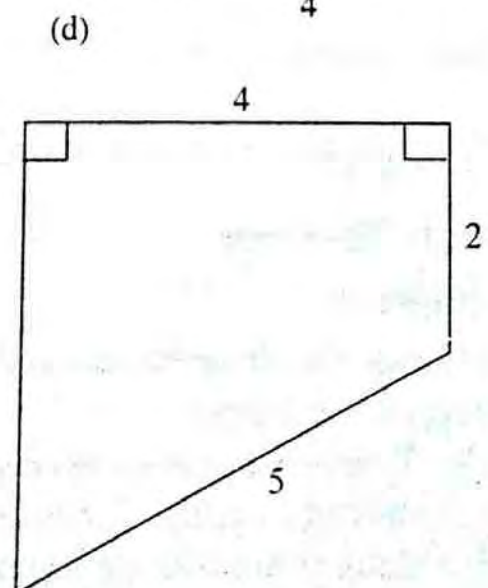
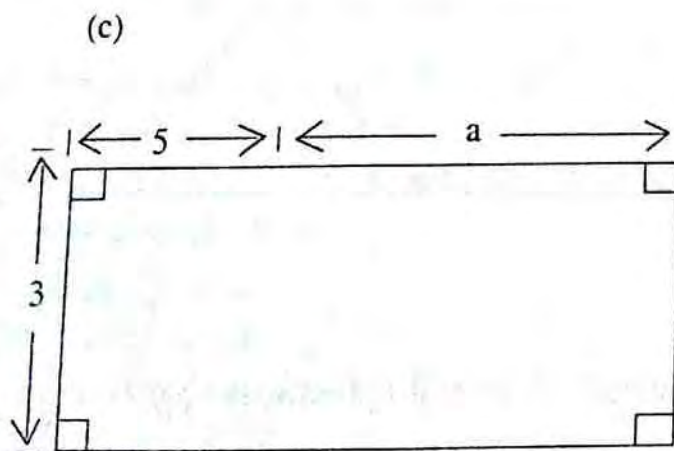
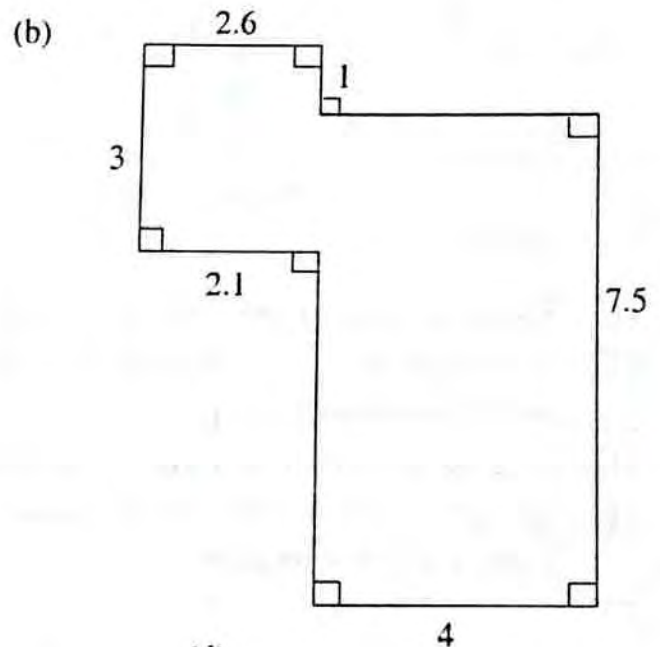
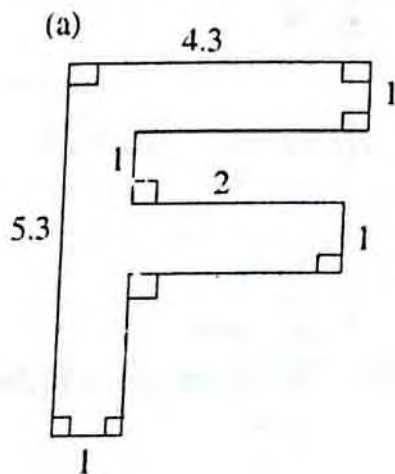


Fig. 12.5

4. How many fencing posts spaced 5 m apart are required to fence a rectangular plot measuring 745 m by 230 m?
5. What is the perimeter of a triangular signboard measuring 46 cm by 42 cm by 38 cm?
6. A rectangular plot measures 100 m by 200 m. Find its perimeter.
7. The perimeter of a parallelogram is 68 cm and one of its side measures 14 cm. Find the length of the other three sides.

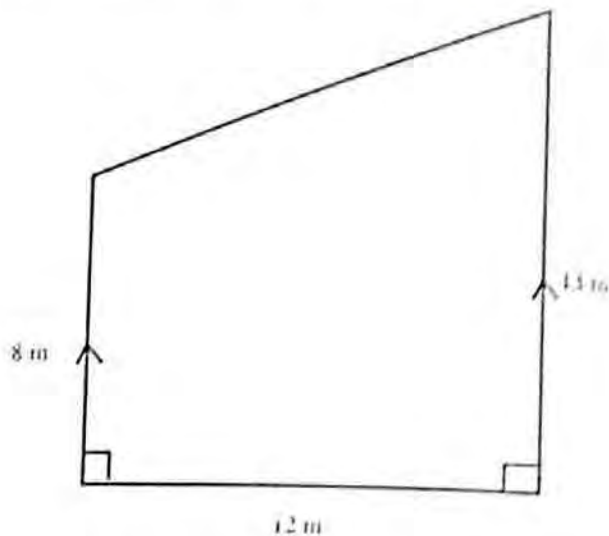


Fig. 12.6

8. Figure 12.6 represents a flower garden. What is its perimeter?
9. The length of a rectangle is three times its breadth. Find its perimeter if the breadth measures b cm.
10. Find the perimeter of a triangle whose sides are x , $2x$ and $x - 5$ cm.
11. In figure 12.7, the trapezium is made up of 3 equilateral triangles of side 6 cm. Find its perimeter.

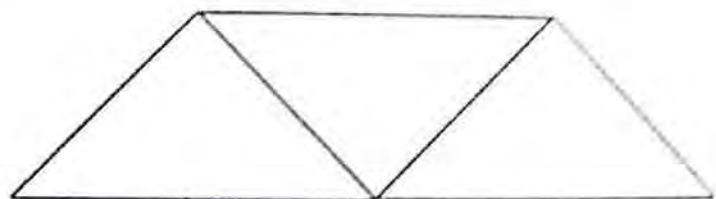


Fig. 12.7

12.4: The Circle

Project

Measure the circumference and diameters of several cylindrical objects, e.g., cups, tins and pipes.

- (i) To obtain the circumference, wrap a strip of paper round the object so that the ends overlap. Make a pin prick, spread out the paper and measure the distance between the two small holes using a ruler.

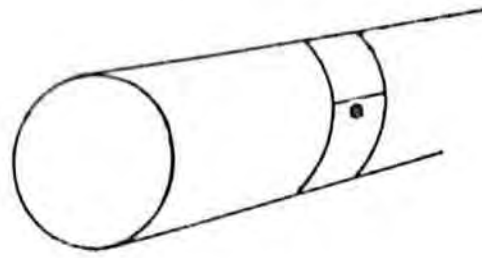


Fig. 12.8

- (ii) To obtain the diameter, place two set squares along the surface of the object, as shown in figure 12.9. Measure AB to obtain the diameter.

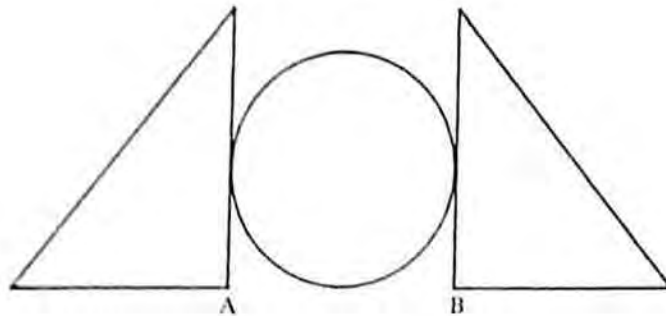


Fig. 12.9

- (iii) Make a table of results as follows:

Object	Circumference (C)	Diameter (d)	$C \div d$
Cup			
Tin			
Pipe			

You should notice from the table that the column $C \div d$ for all cylindrical objects considered gives a value which is approximately 3.14.

The ratio $C \div d$ is constant for all circles. The symbol π (pi) is used to represent this ratio.

$$\text{Thus, } \frac{C}{d} = \pi$$

$$\therefore C = \pi d$$

$$= \pi(2r)$$

$$= 2\pi r$$

The exact value of π is $\frac{22}{7}$.

Example 2

- (a) Find the circumference of a circle of radius 7 cm.
(b) The circumference of a bicycle wheel is 140 cm. Find its radius.

Solution

$$\begin{aligned}\text{(a) } C &= \pi d \\ &= \frac{22}{7} \times 2 \times 7 \\ &= 44 \text{ cm}\end{aligned}$$

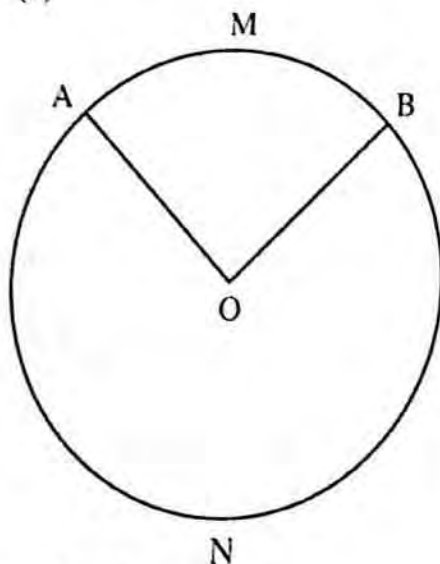
$$\begin{aligned}\text{(b) } C &= \pi d \\ &= 2\pi r \\ &= 2 \times \frac{22}{7} \times r\end{aligned}$$

$$\begin{aligned}\therefore r &= 140 \div \frac{44}{7} \\ &= 22.27 \text{ cm}\end{aligned}$$

Length of an Arc

An arc of a circle is part of its circumference. Figure 12.10 (a) shows two arcs, AMB and ANB. Arc AMB, which is less than half the circumference of the circle, is called the **minor arc**, while arc ANB, which is greater than half the circumference is called the **major arc**. An arc which is half the circumference of a circle is called a **semicircle**.

(a)



(b)

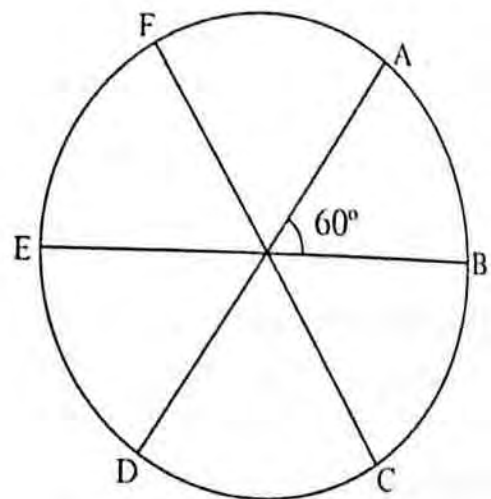


Fig. 12.10

Figure 12.10 (b) shows a circle which has been divided into 6 equal arcs, AB, BC, CD, DE, EF and FA. The length of each arc is $\frac{60^\circ}{360^\circ}$ of the circumference of the circle.

In general, the length l of an arc of a circle which subtends an angle θ at the centre of the circle is given by $l = \frac{\theta}{360^\circ} \times 2\pi r$.

Example 3

An arc of a circle subtends an angle 60° at the centre of the circle. Find the length of the arc if the radius of the circle is 42 cm. (Take $\pi = \frac{22}{7}$).

Solution

The length, l , of the arc is given by;

$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$\theta = 60^\circ, r = 42 \text{ cm}$$

$$\begin{aligned} \text{Therefore, } l &= \frac{60}{360} \times 2 \times \frac{22}{7} \times 42 \\ &= 44 \text{ cm} \end{aligned}$$

Example 4

The length of an arc of a circle is 62.8 cm. Find the radius of the circle if the arc subtends an angle 144° at the centre. (Take $\pi = 3.142$)

Solution

$$l = \frac{\theta}{360} \times 2\pi r = 62.8 \text{ and } \theta = 144^\circ$$

$$\text{Therefore, } \frac{144}{360} \times 2 \times 3.142 \times r = 62.8$$

$$\begin{aligned} r &= \frac{62.8 \times 360}{144 \times 2 \times 3.142} \\ &= 24.98 \text{ cm} \end{aligned}$$

Example 5

Find the angle subtended at the centre of a circle by an arc of length 11 cm if the radius of the circle is 21 cm.

Solution

$$l = \frac{\theta}{360} \times 2 \times \pi r = 11 \text{ cm and } r = 21 \text{ m}$$

$$\text{Therefore, } \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 21 = 11$$

$$\begin{aligned} \text{Thus, } \theta &= \frac{11 \times 360 \times 7}{2 \times 22 \times 21} \\ &= 30^\circ \end{aligned}$$

Exercise 12.3

For questions, 1 – 4 take, $\pi = 3.142$

1. Copy and complete the table below:

Radius (r)	Diameter (d)	Circumference (C)
5	–	–
7.4	–	–
–	14	–
–	7	21.99
–	–	13.20

2. Copy and complete the following table:

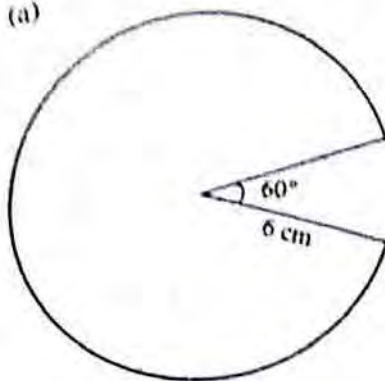
Angle subtended by arc at centre (θ) ^o	Circumference (C) in cm ($2\pi r$)	Arc length in cm $\frac{(\theta \times 2\pi r)}{360}$
30 ^o	48	–
250 ^o	16	–
45 ^o	–	10
90 ^o	60	–
–	30	15

3. Copy and complete the table below:

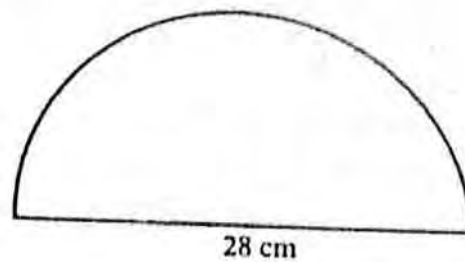
Angle subtended by arc at the centre θ	Radius of the circle r (cm)	length of the arc in cm r
72 ^o	5	–
30 ^o	–	44
–	6	4.4
70 ^o	4.5	5.5
72 ^o	42	52.8

4. Find the perimeter in each of the following figures:

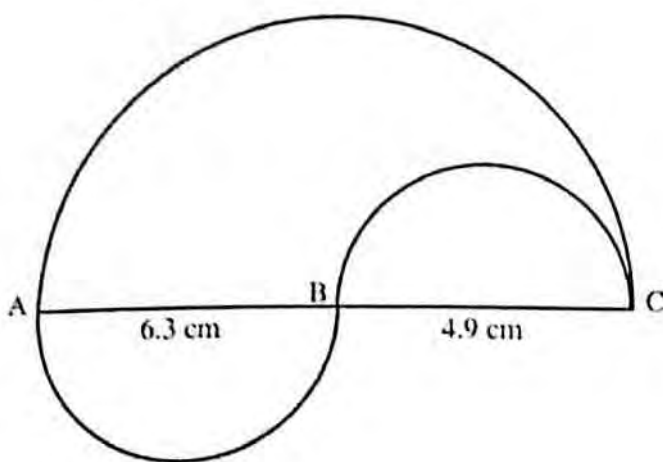
(a)



(b)



(c)



(d)

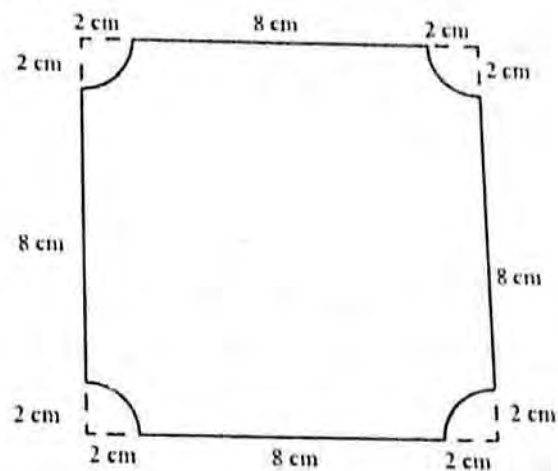


Fig. 12.11

5. Find the perimeter of a semicircular protractor whose radius is 14 cm.
(Take $\pi = \frac{22}{7}$)
6. Find the radius of a bicycle wheel whose circumference is 198 cm.
(Take $\pi = \frac{22}{7}$)
7. An arc PQ of a circle of radius 15 cm subtends an angle of 160° at the centre of the circle. Find the length of the arc PQ. (Take $\pi = 3.142$)
8. The length of an arc of a circle is 9.42 cm. If the diameter of the circle is 10 cm, find the angle subtended by the arc at the centre of the circle.
(Take $\pi = 3.142$)
9. The length of an arc of a circle is one-eighth the circumference of the circle. Find the angle subtended by this arc at the centre if the radius of the circle is 14 cm. (Take $\pi = \frac{22}{7}$)

10. The radius of a wheel is 21 cm. Calculate:
(a) the circumference of the wheel.
(b) The distance covered by the wheel if it turns through an angle of 80°
(Take $\pi = \frac{22}{7}$)
11. The perimeter of a semicircular protractor is 14.28 cm. Find its radius.
12. An arc of a circle is 6 cm long. It subtends an angle of 72° at the centre of the circle. Find the radius of the circle.
13. A wheel of diameter 14 cm is rotates at 2 500 revolutions per minute. Express the speed of a point on the rim in cm per second.
14. Two wire rings of diameter 9 cm and 12 cm are cut and joined to make one large ring. Find the radius of this ring.
15. The wiper of a bus is 40 cm long. It sweeps out through an angle of 120° on a flat windscreen. Calculate the distance moved by the tip of the wiper in one 'sweep'.
16. A bicycle wheel turns 15 times to cover a distance of 66 m. Find the radius of the wheel. (Take $\pi = \frac{22}{7}$)
17. The radius of a circle is 7 cm. Find the circumference of the circle. Hence, find the length of an arc of the circle which subtends an angle of 45° at the centre.
18. An arc of length 'x' cm subtends an angle of $\left(\frac{p}{\pi}\right)^\circ$ at the centre of the circle. Find an expression for the radius, r, of the circle in terms of x and p.
19. Find the length of the minute hand of a wall clock if the tip of the minute hand traces a length of $4p$ cm between 10.15 a.m. and 10.35 a.m. (Give your answer in terms of π).

Chapter Thirteen

AREA

13.1: Units of Area

The area of a plane shape is the amount of surface enclosed within its boundaries. It is normally measured in square units. For example, a square of side 5 cm has an area of $5 \text{ cm} \times 5 \text{ cm} = 25 \text{ cm}^2$.

A square of side 1 m has an area of 1 m^2 , while a square of side 1 km has an area of 1 km^2 .

Conversion of Units of Area

$$\begin{aligned} 1 \text{ m}^2 &= 1 \text{ m} \times 1 \text{ m} \\ &= 100 \text{ cm} \times 100 \text{ cm} \\ &= 10\,000 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 1 \text{ km}^2 &= 1 \text{ km} \times 1 \text{ km} \\ &= 1\,000 \text{ m} \times 1\,000 \text{ m} \\ &= 1\,000\,000 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} 1 \text{ are} &= 10 \text{ m} \times 10 \text{ m} \\ &= 100 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} 1 \text{ hectare (ha)} &= 100 \text{ ares} \\ &= 10\,000 \text{ m}^2 \end{aligned}$$

Exercise 13.1

1. Convert each of the following into the units stated in brackets:

- | | |
|---------------------------------------|--|
| (a) 2 m^2 (cm^2) | (b) 300 cm^2 (m^2) |
| (c) 4 km^2 (m^2) | (d) $5\,000 \text{ m}^2$ (km^2) |
| (e) $9\,000 \text{ m}^2$ (ha) | (f) 200 cm^2 (m^2) |
| (g) 3.4 ha (m^2) | (h) 120 are (ha) |
| (i) 6.8 ha (are) | (j) 0.45 ha (m^2) |

2. Complete the following table:

cm^2	m^2	km^2
120 000	—	—
—	560	—
—	—	0.02

13.2: Area of Regular Plane Figures

Area of a Rectangle

Figure 13.1 is a rectangle of length 5 cm and breadth 3 cm.

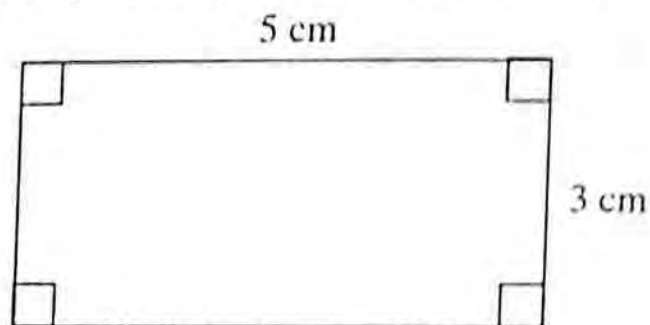


Fig. 13.1

$$\begin{aligned}\text{Its area, } A &= 5 \times 3 \text{ cm}^2 \\ &= 15 \text{ cm}^2\end{aligned}$$

In general, the area of a rectangle, $A = l \times b$ square units, where l is length and b breadth.

Area of a Triangle

Figure 13.2 (a) shows a rectangle which has been divided into two equal triangles.

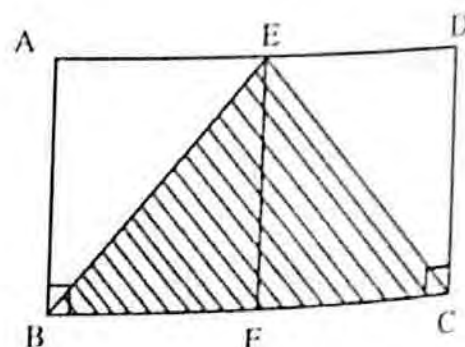
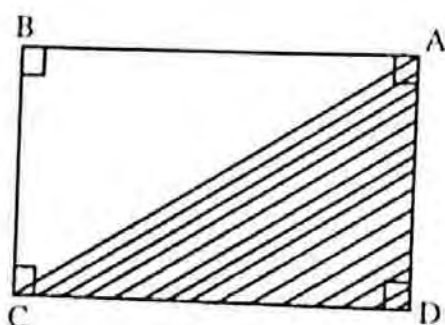


Fig. 13.2

The area of each triangle is half the area of the rectangle. In figure 13.2 (b), rectangle ABCD is made up of rectangles ABFE and EFCD. Triangle EBC is made up of triangles EBF and EFC, each of which is half the area of the corresponding rectangle. EF is the perpendicular height of the triangle. From the illustrations, we can see that the area of a triangle is half that of the rectangle with the same base and same height.

In general, the area A of a triangle with base b units and height h units is given as $A = \frac{1}{2}bh$ square units.

Area of a Parallelogram

The parallelogram ABCD in figure 13.3 is divided into two triangles, ABC and ACD.

Area A of the parallelogram = area of triangle ACD + area of triangle ABC

$$\begin{aligned}\text{That is, } A &= \frac{1}{2}bh + \frac{1}{2}bh \\ &= bh \text{ square units.}\end{aligned}$$

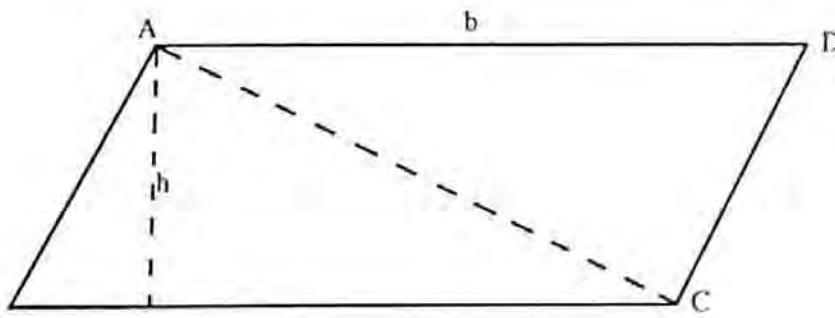


Fig. 13.3

Note:

This formula is also used for a rhombus.

Area of a Trapezium

Figure 13.4 shows a trapezium in which the parallel sides are a units and b units long. The perpendicular distance between the parallel sides is h units.

$$\text{Area of triangle ABD} = \frac{1}{2}ah \text{ square units}$$

$$\text{Area of triangle DBC} = \frac{1}{2}bh \text{ square units}$$

$$\text{Therefore area of trapezium ABCD} = \frac{1}{2}ah + \frac{1}{2}bh$$

$$= \frac{1}{2}h(a + b) \text{ square units.}$$

Thus, the area of a trapezium is given by a half the sum of the lengths of parallel sides multiplied by the perpendicular distance between them.

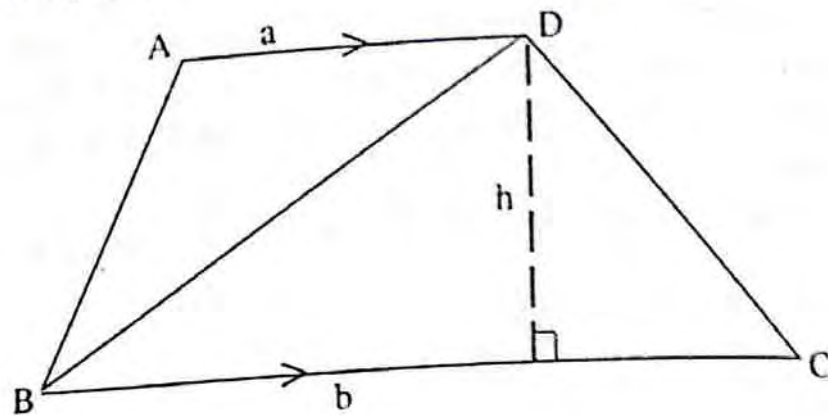


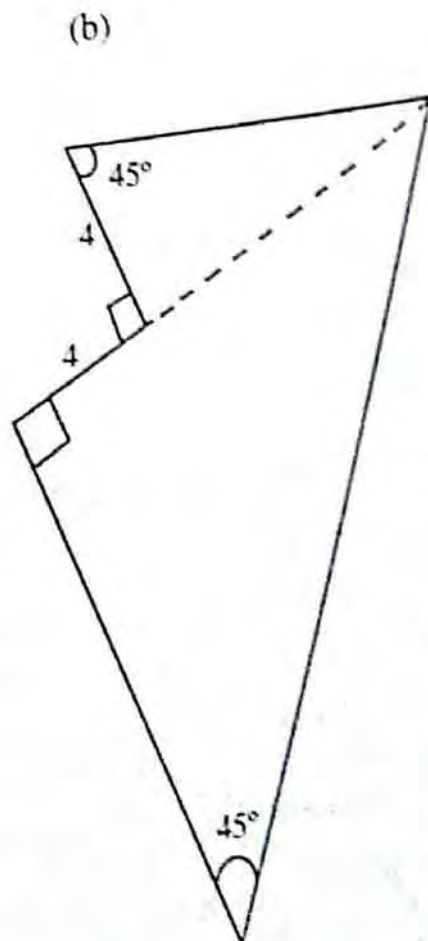
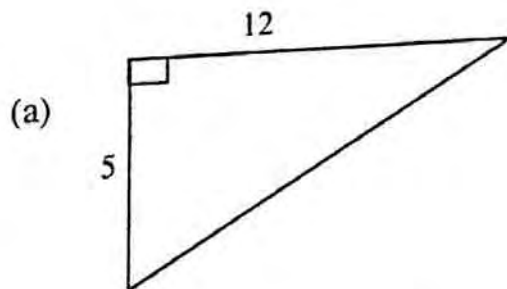
Fig. 13.4

That is, area of trapezium = $\frac{1}{2}(a + b)h$

Exercise 13.2

In questions 1 – 4, the dimensions are in centimetres

- Find the area of each of the following:

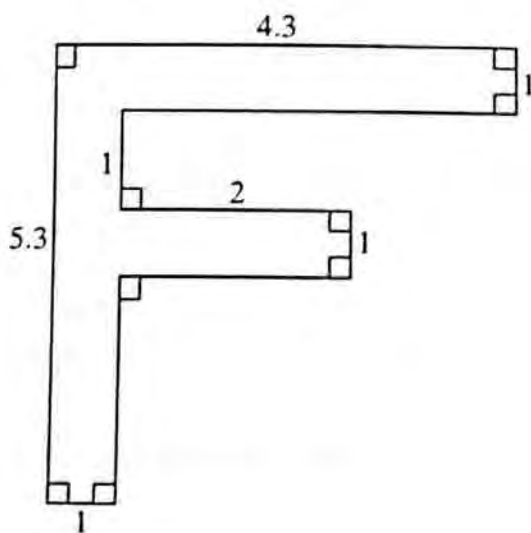


2. Copy and complete the table below for rectangles:

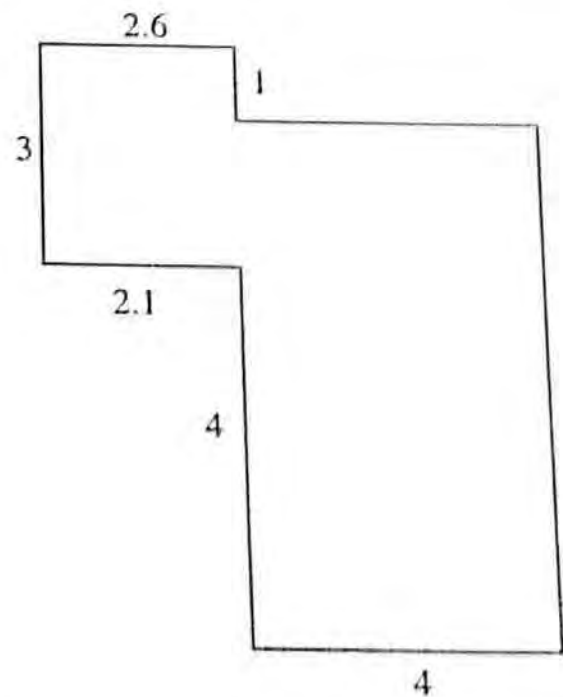
Length	Breadth	Area	Perimeter
15	8	-	-
-	4	-	24
-	-	36	40
9	-	45	-
12.5	-	-	39.6

3. Find the area of each of the following. All dimensions are in centimetres:

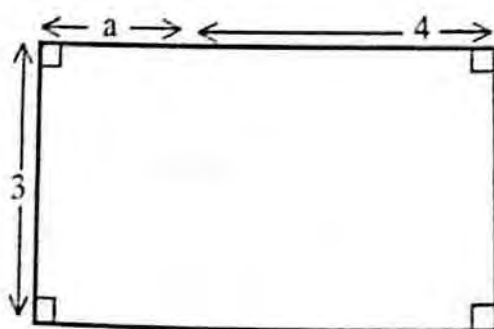
(a)



(b)



(c)



(d)

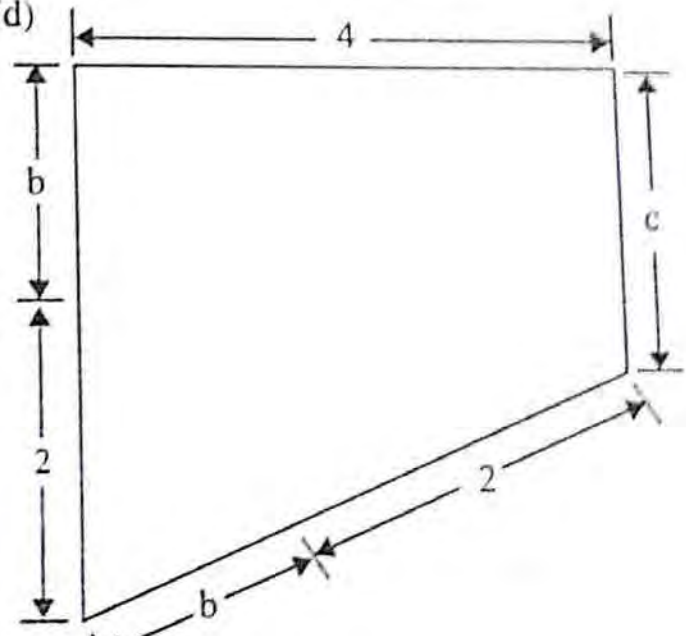


Fig. 13.6

4. Find the area of the shaded region in each of the following. All dimensions are in centimetres:

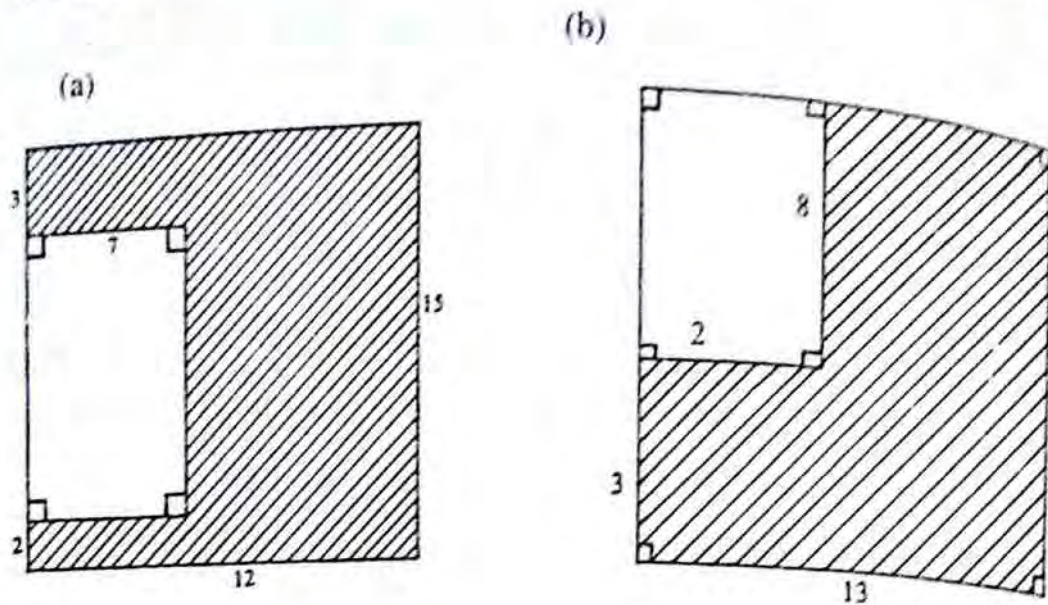


Fig. 13.7

5. A rectangular plot measures 100 m by 200 m. Find its:
 - (a) area in m^2 .
 - (b) area in ha.
6. A photograph measuring 14 cm by 10 cm is fixed inside a rectangular frame of dimensions 24 cm by 18 cm. What is the background area of the space not covered by the photograph?
7. A mat measuring 10 m by 8 m covers an area inside a floor measuring 14 m by 12 m. Find the area not covered by the mat.
8. The floor of a room measures 8 m by 4 m. Square tiles measuring 20 cm by 20 cm are used to cover the floor. How many of these tiles are needed?
9. Figure 13.8 represents a flower garden. What is its:
 - (a) perimeter?
 - (b) area?

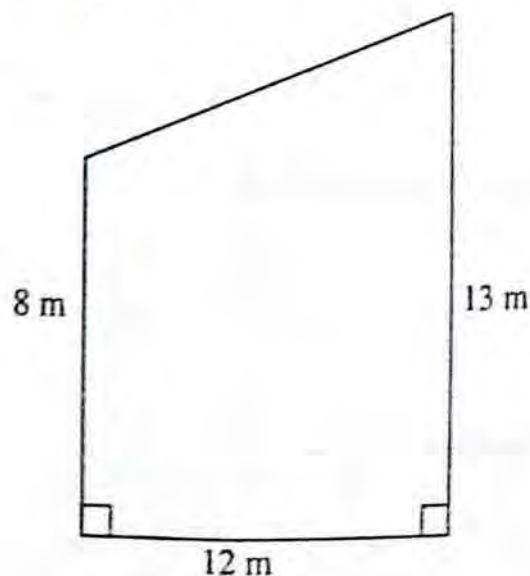


Fig. 13.8

10. The length of a rectangle is three times its breadth. Find its area.
11. The length of a rectangle is three times its breadth. If its perimeter is 24 cm, what is the area of the rectangle?
12. Find the perimeter of a triangle whose sides are x , $2x$ and $x - 5$ cm.
13. A floor is covered by 1 800 rectangular tiles, each measuring 20 cm by 15 cm. Find the total area of the floor in m^2 .
14. The area of 10 square plots is 160 ares. Find the length in metres of the side of each plot.
15. A mat measuring 34 cm by 48 cm is placed centrally in a room measuring 3.6 m by 5.0 m. Find the area of the uncovered floor in cm^2 .
16. The length of a rectangle is twice the width. Its perimeter is 4 cm more than the length. Find its area in cm^2 .
17. The area of a right-angled triangle whose sides are x cm, 5 cm and 13 cm is 30 cm^2 . Find the perimeter of the triangle.
18. How many square tiles of side 15 cm are needed to cover the floor of a room which is 45 m long and 20 m wide?
19. A room has two windows, each measuring 1 m by 1.5 m and a door measuring 2 m by 1 m. The walls are 3 m by 3 m each. Find the cost of painting the inner surface of the walls at sh. 25.00 per m^2 .
20. What is the area in hectares of a rectangular ranch which is 50 km long and 15 km wide?
21. A residential estate is to be developed on a 6 ha piece of land. If 1 500 m^2 is taken up by roads and the rest divided into 40 equal plots, what is the area of each plot?
22. A flower-bed measuring 3 m by 1.5 m is surrounded by a path 1 m wide. Find the area of the path.

13.3: Area of a Circle

Draw a circle of radius 5 cm on a graph paper.

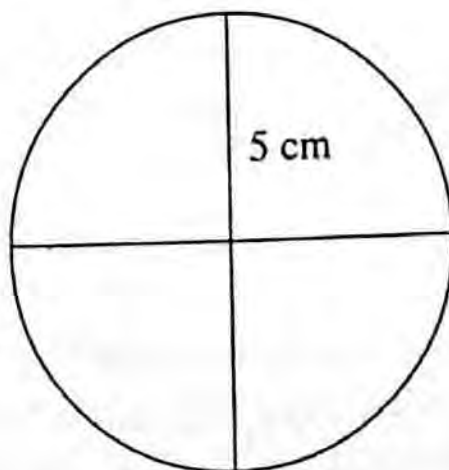


Fig. 13.9

- (i) Estimate the area of the circle by counting the squares enclosed.
- (ii) Divide the estimated area of the circle by the square of the radius.
- (iii) Repeat the procedure for circles of other larger radii and complete the table below:

Radius r	Estimated area of the circle (A)	Square of radius (r^2)	A/r^2
5 cm	74 cm ²	25	2.96
6 cm	110 cm ²	36	3.06
7 cm	—	—	—
8 cm	—	—	—

You should realise that A/r^2 approaches a constant. This constant is π and is approximately equal to 3.142 (3 d.p.). Therefore, the area A of a circle radius r is given by; $A = \pi r^2$.

Note:

Smaller values of the radius do not give good estimates for the area of the circle. Therefore, this will not give a good estimate for the constant π .

Sector of a Circle

A sector is a region bounded by two radii and an arc, see figure 13.10.

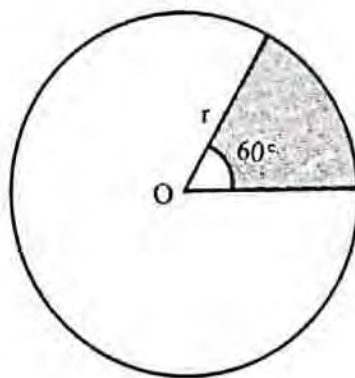


Fig. 13.10

Project

- (i) Draw a circle of radius 4 cm on a manila paper.
- (ii) Divide the circle into 12 equal sectors of angle 30° each.
- (iii) Cut out the sectors and arrange them as shown below:

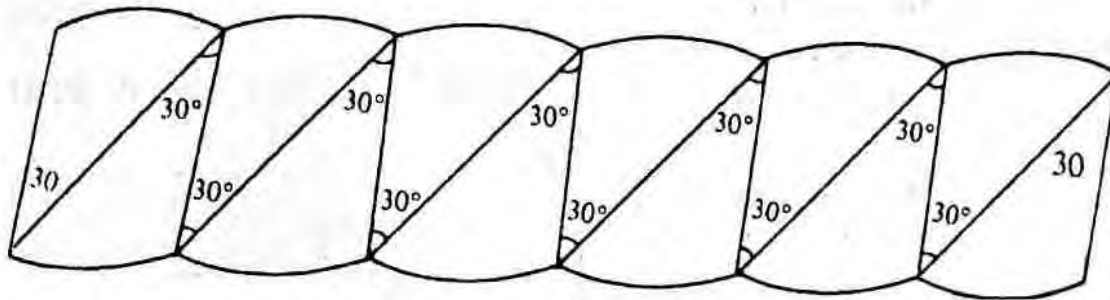


Fig. 13.11

- (iv) The figure formed is roughly a parallelogram whose length is half the circumference and height r . What is the area of this parallelogram?
- (v) Repeat the same procedure, but divide the circular cut-out into more sectors.
- (vi) Notice that the more the sectors, the closer the parallelogram formed approximates to a rectangle. What is the area of this rectangle in terms of r ?

The Area of a Sector

Suppose we want to find the area of the shaded part in figure 13.12.

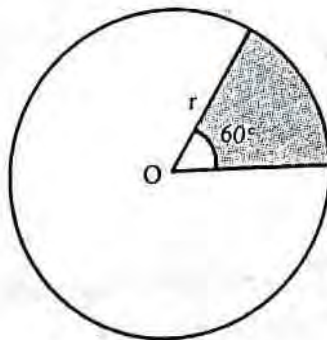


Fig. 13.12

The area of the whole circle is πr^2 .

The whole circle subtends 360° at the centre.

Therefore, 360° corresponds to πr^2 .

1° corresponds to $\frac{1}{360^\circ} \times \pi r^2$.

60° corresponds to $\frac{60^\circ}{360^\circ} \times \pi r^2$

In general, the area of a sector subtending an angle θ (see figure 13.13) at the centre of the circle is given by;

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

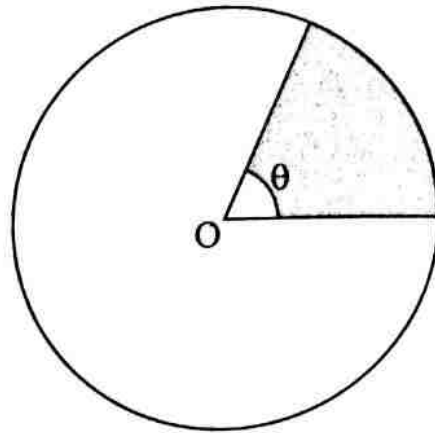


Fig. 13.13

Example 1

Find the area of the sector of a circle of radius 3 cm if the angle subtended at the centre is 140° . (Take $\pi = \frac{22}{7}$)

Solution

Area A of a sector is given by $A = \frac{\theta}{360^\circ} \times \pi r^2$

Here, $r = 3$ cm and $\theta = 140^\circ$

$$\begin{aligned} \text{Therefore, } A &= \frac{140}{360} \times \frac{22}{7} \times 3 \times 3 \\ &= 11 \text{ cm}^2 \end{aligned}$$

Example 2

The area of a sector of a circle is 38.5 cm^2 . Find the radius of the circle if the angle subtended at the centre is 90° . (Take $\pi = \frac{22}{7}$)

Solution

From the formula $A = \frac{\theta}{360} \times \pi r^2$, we get $\frac{90}{360} \times \frac{22}{7} \times r^2 = 38.5$

$$\text{Therefore, } r^2 = \frac{38.5 \times 360 \times 7}{90 \times 22}$$

Thus, $r = 7$ cm

Example 3

The area of a sector of a circle radius 63 cm is $4\,158 \text{ cm}^2$. Calculate the angle subtended at the centre of the circle. (Take $\pi = \frac{22}{7}$)

Solution

Using $A = \frac{\theta}{360} \times \pi r^2$,

$$\theta = \frac{4158 \times 7 \times 360}{22 \times 63 \times 63}$$

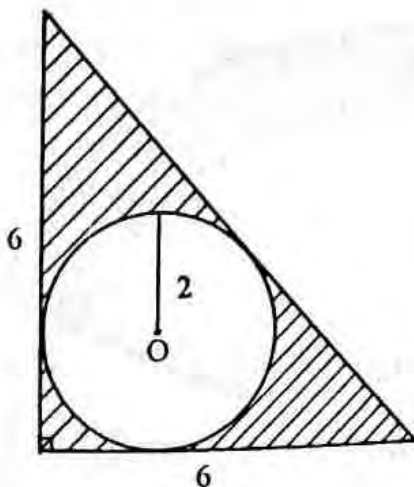
$$= 120^\circ$$

Exercise 13.3

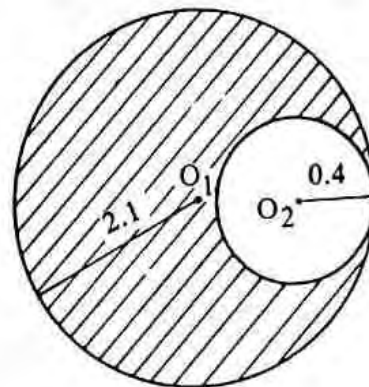
Where necessary, take $\pi = \frac{22}{7}$. In other cases take $\pi = 3.142$.

- Calculate the area of a circle of radius:
 - 3.5 cm
 - 0.49 cm
 - 9.1 cm
 - 12 cm (to 1 d.p.)
 - 9 cm (to 2 d.p.)
 - 2.4 cm (to 4 s.f.)
 - 5 cm (to 5 s.f.)
 - 13 cm (to 2 s.f.)
 - 10.5 cm
- A sector of a circle of radius r makes an angle θ at the centre. Calculate the area of a sector if:
 - $r = 1.4$ cm, $\theta = 30^\circ$
 - $r = 2.1$ cm, $\theta = 45^\circ$
 - $r = 8$ cm, $\theta = 33^\circ$
 - $r = 2$ cm, $\theta = 203^\circ$
 - $r = 9.1$ cm, $\theta = 24^\circ$
 - $r = 8.4$ cm, $\theta = 60^\circ$
 - $r = 4$ cm, $\theta = 259^\circ$
 - $r = 10$ cm, $\theta = 301^\circ$
- Find the areas of the shaded regions in the figures below. Dimensions are in centimetres:

(a)



(b)



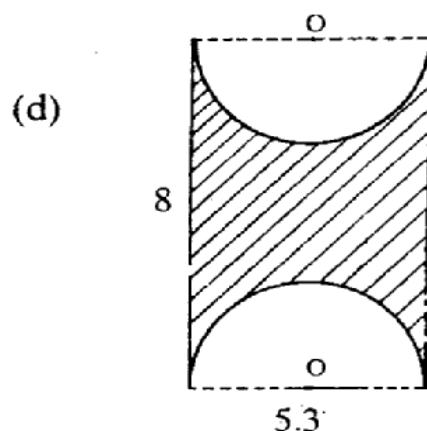
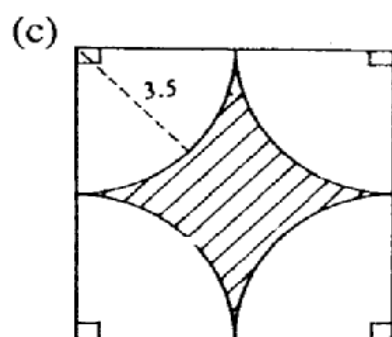


Fig. 13.14

4. Find the area of grass watered by a sprinkler which is capable of spraying water up to a distance of 16 metres.
5. Find the area of the circular face of a coin whose diameter is 2.1 cm.
6. Find the area of a circular parade ground whose radius is 14 m.
7. A goat is tethered to a post by a rope 6.3 m long. Find its maximum grazing area.
8. A circular pond of diameter 7 m is surrounded by a path 2.8 m wide. Find the area of the path.
9. Two equal circular sheets of metal with radius 0.7 m are cut out from a rectangular sheet measuring 2 m by 3 m. Find the area of the remaining sheet.
10. Find the area of the face of a washer whose internal and external radii are 0.5 cm and 1.5 cm respectively.
11. The shaded region in figure 13.15 shows the area swept out on a flat windscreen by a wiper. Calculate the area of the region.

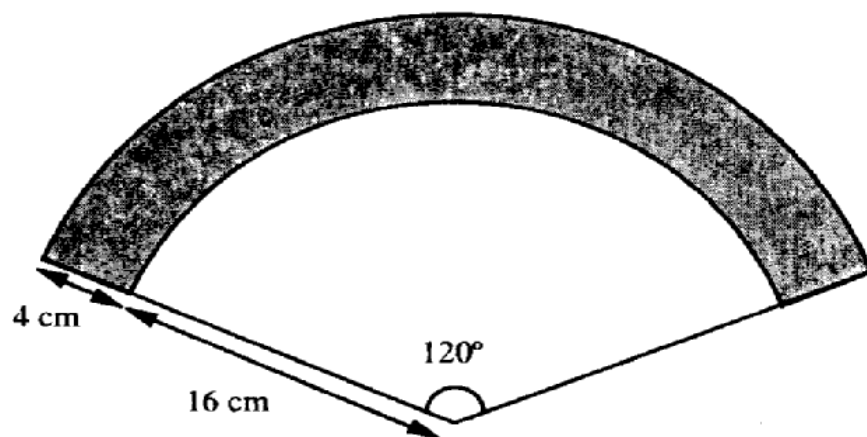


Fig. 13.15

12. Find the difference between the area swept out by the minute hand of a clock which is 3.6 cm long and the hour hand which is 2.9 cm long.

13. The length of a minute hand of a clock is 3.5 cm. Find the angle it turns through if it sweeps an area of 4.8 cm. (Take $\pi = \frac{22}{7}$)
14. The perimeter of a quadrant of a circle is 50 cm². Find the area of the quadrant (a quadrant is a quarter of a circle). (Take $\pi = \frac{22}{7}$)
15. The two arms of a pair of divider are spread so that the angle between them is 45°. Find the area of the sector formed if the length of each arm is 8.4 cm. (Take $\pi = \frac{22}{7}$)

13.4: Surface Area of Solids

Consider a cuboid ABCDEFGH shown in figure 13.16. If the cuboid is cut through a plane parallel to the ends, the cut surface has the same shape and size as the end faces. PQRS is such a plane. The plane is called the **cross-section** of the cuboid.

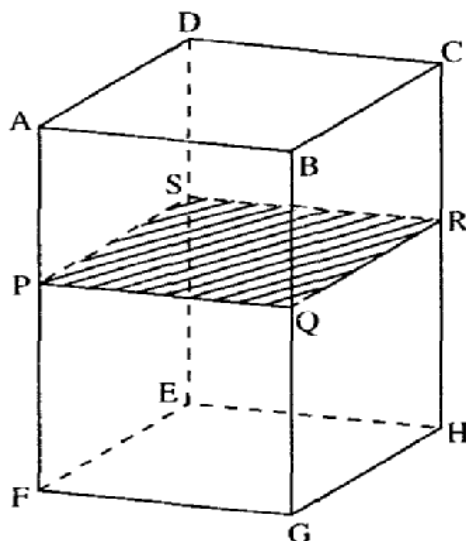
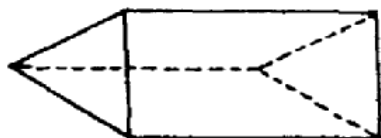


Fig. 13.16

A solid with a uniform cross-section is called a **prism**. The following are some examples of prisms.

(a)



(b)



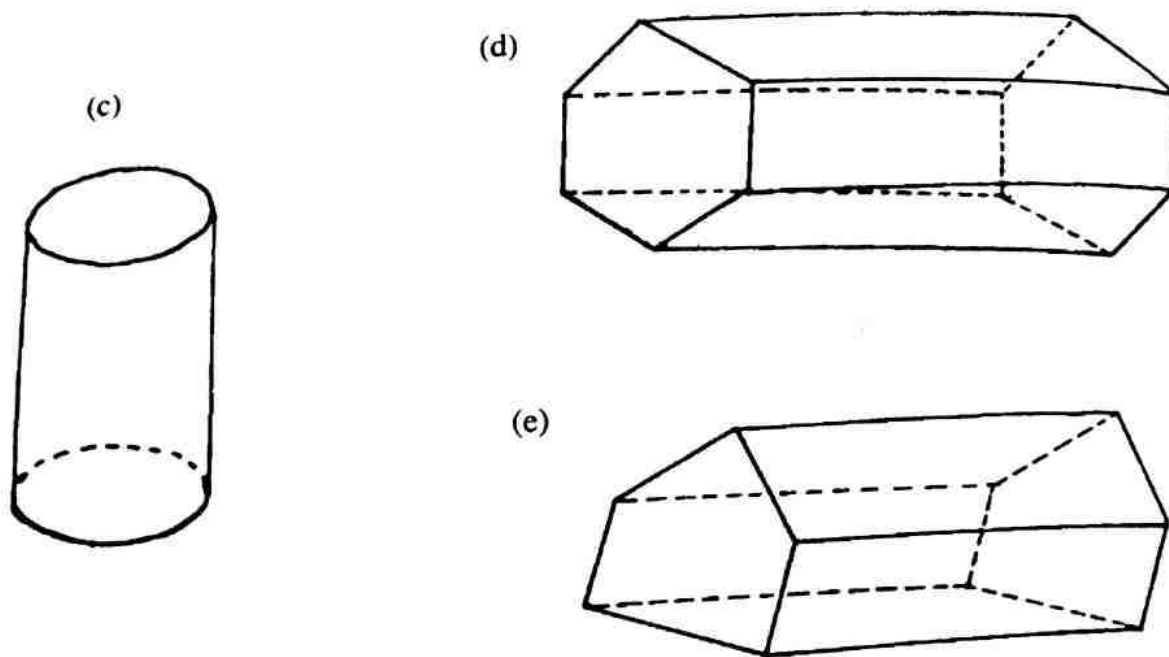


Fig. 13.17

What are the shapes of the cross-sections of the prisms in figure 17 (a) to (e)?
The surface area of a prism is given by the sum of the areas of the faces.

Figure 13.18 shows a cuboid of length l , breadth b and height h . Its total surface area is given by;

$$A = 2lb + 2bh + 2hl$$

$$= 2(lb + bh + hl)$$

For a cube of side 2 cm;

$$A = 2(3 \times 2^2)$$

$$= 24 \text{ cm}^2$$

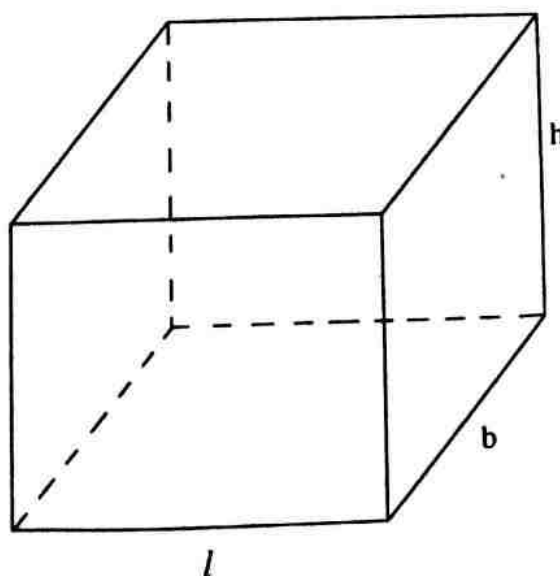


Fig. 13.18

Example 4

Find the surface area of a triangular prism shown in figure 13.19.

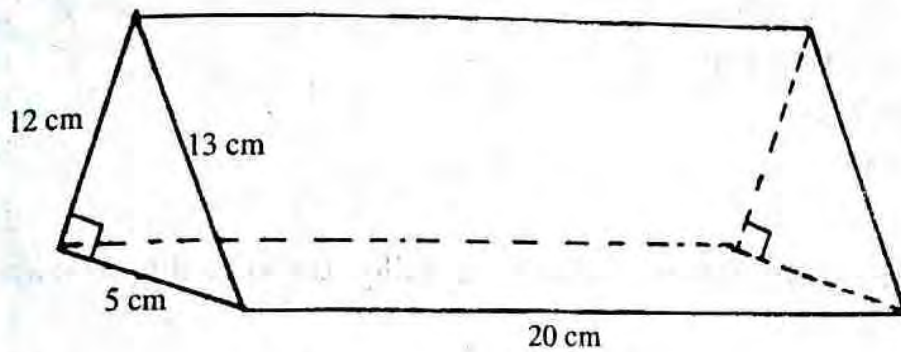


Fig. 13.19

Solution

$$\begin{aligned}\text{Area of the triangular surfaces} &= \frac{1}{2} \times 5 \times 12 \times 2 \text{ cm}^2 \\ &= 60 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of the rectangular surfaces} &= 20 \times 13 + 5 \times 20 + 12 \times 20 \\ &= 260 + 100 + 240 \\ &= 600 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Therefore, the total surface area} &= (60 + 600) \text{ cm}^2 \\ &= 660 \text{ cm}^2\end{aligned}$$

A prism with circular cross-section is called a **cylinder**, see figure 13.20.



Fig. 13.20

If you roll a piece of paper around the curved surface of a cylinder and open it out, you will get a rectangle whose breadth is the circumference and length is the height of the cylinder. The ends are two circles. The surface area S of a cylinder with base radius r and height h is therefore given by;

$$S = 2\pi rh + 2\pi r^2$$

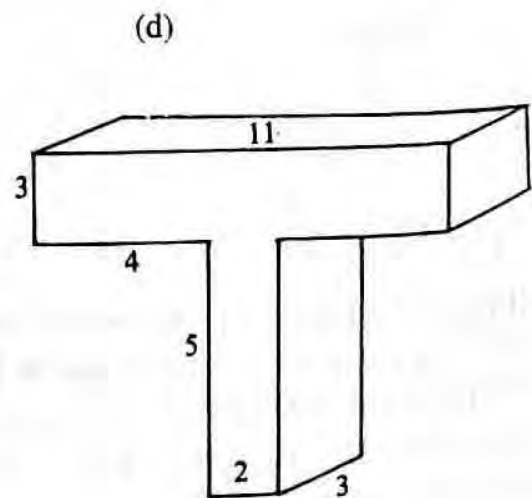
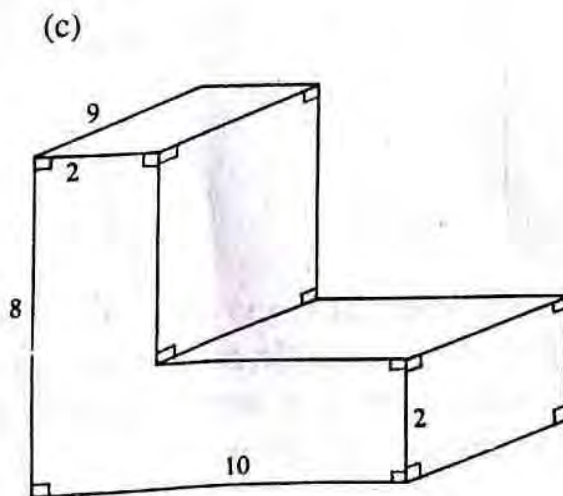
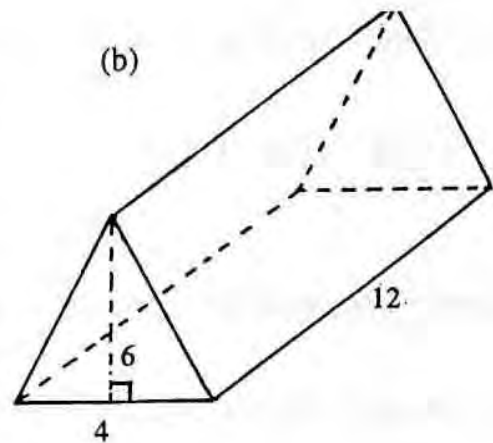
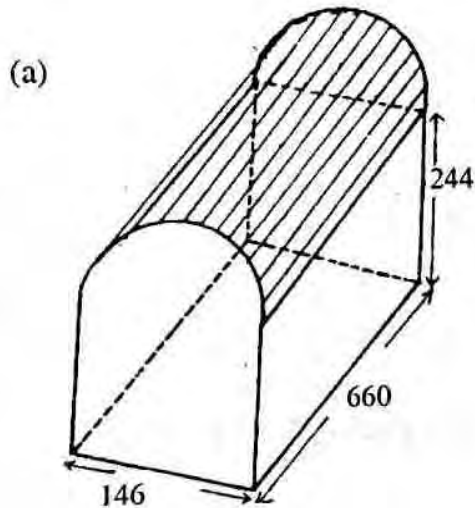
height 12 cm.

Solution

$$\begin{aligned}
 S &= 2\pi(7.7) \times 12 + 2\pi(7.7)^2 \text{ cm}^2 \\
 &= 2\pi(7.7) \times 12 + (7.7)^2 \text{ cm}^2 \\
 &= 2 \times 7.7 \pi(12 + 7.7) \text{ cm}^2 \\
 &= 2 \times 7.7 \times \pi(19.7) \text{ cm}^2 \\
 &= 15.4\pi(19.7) \text{ cm}^2 \\
 &= 953.48 \text{ cm}^2
 \end{aligned}$$

Exercise 13.4

1. Find the surface area of each of the following (the dimensions are in centimetres)



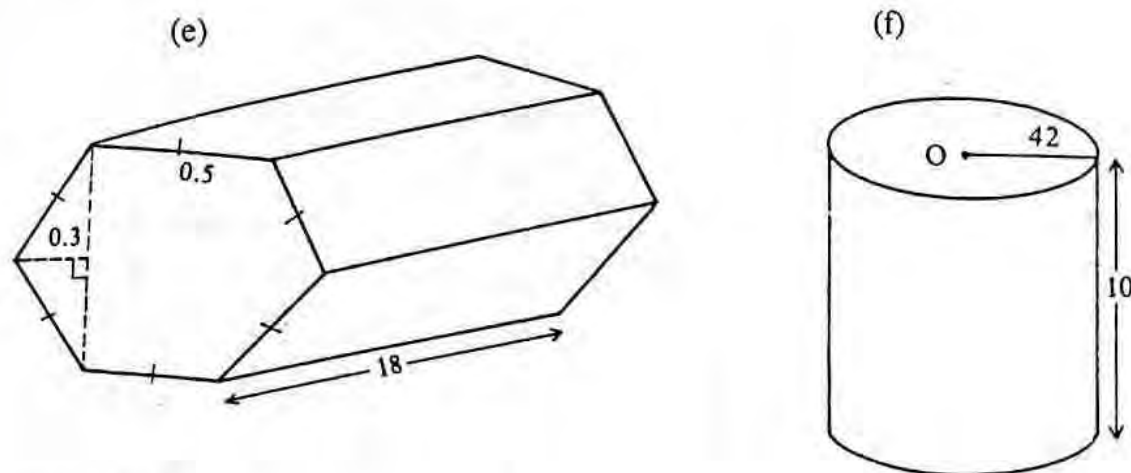


Fig. 13.21

2. Find the surface area of building block measuring 9 cm by 12 cm by 4 cm.
3. What is the surface area of a rectangular eraser which measures 2.3 cm by 2 cm by 0.5 cm?
4. Two metallic pipes, each of length 3 m and external diameter 10 cm, are used as netball posts. Find their total external surface area.
5. An open chalk box is 15 cm long, 12 cm wide and 10 cm high. Find its external surface area.
6. A cylindrical water-tank with no top was constructed at a dining hall corner. If the diameter of the tank was 2.8 m and height 4.8 m, what was its surface area?
7. A cylindrical pipe 3 m long is made from a metallic sheet of thickness 7 mm. Find the total area of the metallic surfaces if the internal radius is 6.3 cm.
8. Find the surface area of a triangular prism of length 25 cm, height 4.5 cm and base 6 cm.
9. Calculate the thickness of a disc of diameter 14 cm and surface area 352 cm^2 .
10. Find the number of revolutions made by a roller of diameter 1.02 m and thickness 1.3 m if it rolls over a surface of 291.72 m^2 .
11. The cost of a rectangular manila paper of length 0.5 m, width 0.3 m and thickness 1 mm is sh. 8 per m^2 . Find the total cost of a pile of similar manilla papers of height 4.4 m.
12. The open corridor in figure 13.22 consists of two walls 244 cm high which are 146 cm apart and a smooth semicircular roof. The length of the corridor is 660 cm. Find the internal surface area of the corridor.

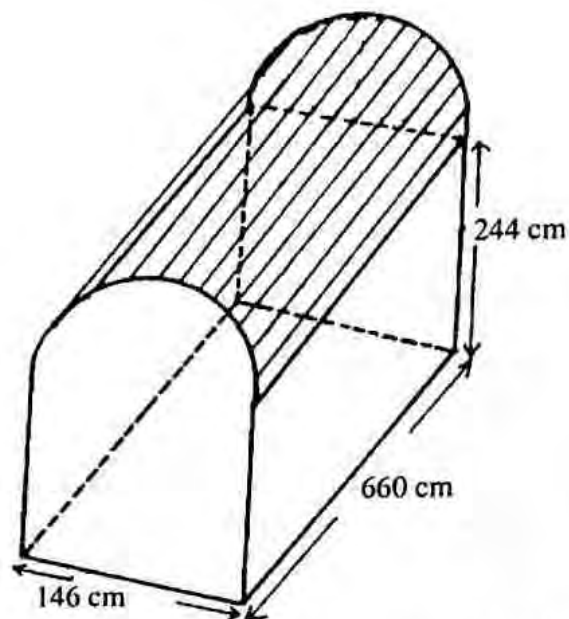


Fig. 13.22

13. The diameter of a cylindrical unsharpened pencil is 8 mm and its length is 17.5 cm. Calculate its surface area.
14. Figure 13.23 shows cross-section of a ruler which is a rectangle of 2.5 cm by 0.2 cm on which is surmounted an isosceles trapezium (one in which the non-parallel sides are of equal length). The shorter of the parallel sides of the trapezium is 0.7 cm long. If the greatest height of the ruler is 0.4 cm and it is 33 cm long, calculate its surface area.

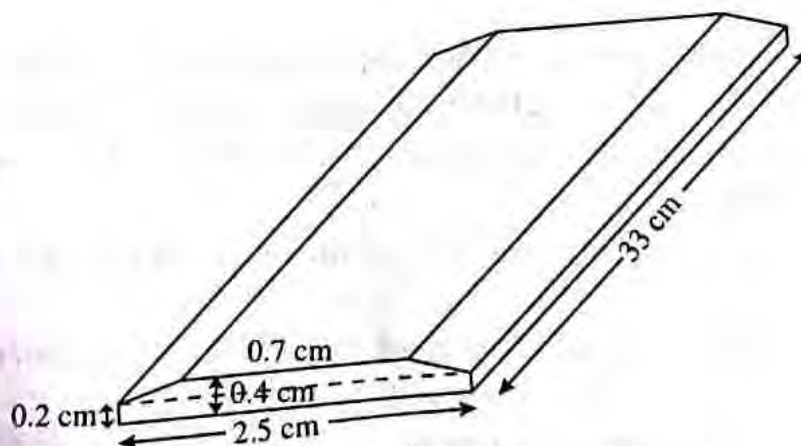


Fig. 13.23

15. Figure 13.24 shows a corrugated iron sheet made of sections, each of which is the minor arc of a circle of radius 4.2 cm subtending an angle of 150° at the centre of the circle. If there are 50 sections and the sheet is 2 m long, calculate the area of the curved top surface of the sheet.

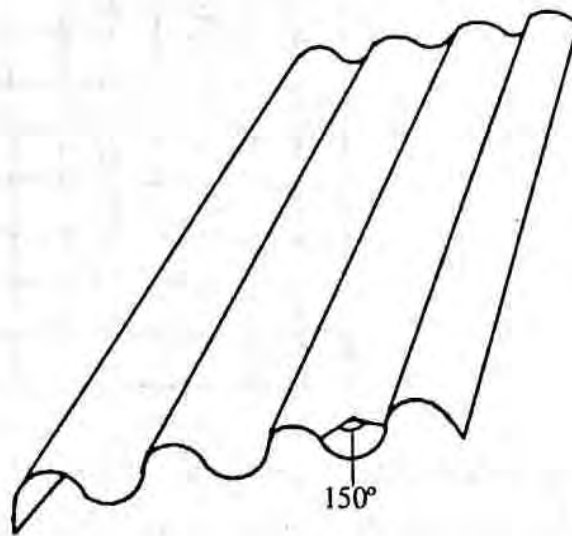


Fig. 13.24

16. The cross-section of a slice of bread consists of a rectangle 5.5 cm by 8 cm and a major segment of a circle of radius 4.2 cm, see figure 13.25. The major arc ABC subtends an angle of 220° at the centre of the circle. Calculate the total surface area of the slice if it is 1.5 cm thick.

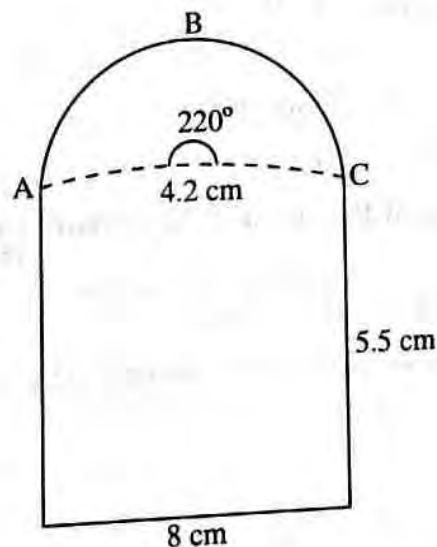


Fig. 13.25

17. An open box has external breadth 12 cm, length 15 cm and height 10 cm. If the thickness of the material of the box is 1 cm, calculate the total surface area of the box. (Hint: there are 14 surfaces).

13.5: Area of Irregular Shapes

The area of an irregular figure cannot be found accurately, but it can be estimated as follows:

- (i) Draw a grid of unit squares on the figure or copy the figure on such a grid, see figure 13.26.

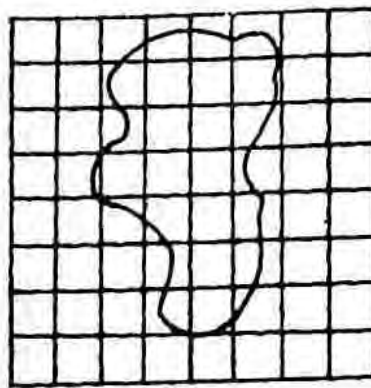


Fig. 13.26

- (ii) Count all the unit squares fully enclosed within the figure.
- (iii) Count all the partially enclosed unit squares and divide the total by two, i.e., treat each one of them as half of a unit square.
- (iv) The sum of the numbers in (ii) and (iii) gives an estimate of the area of the figure.

From the figure, the number of full squares is 9.

Number of partial squares = 18

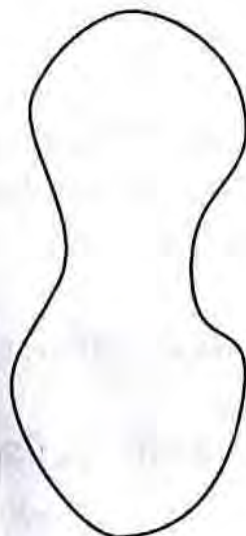
$$\begin{aligned}\text{Total number of squares} &= 9 + \frac{18}{2} \\ &= 18\end{aligned}$$

$\therefore$ Approximate area = 18 sq. units.

Exercise 13.5

1. Trace the outline of the palm of your hand on a graph paper and estimate its area.
2. Repeat question 1 for your foot.
3. Estimate the area of each of the shapes below in cm^2 :

(a)



(b)

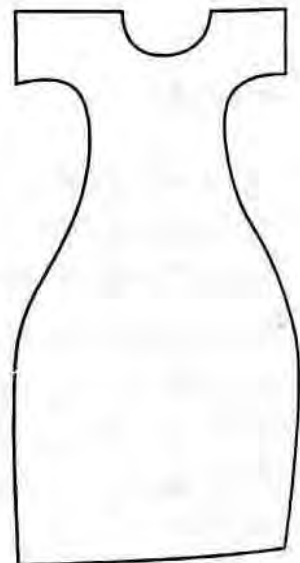


Fig. 13.27

Chapter Fourteen

VOLUME AND CAPACITY

14.1: Introduction

Volume is the amount of space occupied by a solid object. The unit of volume is **cubic units**.

A cube of edge 1 cm has a volume of $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$.

14.2: Conversion of Units of Volume

A cube of side 1 m has a volume of 1 m^3 .

But $1 \text{ m} = 100 \text{ cm}$.

$$\therefore 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$$

$$\text{Thus, } 1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$$

$$\text{or } 1 \text{ cm}^3 = (0.01 \times 0.01 \times 0.01) \text{ m}^3$$

$$= 0.000001 \text{ m}^3$$

$$= 1 \times 10^{-6} \text{ m}^3$$

A cube of side 1 cm has a volume of 1 cm^3 .

But $1 \text{ cm} = 10 \text{ mm}$

$$\therefore 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$$

$$\text{Thus, } 1 \text{ cm}^3 = 1\,000 \text{ mm}^3$$

14.3: Volume of Cubes, Cuboids and Cylinders

Cube

A cube is a solid having six plane square faces in which the angle between two adjacent faces is a right-angle.

$$\text{Volume of a cube} = \text{area of base} \times \text{height}$$

$$= l^2 \times l$$

$$= l^3$$

Cuboid

A cuboid is a solid with six faces which are not necessarily square.

$$\text{Volume of a cuboid} = \text{length} \times \text{width} \times \text{height}$$

$$= A \text{ sq. units} \times h$$

$$= Ah \text{ cubic units.}$$

CHUGU BOYS' HIGH SCHOOL
P O Box 565-60200,
MERU
Tel: +254 716 467 080 / 735 261 382

Cylinders

This is a solid with a circular base.

$$\begin{aligned}\text{Volume of a cylinder} &= \text{area of base} \times \text{height} \\ &= \pi r^2 \times h \\ &= \pi r^2 h \text{ cubic units}\end{aligned}$$

Example 1

Find the volume of a cuboid of length 5 cm, breadth 3 cm and height 4 cm.

Solution

$$\begin{aligned}\text{Area of its base} &= 5 \times 4 \text{ cm}^2 \\ \therefore \text{Volume} &= 5 \times 4 \times 3 \text{ cm}^3 \\ &= 60 \text{ cm}^3\end{aligned}$$

Example 2

Find the volume of a solid whose cross-section is a right-angled triangle of base 4 cm, height 5 cm and length 12 cm.

Solution

$$\begin{aligned}\text{Area of cross section} &= \frac{1}{2} \times 4 \times 5 \\ &= 10 \text{ cm}^2 \\ \text{Therefore, volume} &= 10 \times 12 \\ &= 120 \text{ cm}^3\end{aligned}$$

Example 3

Find the volume of a cylinder with radius 1.4 m and height 13 m.

Solution

$$\begin{aligned}\text{Area of cross-section} &= \frac{22}{7} \times 1.4 \times 1.4 \\ &= 6.16 \text{ m}^2 \\ \therefore V &= 6.16 \times 13 \\ &= 80.08 \text{ m}^3\end{aligned}$$

In general, the volume V of a cylinder of radius r and length l is given by $V = \pi r^2 l$

Exercise 14.1

1. Express in m^3 :

(a) 105 cm^3

(b) 19.7 cm^3

(c) 750 mm^3

(d) 7.5 Dm^3

(e) 0.2 Hm^3

(f) 0.01 km^3

2. A cylindrical jar of diameter 9 cm and depth 12 cm is full of milk. The milk is poured into a cylindrical glass of diameter 6 cm. What is the depth of the milk in the glass?
3. A cylindrical can of diameter 20 cm and height 60 cm is filled with water using a cylindrical jar of diameter 12 cm and height 8 cm. How many jarfuls will fill the can?
4. Small cubes of edge 2 cm are to be packed into a rectangular container measuring 6 cm by 5 m and 4 m. How many cubes are required?
5. What is the volume of the material used in making a PVC pipe 50 m long and 0.3 cm thick if the internal diameter of the pipe is 100 cm? Give your answer to 4 d.p. in m^3 .
6. Find the volume of earth excavated to make a pit latrine 9 m deep, 1.3 m long and 0.7 m wide.
7. What is the maximum number of reams of foolscaps measuring 5 cm by 22 cm by 30 cm that can be packed in a carton measuring 0.55 m by 0.8 m by 1.5 m.
8. Calculate the volume of concrete required to make a culvert of internal radius 0.36 m, thickness 0.07 m and length 2 m. Give your answer to 3 d.p. in m^3 .
9. The head of the axe shown in figure 14.1 is a wedge of length 18.3 cm and width 12 cm. It has a cylindrical hole of diameter 4 cm and its greatest thickness is 6 cm. Find the volume of the metal used in making the axe.

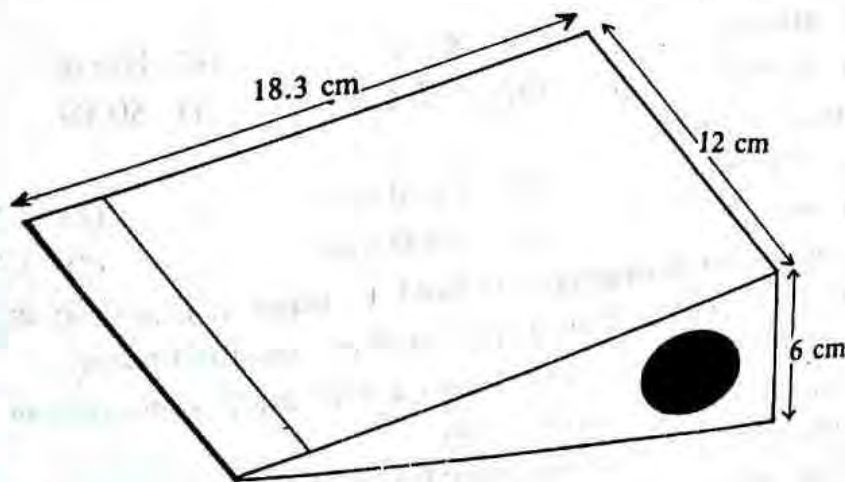


Fig. 14.1

10. The inside of a metal tin is 20 cm long, 15 cm wide and 40 cm deep. The thickness of the tin is 0.2 cm. Calculate the volume of metal used to make the tin.

14.4: Capacity

Capacity is the ability of a container to hold fluids. The SI unit of capacity is the litre (l).

Conversion of Units of Capacity

1 centilitre (cl)	=	10 millilitres (ml)
1 decilitre (dl)	=	10 centilitres (cl)
1 litre (l)	=	10 decilitres (dl)
1 Decalitre (Dl)	=	10 litres (l)
1 hectolitre (Hl)	=	10 Decalitres (Dl)
1 kilolitre (kl)	=	10 Hectolitres (Hl)
1 kilolitre (kl)	=	1 000 litres (l)
1 litre (l)	=	1 000 millilitres (ml)

Relationship between Volume and Capacity

A cube of edge 10 cm holds 1 litre of liquid.

$$1 \text{ litre} = 10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} \\ = 1\,000 \text{ cm}^3$$

$$1 \text{ m}^3 = 10^6 \text{ cm}^3$$

$$1 \text{ m}^3 = 10^3 \text{ litres}$$

Exercise 14.2

- Express in litres:

(a) 400 ml	(b) 80 cl	(c) 100 dl
(d) 536 ml	(e) 375 Hl	(f) 50 Dl
- Express in litres:

(a) 724 cm ³	(b) 1 420 cm ³	(c) 1.5 m ³
(d) 30 cm ³	(e) 3 400 mm ³	(f) 170 dm ³
- A cylindrical container can hold 12 litres of liquid. If the height of the container is 0.4 m, find its radius to one decimal place.
- One litre of milk is poured into a cylindrical container of radius 10 cm. Calculate the depth of the milk.
- Find the width of a room which is 8 m long and 5 m high, if it contains 120 m³ of air.

6. A rectangular tin measures 20 cm by 20 cm by 30 cm. What is its capacity in litres?
7. How many kilolitres of water are there in a full rectangular tank $4\frac{1}{2}$ m long, 4 m wide and $2\frac{1}{8}$ m deep?
8. (a) A school water tank has a radius of 2.1 m and a height of 450 cm. How many litres of water does it carry when full?
(b) If the school uses 5 000 litres of water a day, approximately how many days will the full tank last?

Chapter Fifteen

MASS, WEIGHT AND DENSITY

15.1: Mass

The mass of an object is the quantity of matter in it. Mass is constant quantity, wherever the object is, and matter is anything that occupies space. The three states of matter are solid, liquid and gas.

The SI unit of mass is the kilogram. Other common units are tonne, gram and milligram.

The following table shows units of mass and their equivalent in kilograms.

Unit	Abbreviation	Equivalence in kg
1 hectogram	Hg	0.1 kg
1 decagram	Dg	0.01 kg
1 gram	g	0.001 kg
1 decigram	dg	0.0001 kg
1 centigram	cg	0.00001 kg
1 milligram	mg	0.000001 kg
1 microgram	μg	0.000000001 kg
1 megagram	Mg	1 000 kg
1 tonne (1 megagram)	t	1 000 kg

Exercise 15.1

- Express each of the following masses in kilograms:
 - 20 Hg
 - 45 Dg
 - 800 g
 - 435 dg
 - 3 406 cg
 - 7 889 mg
 - 24 072 μg
 - 39 Mg
 - 4 tonnes
- John requires 2 100 kg of sand to construct his house. How many lorries of sand will he buy if 1 lorry carries 7 tonnes of sand?
- Express each of the following in grams:
 - 4 538 μg
 - 4 626 mg
 - 428 cg
 - 324 dg
 - 46 Dg
 - 892 Hg

4. Mary bought $2\frac{1}{2}$ kg of meat. Half of the meat was cooked for supper and a quarter of the remainder used to make burgers for the following day's breakfast. How much meat in grams was left?
5. A textbook has 268 leaves. Each leaf has a mass of 50 cg and the cover 20 g. Find the mass of the book in kilograms.

15.2: Weight

The weight of an object on earth is the pull of the earth on it. The weight of any object varies from one place on the earth's surface to the another. This is because the closer the object is to the centre of the earth, the more the gravitational pull, hence the more its weight. For example, an object weighs more at sea level than on top of a mountain.

Units of Weight

The SI unit of weight is the Newton. The pull of the earth, sun and moon on an object is called **the force of gravity** due to the earth, sun and moon respectively. The force of gravity due to the earth on an object of mass 1 kg is approximately equal to 9.8 N. The strength of the earth's gravitational pull (symbol 'g') on an object on the surface of the earth is about 9.8 N/kg.

Weight of an object = mass of object $\times$ gravitation

Weight (N) = mass (kg) $\times$ g N/kg

Exercise 15.2

1. Find the weight of each of the following objects on the earth's surface:
(Take $g = 9.8$ N/kg)
 - (a) 38 kg (b) 407 g
 - (c) 3 tonnes (d) 52 Dg
2. Jane's weight is 556 N. What is her mass in kg?
3. The acceleration due to gravity on the moon is 1.6 N/kg. An astronaut weighs 670 N on the earth's surface. Find:
 - (a) his mass.
 - (b) his weight on the moon's surface.
4. The force of gravity on the moon is one-sixth of that on earth. What would a mass of 50 kg weigh:
 - (a) on the earth?
 - (b) on the moon?

15.3: Density

The density of a substance is the mass of a unit cube of the substance. A body of mass (m) kg and volume (V) m^3 has:

- (i) Density (d) = $\frac{\text{mass (m)}}{\text{volume (V)}}$
(ii) Mass (m) = density(d) x volume (V)
(iii) Volume (V) = $\frac{\text{mass (m)}}{\text{density (d)}}$

Units of Density

The SI unit of density is kg/m^3 . The other common unit is g/cm^3 .

$$1 \text{ g/cm}^3 = 1\,000 \text{ kg/m}^3$$

Example 1

Find the mass of an ice cube of side 6 cm, if the density of ice is 0.92 g/cm^3 .

Solution

$$\text{Volume of cube} = 6 \times 6 \times 6 = 216 \text{ cm}^3$$

$$\begin{aligned}\text{Mass} &= \text{density} \times \text{volume} \\ &= 216 \times 0.92 \\ &= 198.72 \text{ g}\end{aligned}$$

Example 2

Find the volume of cork of mass 48 g, given that density of cork is 0.24 g/cm^3 .

Solution

$$\begin{aligned}\text{Volume} &= \frac{\text{mass}}{\text{density}} \\ &= \frac{48}{0.24} \\ &= 200 \text{ cm}^3\end{aligned}$$

Example 3

The density of iron is 7.9 g/cm^3 . What is this density in kg/m^3 ?

Solution

$$\begin{aligned}1 \text{ g/cm}^3 &= 1\,000 \text{ kg/m}^3 \\ 7.9 \text{ g/cm}^3 &= \frac{7.9 \times 1000}{1} \\ &= 7\,900 \text{ kg/m}^3\end{aligned}$$

Example 4

A rectangular slab of glass measures 8 cm by 2 cm by 14 cm and has a mass of 610 g. Calculate the density of glass in kg/m^3 .

Solution

$$\begin{aligned}\text{Volume of the slab} &= 8 \times 2 \times 14 \\ &= 224 \text{ cm}^3\end{aligned}$$

$$\text{Mass of the slab} = 610 \text{ g}$$

$$\begin{aligned}\therefore \text{Density} &= \frac{610}{224} \\ &= 2.5 \times 1\,000 \text{ kg/m}^3 \\ &= 2\,500 \text{ kg/m}^3\end{aligned}$$

Exercise 15.3

1. (a) Express in g/cm^3 :

- | | |
|-------------------------------|-------------------------------|
| (i) 1.3 kg/m^3 | (ii) 250 kg/m^3 |
| (iii) $8\,900 \text{ kg/m}^3$ | (iv) $19\,300 \text{ kg/m}^3$ |
| (v) $2\,700 \text{ kg/m}^3$ | (vi) $11\,500 \text{ kg/m}^3$ |

(b) Express in kg/m^3 :

- | | |
|-----------------------------|-------------------------------|
| (i) 0.8 g/cm^3 | (ii) 0.0078 kg/cm^3 |
| (iii) 11.4 g/cm^3 | (iv) 0.01 kg/cm^3 |
| (v) 450 g/m^3 | (vi) 0.0025 kg/cm^3 |

- What is the mass of water that can fill a cylindrical tank whose diameter and height are 2.8 m and 3 m respectively. (Take density of water as 1 kg/l)
- A cylindrical milk churn contains 15 litres of milk. Find the density of milk in g/cm^3 if the total mass of milk in the churn is 14 kg.
- The reading of liquid in a measuring cylinder is 45 cm^3 . A solid of mass 150 g is put into the container. If the density of the solid is 8.6 g/cm^3 , find the new reading.
- A right-angled triangular prism has length 3 m, breadth 2 m and height 2.5 m. If the mass of the prism is 3.4 kg, find its density.
- The density of a certain type of wood is 0.48 g/cm^3 . Find the mass of a log of this wood with diameter 49 cm and length 3 m.
- A wooden block measuring 20 cm by 30 cm by 50 cm has a mass of 22.5 kg. Find the density of this wood in g/cm^3 .
- Find the density in kg/m^3 of petrol if the mass of 1.5 litres of petrol is 1.2 kg.
- Calculate the mass in grams of 205 cm^3 of steel if it has a density of $7\,800 \text{ kg/m}^3$.
- The density of gold is 19.3 g/cm^3 . Calculate the volume, in m^3 , of a golden ring mass of 57.9 g.
- $2\,000 \text{ cm}^3$ of a mixture consists of 2.5 kg of substance A and 7.5 kg of substance B. Find the density of the mixture in g/cm^3 .

12. Calculate the volume of 1.5 kg of cork if the density of cork is 0.25 g/cm^3 .
13. Calculate the mass of air in a room with internal measurements 6 m by 4 m by 9 m, if the density of air is 1.3 kg/m^3 .
14. A cylindrical column of fat has diameter 17.5 cm and height 10 cm. Calculate the density of fat if the column has a mass of 2 kg.
15. 1.5 litres of water (density 1 g/cm^3) is added to 5 litres of alcohol (density 0.8 g/cm^3). Calculate the density of the mixture.

Chapter Sixteen

TIME

16.1: Units of Time

$$1 \text{ week} = 7 \text{ days}$$

$$1 \text{ day} = 24 \text{ hours}$$

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ min} = 60 \text{ seconds}$$

Example 1

How many hours are there in one week?

Solution

$$1 \text{ week} = 7 \text{ days}$$

$$1 \text{ day} = 24 \text{ hours}$$

$$\begin{aligned}\therefore 1 \text{ week} &= (7 \times 24) \text{ hours} \\ &= 168 \text{ hours}\end{aligned}$$

Example 2

Convert 3 h 45 min into minutes.

Solution

$$1 \text{ h} = 60 \text{ min}$$

$$3 \text{ h} = (60 \times 3) \text{ min}$$

$$\begin{aligned}3 \text{ h } 45 \text{ min} &= [(60 \times 3) + 45] \text{ min} \\ &= (180 + 45) \text{ min} \\ &= 225 \text{ min}\end{aligned}$$

Example 3

Express 4 h 15 min in seconds.

Solution

$$1 \text{ hour} = 60 \text{ min}$$

$$1 \text{ min} = 60 \text{ sec}$$

$$4 \text{ h } 15 \text{ min} = (4 \times 60 + 15) \text{ min}$$

$$\begin{aligned} &= (240 + 15) \text{ min} \\ &= 255 \text{ min} \\ &= 255 \times 60 \text{ sec} \\ &= 15\,300 \text{ sec} \end{aligned}$$

Exercise 16.1

1. Express each of the following in minutes:
(a) 5 h 10 min (b) 4 h 08 min (c) 3 h 25 min
(d) 6 h 14 min (e) 2 h 38 min (f) 8 h 24 min
2. Convert each of the following into seconds:
(a) 58 min (b) 1 h 34 min (c) 2 h 16 min
(d) 3 h 5 min 5 sec (e) 4 h 20 min 3 sec (f) 7 h 15 min 48 sec
3. Find how many hours there are in:
(a) 2 weeks (b) 1 week, 20 hours
(c) 4 days (d) 2 weeks and 1 day
(e) 8 days (f) 10 days, 4 hours
4. Convert each of the following into hours and minutes:
(a) 72 min (b) 108 min (c) 240 min
(d) 720 min (e) 842 min (f) 630 min
5. Express each of the following in minutes and seconds:
(a) 120 s (b) 256 s (c) 437 s
(d) 504 s (e) 372 s (f) 275 s
6. Convert each of the following into hours, minutes and seconds:
(a) 7 552 s (b) 9 720 s (c) 2 473 s
(d) 1 548 s (e) 18 170 s (f) 32 449 s
7. Find how many minutes there are in:
(a) 2 days (b) 5 days (c) 3 weeks
(d) 5 weeks (e) 16 days (f) 24 days

16.2: The 12 and 24-Hour Systems

In the 12-hour system, time is counted from midnight. The time from midnight to midday is written as a.m. while that from midday to midnight is written as p.m.

In the 24 hour-system, time is counted from midnight to midnight and expressed in hours.

Exercise 16.2

- Express each of the following in the 24-hour system:
 - 2.30 a.m.
 - 6.50 p.m.
 - 1.15 p.m.
 - 7.10 a.m.
 - 11.20 a.m.
 - 8.30 p.m.
 - 9.47 p.m.
 - 11.50 p.m.
 - 12.30 a.m.
 - 4.19 p.m.
 - 2.45 p.m.
 - 6.45 a.m.
- Express in a.m. or p.m.:
 - 0825 h
 - 1615 h
 - 1317 h
 - 0005 h
 - 1420 h
 - 1450 h
 - 2230 h
 - 2340 h
 - 0611 h
 - 1836 h
 - 0245 h
 - 1247 h
- A session started at 2000 h and lasted 10 hours. At what time did it end? Express your answer in both the 24-hour system and a.m./p.m.
- A 35-minute lesson ended at 1215 h. At what time did it begin?
- How long do the following journeys take?

<i>Time of departure</i>	<i>Time of arrival</i>	<i>Time taken</i>
8.15 a.m.	7.15 p.m.	
6.40 p.m.	9.45 a.m.	
8.30 p.m.	6.15 a.m., next day	
9.15 a.m.	2.17 p.m.	
1222 h	1402 h	
2235 h	0615 h, next day	

- A football match lasts 90 minutes with a break of 15 minutes at half-time. If a referee allows five minutes extra for injuries and stoppages, what time does a match which kicks off at 4.30 p.m. end?
- A boxing round lasts 3 minutes with a break of one minute between rounds.
 - How long does a 15-round contest which goes the whole length take?
 - If the first bell goes at 0815 h, at what time does the contest end?
- Classes in a school start at 8.20 a.m. and end at 4.20 p.m. Tea and lunch breaks take a total of two hours. How long do the lessons take? If there are eight equal lessons in a day, how long is each lesson?
- A bus left Nakuru at 2230 h and arrived in Nairobi at 0135 h the next day. How long did the journey take?

10. A church service lasted 2 hours and 25 minutes. What time did it start if it ended at 12.15 p.m?
11. Otieno leaves home for school at 8.15 a.m. He runs for six minutes and walks the remaining distance for 15 minutes. At what time does he report to school?
12. A bus which left Nairobi at 9.15 p.m. took 5 hours 17 minutes to reach its destination. Find the time it arrived.
13. A school assembly which starts at 7.45 a.m. lasts for 25 minutes after which students break for lessons. At what time do they start lessons?
14. An athlete took 5 minutes 39 seconds to complete a 3 000 m race. If the athlete crossed the finishing line 4.17 p.m., at what time did the race start?
15. A service vehicle which left Mombasa for Nairobi at 1000 h had a puncture after travelling for 4 h 20 min. Fixing a new tyre took 33 minutes. The vehicle then travelled for another 1 hour 20 minutes before reaching Nairobi. At what time did it arrive?

16.3: Travel Timetables

Travel timetables show the expected arrival and departure time for vehicles, ships, aeroplanes, trains.

Example 4

The table below shows a timetable for a public service vehicle plying between two towns A and D via towns B and C.

<i>Town</i>	<i>Arrival time</i>	<i>Departure time</i>
A		8.20 a.m.
B	10.40 p.m.	11.00 a.m.
C	2.30 p.m.	2.50 p.m.
D	4.00 p.m.	

- (a) At what time does the vehicle leave town A?
- (b) At what time does it arrive in town D?
- (c) How long does it take to travel from town A to D?
- (d) What time does the vehicle take to travel from town C to D?

Solution

- (a) 8.20 a.m.
 (b) 4.00 p.m.
 (c) Arrival time in town D was 4.00 p.m. Departure from town A was 8.20 a.m.
 $\text{Time taken} = (12.00 - 8.20 + 4 \text{ h})$
 $= 3 \text{ h } 40 \text{ min} + 4 \text{ h}$
 $= 7 \text{ h } 40 \text{ min}$
 (d) The vehicle arrived in town D at 4.00 p.m. It departed from town C at 2.50 p.m.
 $\text{Time taken} = 4.00 - 2.50$
 $= 1 \text{ h } 10 \text{ min}$

Exercise 16.3

1. Below is a travel timetable for a minibus:

<i>Town</i>	<i>Arrival time</i>	<i>Departure time</i>
X		0830 h
Y	1115 h	1130 h
Z	1410 h	

- (a) How long does the minibus take to travel from town X to town Y?
 (b) How long does the minibus take at town Y?
 (c) What time does it take to travel from town Y to town Z?
2. Below is a travel timetable for a vehicle operating between towns A and D, seventy kilometres apart:

<i>Town</i>	<i>Arrival</i>	<i>Departure</i>
A		10.10 a.m.
B	10.30 a.m.	10.40 a.m.
C	11.00 a.m.	11.05 a.m.
D	11.20 a.m.	

- (a) At what time does the vehicle depart from town A?
 - (b) How long does it take to travel from town A to town B?
 - (c) For how long does it stay in town B?
 - (d) At what time does it arrive in town D?
 - (e) What is the average speed for the whole journey?
3. The travel timetable below shows the departure and arrival times for a bus plying between two towns M and R, 300 kilometres apart:

<i>Town</i>	<i>Arrival</i>	<i>Departure</i>
M		0830 h
N	1000 h	1020 h
P	1310 h	1340 h
Q	1510 h	1520 h
R	1600 h	

- (a) How long does the bus take to travel from town M to N?
 - (b) What time does it take at town P?
 - (c) At what time does it arrive at town R?
 - (d) What is its average speed for the whole journey?
4. The table below shows bus fares in shillings between Mukurwe and Thika:

Mukurwe				
10	Kangoo			
20	10	Mangu		
30	20	10	Gatukuyu	
60	50	40	30	Thika

Cyprian travelled from Mukurwe to Gatukuyu and later from Gatukuyu to Thika.

- (a) How much money did he spend as fare?
- (b) If he travelled from Mukurwe to Thika without alighting, how much would he have paid?

5. The table below represents bus fares in shillings between towns A and E via towns B, C and D:

A				
20	B			
25	10	C		
40	35	30	D	
50	40	35	10	E

Use the table to answer the following questions:

- (a) A trader travelled from town A to town E. How much did he pay as bus fare?
- (b) If the trader decided to alight at town D and after sometime proceed to town E, how much would he have paid?

CHIGU BOYS' HIGH SCHOOL
P.O. Box 565-60200,
MERU
Tel: +254 716 457 080 / 735 261 362

Chapter Seventeen

LINEAR EQUATIONS

17.1: Introduction

Linear equations are straight line equations involving one or two unknowns. In this chapter, we will deal with the formation and solving of such equations. Consider the following cases:

Example 1

Solve for the unknowns in each of the following equations:

(a) $3x + 4 = 10$

(b) $\frac{x}{3} - 2 = 4$

(c) $\frac{p+5}{3} = \frac{5}{4}$

Solution

(a) $3x + 4 = 10$

$$3x + 4 - 4 = 10 - 4$$

$$3x = 6$$

$$x = 2$$

(b) $\frac{x}{3} - 2 = 4$

$$\frac{x}{3} - 2 + 2 = 4 + 2$$

$$\frac{x}{3} = 6$$

$$3 \times \frac{x}{3} = 6 \times 3$$

$$x = 18$$

(c) $\frac{p+5}{3} = \frac{5}{4}$

$$3 \times \left(\frac{p+5}{3} \right) = \frac{5}{4} \times 3$$

$$p + 5 = \frac{15}{4}$$

$$4(p + 5) = \frac{15}{4} \times 4$$

$$4p + 20 = 15$$

$$4p = -5$$

$$p = -\frac{5}{4}$$

$$= -1\frac{1}{4}$$

Example 2

Solve for the unknown in each of the following equations:

(a) $\frac{x+1}{2} - \frac{x-2}{3} = \frac{1}{8}$

(b) $1 - \frac{y}{2} - 2\left(\frac{y-3}{2}\right) = 0$

(c) $\frac{3y}{2} - \frac{14y-3}{5} = \frac{y-4}{4}$

Solution

(a) $\frac{x+1}{2} - \frac{x-2}{3} = \frac{1}{8}$

$$\frac{(x+1)}{2} \times 24 - \frac{(x-2)}{3} \times 24 = \frac{1}{8} \times 24 \text{ (Multiply both sides by L.C.M. of 2, 3 and 8)}$$

$$12(x+1) - 8(x-2) = 3$$

$$12x + 12 - 8x + 16 = 3$$

$$4x + 28 = 3$$

$$4x = -25$$

$$x = -\frac{25}{4}$$

$$= -6\frac{1}{4}$$

(b) $1 - \frac{y}{2} - 2\left(\frac{y-3}{2}\right) = 0$

$$(1 \times 2) - \left(\frac{y}{2} \times 2\right) - 2\left(\frac{y-3}{2}\right) \times 2 = 0 \times 2$$

$$2 - y - 2(y-3) = 0$$

$$2 - y - 2y + 6 = 0$$

$$8 - 3y = 0$$

$$3y = 8$$

$$y = \frac{8}{3}$$

$$= 2\frac{2}{3}$$

(c) $\frac{3y}{2} - \frac{14y-3}{5} = \frac{y-4}{4}$

$$\frac{3y}{2} \times 20 - \frac{(14y-3)}{5} \times 20 = \left(\frac{y-4}{4}\right) \times 20 \text{ (Multiply both sides by L.C.M. of 2, 5 and 4).}$$

$$30y - 4(14y - 3) = 5(y - 4)$$

$$30y - 56y + 12 = 5y - 20$$

$$-26y + 12 = 5y - 20$$

$$31y = 32$$

$$y = \frac{32}{31}$$

$$= 1 \frac{1}{31}$$

Exercise 17.1

Solve for the unknown in each of the following equations:

1. (a) $\frac{y+3}{3} - \frac{y-3}{4} = \frac{1}{12}$

(b) $15x = \frac{31-2x}{4}$

2. (a) $\frac{x+2}{4} - \frac{x+3}{5} = \frac{x+4}{6}$

(b) $\frac{0.1}{x} + \frac{3.9}{x} = 12$

3. (a) $\frac{0.5}{x} - 3 = 1 + \frac{8}{x}$

(b) $\frac{3}{x-4} = \frac{3}{6}$

4. (a) $\frac{x-4}{2} = \frac{x+6}{3}$

(b) $\frac{z+1}{3} = \frac{z-4}{9}$

5. (a) $\frac{2}{x} + 1 = \frac{3}{4}$

(b) $\frac{x-7}{5+x} = 2$

6. (a) If $l = \frac{4a+6}{3} + \frac{3b+2a}{5}$, find a when $b = 2$ and $l = 10$.

17.2: Word Problems

Equations arise in everyday life. Consider the following example;

Mary bought a number of oranges from Anita's kiosk. She then went to Mark's kiosk and bought the same number of oranges. Mark then gave her three more oranges. The oranges from the two kiosks were wrapped in different paper bags. On reaching her house, she found that a quarter of the first lot of oranges and a fifth of the second were bad. If in total six oranges were bad, find how many oranges she bought from Anita's kiosk.

Let the number of oranges bought at Anita's kiosk be x . Then, the number of oranges obtained from Mark's kiosk will be $x + 3$.

Number of bad oranges from Anita's kiosk was $\frac{x}{4}$.

Number of bad oranges from Mark's kiosk was $\frac{x+3}{5}$.

$$\therefore \text{Total number of bad oranges} = \frac{x}{4} + \frac{x+3}{5}$$

Thus, $\frac{x}{4} + \frac{x+3}{5} = 6$

Multiply each term of the equation by 20 (L.C.M. of 4 and 5) to get rid of the denominator.

$$20 \times \frac{x}{4} + 20 \left(\frac{x+3}{5} \right) = 6 \times 20$$

$$5x + 4(x + 3) = 120$$

$$5x + 4x + 12 = 120 \text{ (Removing brackets)}$$

Subtracting 12 from both sides,

$$9x = 108$$

$$\therefore x = 12$$

Thus, the number of oranges bought from Anita's kiosk was 12.

Solve the same problem by representing the number obtained from Mark's kiosk by x . **Generally, if any operation is performed on one side of an equation, it must also be performed on the other side.**

Example 3

Solve for x in the equation: $\frac{x+3}{2} - \frac{x-4}{3} = 4$

Solution

Eliminate the fractions by multiplying each term by 6 (L.C.M. of 2 and 3);

$$6 \left(\frac{x+3}{2} \right) - 6 \left(\frac{x-4}{3} \right) = 4 \times 6$$

$$3(x + 3) - 2(x - 4) = 24$$

$$3x + 9 - 2x + 8 = 24$$

$$x + 17 = 24$$

$$x = 7$$

Note the change in signs when brackets are removed.

Exercise 17.2

1. A piece of wire 200 cm long is bent to form a rectangular shape. One side of the rectangle is 4 cm longer than the other. Find the dimensions of the rectangle.
2. A man earns x shillings while his wife earns $\frac{1}{3}$ of this. After spending a third of their combined income, they have sh. 2 400 left. How much money does the man earn?

3. A has twenty shillings more than B. After A spends $\frac{1}{4}$ of his money and B $\frac{1}{5}$ of her's, they find that B has ten shillings more than A. How much money did each person have at the beginning?
4. In figure 17.1 $AB = AC$, $\angle ACB = (x + 5)^\circ$ and $\angle BCD = 4x^\circ$. Find all the angles of triangle ABC.

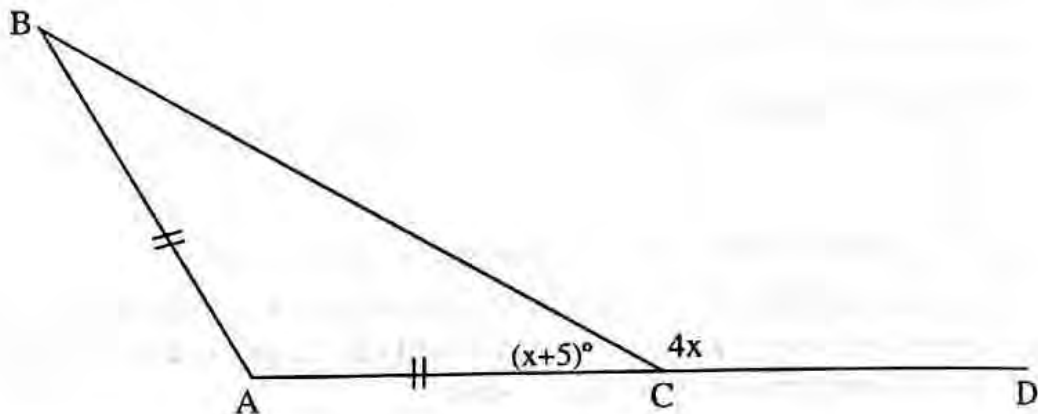


Fig. 17.1

5. The sum of four consecutive integers is -22 . Find the integers.
6. Of three consecutive odd integers, k is the middle one. If the difference between the products of the last two and first two is 28, find the three integers.
7. A man divides his savings amongst his son, daughter and wife. His wife gets sh. 6 000 more than the daughter and the son receives twice as much as his mother. If the total savings is sh. 126 000, find how much each gets.
8. A school Physics laboratory has two kinds of thermometers. One shows the temperature in degrees Farenheit ($^\circ\text{F}$) and the other in degrees Celcius ($^\circ\text{C}$). If $F = 32 + \frac{9C}{5}$, find:
 - (a) the reading in degrees Celcius when reading on the Farenheit thermometer is 212° .
 - (b) the temperature at which the two thermometers will show the same reading.
9. A man is 24 years older than his son. After 10 years he will be three times as old as his son. How old is the son?
10. The length of a rectangle is $(x + 3)$ cm. If the width of the rectangle is two thirds its length and the perimeter is 27 cm, find the area.

11. A farmer requires four bags of fertilizer and three packets of maize seed for his plot. The total cost for both is sh. 1 500. If a bag of fertilizer costs sh. 200 more than a packet of maize seed, calculate the price of:
- a bag of fertilizer.
 - a packet of maize seed.

17.3: Linear Equations in Two Unknowns

In the last section, we dealt with equations in only one unknown. We shall now deal with equations in two unknowns. Consider the following situation;

The cost of two skirts and three blouses is sh. 600. If the cost of one skirt and two blouses of the same quality is sh. 350, find the cost of each item.

Let the cost of one skirt be x shillings and that of one blouse be y shillings. The cost of two skirts and three blouses is $2x + 3y$ shillings. The cost of one skirt and two blouses is $x + 2y$ shillings.

So, $2x + 3y = 600$ (i)

$x + 2y = 350$ (ii)

Multiplying equation (ii) by 2 to get equation (iii);

$2x + 4y = 700$ (iii)

$2x + 3y = 600$ (i)

Subtracting equation (i) from (iii);

$y = 100$

From equation (ii), $x + 2y = 350$

But $y = 100$

$\therefore x + 200 = 350$

$x = 150$

Thus, the cost of one skirt is sh. 150 and that of a blouse is sh. 100.

In solving the problem above, we have reduced the equations from two unknowns to a single unknown in y by eliminating x . This is the **elimination method** of solving simultaneous equations.

Example 4

Solve the following simultaneous equations by elimination method;

(a) $a + b = 7$

$a - b = 5$

(b) $3a + 5b = 20$

$6a - 5b = 12$

(c) $3x + 4y = 18$

$5x + 6y = 28$

Solution

(a) $a + b = 7$ (i)

$a - b = 5$ (ii)

Adding (i) to (ii);

$$2a = 12 \Rightarrow a = 6$$

Subtracting (ii) from (i);

$$2b = 2$$

$$b = 1$$

(b) $3a + 5b = 20$ (i)

$6a - 5b = 12$ (ii)

In this example, it is easier to eliminate b.

Adding the two equations;

$$3a + 5b = 20$$
 (i)

$$6a - 5b = 12$$
 (ii)

$$9a = 32$$

$$\therefore a = \frac{32}{9}$$

Find the value of b.

(b) $3x + 4y = 18$ (i)

$5x + 6y = 28$ (ii)

In this example, it is not immediately obvious which unknown is easier to eliminate.

To eliminate x;

Multiplying (i) by 5 and (ii) by 3 to get (iii) and (iv) respectively and subtracting (iv) from (iii);

$$15x + 20y = 90$$
 (iii)

$$15x + 18y = 84$$
 (iv)

$$2y = 6$$

$$\therefore y = 3$$

Substituting $y = 3$ in (i);

$$3x + 12 = 18$$

$$\therefore x = 2$$

Note that the L.C.M. of 3 and 5 is 15.

To eliminate y;

Multiplying (i) by 3, (ii) by 2 to get (v) and (vi) and subtracting (v) from (vi);

$$9x + 12y = 54$$
 (v)

$$10x + 12y = 56$$
 (vi)

$$x = 2$$

Substituting $x = 2$ in (ii);

$$10 + 6y = 28$$

$$6y = 18$$

$$\therefore y = 3$$

Note:

- (i) It is advisable to study the equations and decide which variable is easier to eliminate.
- (ii) It is necessary to check your solution by substituting into the original equations. For example, in the last problem, our solution was $x = 2, y = 3$.

The original equations were:

$$3x + 4y = 18 \dots\dots\dots (i)$$

$$5x + 6y = 28 \dots\dots\dots (ii)$$

Putting $x = 2$ and $y = 3$ in equation (i);

$$\begin{aligned} 3 \times 2 + 4 \times 3 &= 6 + 12 \\ &= 18 \text{ (L.H.S)} \end{aligned}$$

Thus, L.H.S = R.H.S

Similarly, in equation (ii);

$$\begin{aligned} 5 \times 2 + 6 \times 3 &= 10 + 18 \\ &= 28 \text{ (L.H.S)} \end{aligned}$$

Exercise 17.3

Solve the following simultaneous equations by elimination method, then check your answers:

$$\begin{array}{ll} 1. \text{ (a) } m + n = 8 & \text{(b) } 2m + n = 7 \\ & m - n = 4 \end{array}$$

$$\begin{array}{ll} 2. \text{ (a) } 4x - 2y = -3 & \text{(b) } r - s = 1 \\ & 3x + y = 3 \end{array}$$

$$\begin{array}{ll} 3. \text{ (a) } p - q = 3 & \text{(b) } 3r - s = 8 \\ & p + q = -3 \end{array}$$

$$\begin{array}{ll} 4. \text{ (a) } \frac{1}{2}r + p = 3 & \text{(b) } 2u - v = 11 \\ & r - p = 1 \end{array}$$

$$\begin{array}{ll} 5. \text{ (a) } 5m + 2n = 19 & \text{(b) } 3p - 11q = 0 \\ & 3m - 4n = 1 \end{array}$$

$$\begin{array}{ll} 6. \text{ (a) } 3x + 4y = 6 & \text{(b) } x - 15y = 3 \\ & 4x - 6y = 11 \end{array}$$

$$\begin{array}{ll} 7. \text{ (a) } \frac{1}{u} + \frac{1}{v} = \frac{1}{5} & \text{(b) } \frac{x+y}{3} - \frac{x-y}{4} = \frac{2}{3} \end{array}$$

$$\begin{array}{ll} \frac{1}{u} - \frac{1}{v} = \frac{3}{5} & \frac{2x-3}{3} - \frac{2y+3}{4} = \frac{19}{12} \end{array}$$

$$8. \quad (a) \quad \frac{u+1}{6} - \frac{v-u}{2} = 6 \qquad (b) \quad \frac{3y-2x}{15} + \frac{2x+1}{3} = 2$$

$$\frac{2u-4}{3} + \frac{v-u}{2} = 2 \qquad \frac{2x-3y}{15} - \frac{1-x}{3} = 3$$

For each of the following questions, obtain a pair of simultaneous equations and solve them

9. The sum of two numbers is 201 and their difference is 69. Find the numbers.
10. The price of four handkerchiefs and three pairs of socks is sh. 340. If three handkerchiefs and four pairs of socks cost sh. 395, find the price of each item.
11. Biscuits are sold in two types of packets, **A** and **B**. Six packets of **A** and seven packets of **B** contain 84 biscuits, while three packets of **A** and two packets of **B** contain 33 biscuits. Find the number of biscuits in each type of packet.
12. Kirui and Karoki formed a trading partnership. In the first month, Kirui contributed $\frac{1}{12}$ of his salary towards the cost of the company. Karoki's contribution was $\frac{1}{10}$ of his monthly salary, giving a total of sh. 980. Their respective contributions the following month were $\frac{1}{8}$ and $\frac{1}{6}$ of their salaries. If the total contribution for the second month was sh. 1 550, what were their monthly salaries?

17.4: Solution by Substitution

In the example on the cost of skirts and blouses, we obtained the following equations:

$$2x + 3y = 600 \dots\dots\dots (i)$$

$$x + 2y = 350 \dots\dots\dots (ii)$$

Taking equation (ii) alone;

$$x + 2y = 350$$

Subtracting $2y$ from both sides;

$$x = 350 - 2y \dots\dots\dots (iii)$$

Substituting this value of x in equation (i);

$$2(350 - 2y) + 3y = 600$$

$$700 - 4y + 3y = 600$$

$$y = 100$$

Substituting this value of y in equation (iii);

$$\begin{aligned} x &= 350 - 2y \\ &= 350 - 200 \end{aligned}$$

$$x = 150$$

This method of solving simultaneous equations is called the **substitution method**.

Exercise 17.4

1. (a) $2x + 3y = 10$

Express x in terms of y .

(b) $\frac{1}{2}p - 3q = 7$

Express q in terms of p .

2. (a) $\frac{a}{3} + \frac{2b}{5} = 13$

Express b in terms of a .

(b) $0.5 - \frac{r-s}{4} = 3$

Express r in terms of s .

Solve each of the following equations by substitution method:

3. (a) $5x + 6y = 22$

$$2x + 10y = 10$$

(b) $2p + 7q = -19$

$$\frac{1}{2}p - 3q = 7$$

4. (a) $\frac{a}{3} + \frac{2b}{5} = -3$

$$\frac{a}{5} + \frac{b}{5} = 6$$

(b) $0.5 - \frac{r-s}{4} = 3$

$$0.7 - \frac{r+s}{3} = 10$$

5. (a) $x + y = 7$

$$3x + y = 15$$

(b) $2x + y = 1$

$$10x + 3y = 17$$

6. (a) $x + 7y = -3$

$$17x - 3y = 10$$

(b) $7x + 15y = 22$

$$8x + 17y = 25$$

7. (a) $9x - y + 7 = 0$

$$13x - 4y + 5 = 0$$

(b) $\frac{x+y}{6} - \frac{x+y}{4} = \frac{-1}{4}$

$$\frac{x+y}{5} + \frac{x+y}{6} = 1\frac{1}{10}$$

8. $\frac{1}{u} - \frac{1}{v} = \frac{1}{8}$

$$\frac{1}{u} + \frac{1}{v} = \frac{3}{8}$$

Exercise 17.5

Solve each of the following using any suitable method:

1. (a) $x + 3y = 8$

$$5x + 7y = 24$$

(b) $\frac{1}{2}x + \frac{1}{2}y = 2.5$

$$\frac{2}{5}x + y = 3.5$$

2. (a) $0.2x + 0.5y = 3.5$
 $7x - 6y = 5$ (b) $2p + 4q = 8$
 $0.5p - 32q = -10$
3. (a) $5x - 8y = 2$
 $7x + 3y = 17$ (b) $5x - 8y = \frac{1}{2}$
 $3x + 2y = 2$
4. (a) $3x - 15y = 1$
 $4x + 5y = 2$ (b) $5x + 3y = 3.4$
 $7x - 4y = 2.3$
5. (a) $16x - 15y = 1$
 $2x + 3y = \frac{11}{16}$ (b) $\frac{1}{3}(x + y) = 2$
 $\frac{1}{4}(x - y) = 1$
6. $0.5x + 0.25y = 4$
 $0.3x + 1.4y = 5.6$
7. The total number of pupils who play tennis and volleyball in a class is 14. If there are six more pupils playing volleyball than tennis, find the number of pupils in each team, given that no pupil plays more than one game.
8. The cost of three sandwiches and two cups of tea is 60 shillings. If two sandwiches and three cups of tea cost 65 shillings, find the cost of:
 (a) a sandwich.
 (b) a cup of tea.
9. Musa spent sh. 207 to buy seven exercise books and four pens while Allan spent sh. 165 to buy five exercise books and five pens of the same type. Find the cost of each item.
10. The mass of two bags of beans and three bags of salt is 410 kg. If the mass of three bags of beans and two bags of salt is 390 kg, find the mass of each bag.
11. A courier firm which deals with parcels charges a fixed amount of money on the first 1 kg. For every 1 kg extra or part thereof, the rate per kilogram is different. Jane posts a parcel weighing 6 kg at a cost of sh. 350 while her friend Mary pays sh. 230 to post a parcel of mass 4 grams. Find:
 (a) the charge on the first 1 kg of parcel weight.
 (b) the charge on each subsequent kilogram for any parcel.
12. The distance from A to B is d km and that from B to C is x km. If a bus maintains an average speed of 50 km/h between A and B and 60 km/h between B and C, it takes 3 hours to travel from A to C. If it maintains 60 km/h between A and B and 50 km/h between B and C, the journey takes 8 minutes less. What is the distance from A to C via B?

Chapter Eighteen

COMMERCIAL ARITHMETIC

18.1: Introduction

In this section, we shall be dealing with calculations pertaining to business transactions. The medium of any business transaction is usually called the currency. The Kenya currency consists of a basic unit called a shilling. 100 cents are equivalent to one Kenya shilling, while a Kenyan pound is equivalent to twenty Kenya shillings. The following abbreviations will be adopted for use in connection with the Kenya currency: cent abbreviated as ct., shilling as sh, and pound as £. The letter K is put before each of these abbreviations to distinguish the Kenya currency from that of other countries.

Exercise 18.1

1. Express sh. 5.75 as a percentage of one Kenya pound.
2. Calculate $33\frac{1}{3}\%$ of K£3.6.
3. How many rubbers can be bought with sh. 100, if each costs sh. 2.75?
How much balance is expected correct to the nearest cent?
4. How many pens, each costing sh. 8.00, can be purchased with sh. 160?
5. Otieno spent sh. 800 in three days. On the first day, he spent sh. 200 more than the second day and sh. 100 less than the third day. What was his total expenditure on the second and third day?
6. A man's annual earnings is K£5 400. Calculate his monthly earnings in shillings.
7. If a man earns sh. 235 per day, how much was he paid for the month of February 2003 if he was absent for three days and was not expected to work on Saturdays and Sundays?
8. A pupil bought the following items:
6 books @ sh. 20.00
2 pencils @ sh. 10.00
A school bag @ sh. 250.00
A rubber @ sh. 10.50.
Calculate the total expenditure on the items.
9. Mary bought x mangoes at n cents each and y oranges at m cents each.
How much money did she spend:

- (a) in shillings?
 - (b) in cents?
 - (c) If she had p shillings initially, how much balance did she get after the purchase?
10. Kilimo bought meat at sh. 37.50, vegetables at sh. 15.00 and maize meal at sh. 38.50. How much balance did she get from a hundred-shilling note?
11. Jane bought the following items:
3 kilograms of sugar @ sh. 45.50
3 packets of milk @ sh. 25.00
 $1\frac{1}{2}$ loaves of bread @ sh. 22.00
Tea leaves for sh. 46.00
How much balance did she get from sh. 300?
12. The cost of a goat is sh. 1 200, that of a ram sh. 1 400 and that of one bull sh. 12 700. If Mr. Nyang'au bought 7 goats, 12 rams and 6 bulls, how much money did he spend in total?
13. At the close of business on a certain day, a kiosk owner realised that he had sold the following items from his stock:
7 crates of soda @ sh. 480
50 buns @ sh. 7.50
13 trays of eggs @ 145.00
30 loaves of bread @ sh. 22.00
25 packets of milk @ sh. 25.00.
What was his total collection on that day?
14. A farmer bought 45 iron sheets at sh. 450.00 each, 30 bags of cement at sh. 550.00 a bag, 25 kilograms of nails at sh. 150.00 a kilogram, 60 tins of paint at sh. 520.00 a tin, 28 hardboard sheets at sh. 500.00 a sheet and 16 rolls of barbed wire at sh. 3 000.00 a roll. How much money did he spend on the items altogether?
15. A school decides to equip its wood workshop with the following items:
25 jack planes at sh. 3 000 each, 30 hacksaws at sh. 300 each, 26 hammers at sh. 250 each, 25 screw-drivers at sh. 80 each and 11 vices at sh. 2 500 each. Calculate the amount of money required for this equipment.

18.2: Currency Exchange Rates

The Kenya currency cannot be used for business transactions in other countries. To facilitate international trade, many currencies have been given different values relative to one another. These are known as **exchange rates**.

The table below shows the exchange rates of major international currencies at the close of business on a certain day in the year 2002. The buying and selling column represent the rates at which banks buy and sell these currencies.

Currency	Buying	Selling
1 US Dollar (\$)	78.4133	78.4744
1 Sterling Pound (£)	114.1616	114.3043
1 Euro (€)	73.4226	73.52953
1 South African Rand	7.8842	7.9141
1 Ksh/Ush	22.8915	23.0011
1 Ksh/Tsh	12.2428	12.3607
1 UAE Dirham	21.3480	21.3670
100 Japanese Yen (¥)	62.8011	62.8822
1 Indian Rupee (Rs)	1.5986	1.5999
1 Saudi Riyal	20.9080	20.9259
1 Swiss Franc	49.8800	49.9301
1 Canadian Dollar (Can \$)	52.0784	52.1572
1 Australian Dollar (Aus \$)	44.7997	44.8960

The rates are not fixed. When changing the Kenyan currency to foreign currency, the bank sells to you. Therefore, we use the selling column rate. Conversely, when changing foreign currency to Kenya currency, the bank buys from you, so we use the buying column rate.

Example 1

Convert each of the following currencies to its stated equivalent:

- US \$ 305 to Ksh.
- 530 Dirham to €.

Solution

- The bank buys US \$ 1 at Ksh. 78.4133.

$$\begin{aligned}
 \therefore \text{US \$ 305} &= \text{Ksh. } (78.4133 \times 305) \\
 &= \text{Ksh. } 23916.0565 \\
 &= \text{Ksh. } 23916.05 \text{ (to the nearest 5 cents)}
 \end{aligned}$$

- The bank buys 1 Dirham at Ksh. 21.3480

CHUGU BOYS' HIGH SCHOOL

P. O. Box 565-60200,

MUJI

Tel: +254 716 457 080 / 735 261

$$\begin{aligned}
 \therefore 530 \text{ Dirham} &= \text{Ksh. } (21.3480 \times 530) \\
 &= \text{Ksh. } 11314.44 \\
 &= \text{Ksh. } 11314.40 \text{ (round off downwards to the nearest } 0.05)
 \end{aligned}$$

The bank sells 1 € at Ksh. 73.52953

$$\begin{aligned}
 \therefore 530 \text{ Dirham} &= € \frac{11314.44}{73.52953} \\
 &= € 153.876
 \end{aligned}$$

Example 2

A lady bought US \$ 5 000. From this, she spent US \$ 1 000 on a return ticket and US \$ 1 750 while in USA. Upon her return, she sold the remaining dollars.

- How much did she pay to the bank in Kenya shillings to get the US \$ 5 000?
- How much, in Kenya shillings, did she get after selling the remaining amount to the bank?

Solution

- The bank sells US \$ 1 at Ksh. 78.4744.

$$\begin{aligned}
 \text{Therefore, US \$ 5 000} &= \text{Ksh. } (78.4744 \times 5\,000) \\
 &= \text{Ksh. } 392\,372
 \end{aligned}$$

- The lady had US \$ 5 000.

$$\text{Total expenditure US \$ } (1\,000 + 1\,750) = \text{US \$ } 2\,750$$

$$\begin{aligned}
 \text{Balance} &= \text{US\$ } (5\,000 - 2\,750) \\
 &= \text{US\$ } 2\,250
 \end{aligned}$$

The bank buys US \$ 1 at Ksh. 78.4133.

$$\begin{aligned}
 \therefore \text{US \$ } 2\,250 &= \text{Ksh. } 78.4133 \times 2\,250 \\
 &= \text{Ksh. } 176\,429.925 \\
 &= \text{Ksh. } 176\,429.90 \text{ (to the nearest 10 cents)}
 \end{aligned}$$

Exercise 18.2

- For each of the following currencies, obtain the stated equivalent:
 - £ 265 Sterling in Kenya shillings.
 - € 238 in Kenya shillings.
 - US \$ 259 to Euro.
 - Ksh. 44 155 to US Dollars.
 - 389 Swiss Francs to Sterling Pounds.
 - 1 000 000 Japanese Yen to Kenya shillings.

- (g) US \$ 5 200 to Kenya shillings.
 - (h) Ksh. 4 500 to Japanese Yen.
 - (i) £ 5 080 Sterling to US Dollars.
 - (j) € 500 to Sterling pounds.
 - (k) 500 Dirham to Kenya shillings.
 - (l) Ksh. 5 000 to Canadian Dollars.
 - (m) 200 Saudi Riyal to Kenya shillings.
2. The price of factory equipment to be imported from the Netherlands was quoted at 7 805 000 Euros. Express this in Kenya shillings.
 3. A tourist visited Kenya and sold 500 Euros for his use in the country. How much Kenya shillings did he get?
 4. It costs US \$ 500 for an air ticket to USA. How much will a businessman spend in buying the dollars in Kenya shillings?
 5. An exporter bought Sterling Pounds equivalent to Ksh. 500 000. After settling bills worth £ 1 000 Sterling, he exchanged the balance for Euros. If he purchased goods worth 100 Euros, calculate his balance in Kenya shillings.
 6. Four businesswomen were to go on a trip to Britain. Calculate the cost of this trip in Kenya Shillings if each person required:
 - breakfast for three days at £3 sterling per breakfast.
 - 4 lunches at £ 5.50 Sterling each,
 - 3 dinners at £ 6.00 Sterling each,
 - 4 return tickets at Ksh. 48 000 each,
 - other miscellaneous expenditure worth £ 500 Sterling per person.
 7. A car was bought in Japan at a factory price of 500 000 Japanese Yen and imported into Kenya. The cost of shopping and insurance on this car was 80% of its price and the duty charge a further 100% of the cost. Calculate, in Kenya shillings, what the importer paid for the car.
 8. John intended to import a car worth € 15 000 from France.
 - (a) How much did he pay in Kenya shillings to acquire the Euros?
 - (b) If he later changed his mind and instead re-converted the money to Kenya shillings, how much did he end up with?

18.3: Profit and Loss

The difference between the cost price and the selling price is either profit or loss. If the selling price is greater than the cost price, the difference is a profit and if the selling price is less than the cost price, the difference is a loss.

Note:

Selling price – cost price = profit

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100$$

Cost price – selling price = loss

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100$$

Example 3

Tirop bought a cow at sh. 18 000 and sold it at sh. 21 000. What percentage profit did he make?

Solution

Selling price = sh. 21 000

Cost price = sh. 18 000

Profit = sh. (21 000 – 18 000)
= sh. 3 000

$$\begin{aligned}\text{Percentage profit} &= \frac{3\,000}{18\,000} \times 100 \\ &= 16\frac{2}{3}\%\end{aligned}$$

Example 4

Jane bought a dress at sh. 3 500 and later sold it at sh. 2 800. What percentage loss did she incur?

Cost price = sh. 3 500

Selling price = sh. 2 800

Loss = sh. (3 500 – 2 800)
= sh. 700

$$\begin{aligned}\text{Percentage loss} &= \frac{700}{3\,500} \times 100 \\ &= 20\%\end{aligned}$$

Exercise 18.3

1. Abdi bought a pair of trousers for sh. 650 and later sold it at sh. 720. What profit did he make?
2. A trader bought a 50 kg bag of sugar at sh. 2 100. She sold the sugar at sh. 50 per kilogram. What was the percentage profit?

3. Parpai bought a textbook at sh. 450 and later sold it at sh. 400. What was the percentage loss?
4. A businessman bought a bag containing 50 mangoes for sh. 250. He sold the mangoes at sh. 10 each. If 5 mangoes were bad, what was his percentage profit?
5. A lady sells blouses at sh. 1 500 each. She makes a profit of sh. 150 for each blouse. How much does she pay for each?
6. Tina bought a bag containing 80 tomatoes for sh. 270. She sold the tomatoes in piles of four, making a profit of 50%. For how much did she sell each pile?

18.4: Discount

A shopkeeper may decide to sell an article at reduced price. The difference between the marked price and the reduced price is then referred to as the discount. The discount is usually expressed as a percentage of the actual price.

Example 5

The price of an article is marked at sh. 120.00. A discount is allowed and the article sold at sh. 96.00. Calculate the percentage discount.

Solution

Actual price = sh. 120.00

Reduced price = sh. 96.00

Discount = sh. (120.00 – 96.00)
= sh. 24

Percentage discount = $\frac{24}{120} \times 100$
= 20%

Exercise 18.4

1. The marked price of a shirt was sh. 500.00. The shopkeeper offered a discount and sold it at sh. 480. Calculate the percentage discount.
2. Mama Mwanyumba bought the following goods from a supermarket:
3 kg of sugar @ sh. 46.00
2 loaves of bread @ sh. 22.50
4 packets of milk @ sh. 25.50
 - (a) How much did she pay for the goods?
 - (b) How much would she have paid for the goods had she been allowed a 10% discount?

3. Jane paid sh. 12 000 for a T.V. set after she was allowed a discount of $16\frac{2}{3}\%$. What was the marked price of the T.V?
4. A school bought textbooks worth sh. 27 027 from a bookseller. If the bookseller allowed a discount of 10%, what was the cost of the books without the discount?
5. A farmer was allowed a cash discount of sh. 175 on farm implements worth sh. 3 500. What was the percentage discount?
6. The following items were displayed in a hardware shop:

<i>Item</i>	<i>Price per unit (sh.)</i>	<i>Discount allowed %</i>
Panga	180	8
Jembe	350	5
Bow-saw	250	5
Tool box	2 500	$7\frac{1}{2}$
Fork jembe	400	5

If a farmer bought three pangas, four jembes, one bow-saw, three fork jembes and one tool box, calculate:

- (a) The total cash discount on all the items bought.
 - (b) The total amount paid for the items.
 - (c) The amount of money he would have paid if no discount was allowed.
 - (d) The total percentage discount.
7. Calculate the marked price on a bag of cement selling at sh. 570 after a discount of 5% is offered.
 8. An umbrella and a pen are sold at a discount of 8% and 3% respectively. Calculate the overall discount offered on the two commodities, if the cost of the umbrella is four times that of the pen.

18.5: Commission

A commission is an agreed rate of payment, usually expressed as a percentage, to an agent for his services.

Example 6

Mr Nyongesa, a salesman in a soap industry, sold 250 pieces of toilet soap at sh. 45.00 and 215 packets of detergent at sh. 75.00 per packet. If he got a 5% commission on the sales, how much money did he get as commission?

Solution

Sales for the toilet soap was $250 \times 45 = \text{sh. } 11\,250$

Sales for the detergent was $215 \times 75 = \text{sh. } 16\,125$

$$\begin{aligned}\text{Commission} &= \frac{5}{100} (11\,250 + 16\,125) \\ &= \frac{5}{100} \times 27\,375 \\ &= \text{sh. } 1368.75\end{aligned}$$

Exercise 18.5

1. Miss Onyango sold goods worth sh. 12 000 at a commission of 5%. How much commission did she get?
2. Chris works as a salesman. He is paid a salary of sh. 24 000 per month plus a commission of $2\frac{1}{2}\%$ of his sales. In one month, he sold goods worth sh. 100 000. How much did he earn altogether during that month?
3. A salesman is paid a salary of sh. 12 000 per month. He is also paid a commission of 2% on sales up to sh. 15 000 and $2\frac{1}{2}\%$ on sales above that amount. In one month, he sold goods worth sh. 25 000. How much was he paid that month?
4. Simon earned sh. 400 as a commission for a sale of goods worth sh. 16 000. What would be his earnings for a total sale of sh. 7 000?
5. A saleswoman was paid a monthly salary of sh. 20 000 plus commission on goods sold. In one month, she sold goods worth sh. 40 000. At the end of that month, her total earnings were sh. 21 200. What percentage commission was she given?
6. A saleslady was paid a monthly salary plus a commission of 8% on goods sold. In one month, she sold goods worth sh. 64 000 and her total earnings were sh. 23 120. What was her basic salary without commission?
7. A salesman earns 25% commission. His sales amounted to sh. 2 450 after giving buyers a 2% discount. Calculate his commission. Suppose all the goods were sold at the marked price, what would be his earnings?

Mixed Exercise 2

1. Solve the following pairs of simultaneous equations:

(a) $3x - \frac{1}{3}y = \frac{1}{2}$

(b) $7x + \frac{1}{5}y = 0.4$

$$x + \frac{1}{3}y = \frac{5}{6}$$

$$x + \frac{1}{3}y = \frac{1}{3}$$

(c) $25x - 16y = 36$

(d) $4x - \frac{3}{4}y = 5.1$

$$2y - x = 2\frac{3}{8}$$

$$\frac{1}{3}x - 5y = -5.5$$

2. The ratio of the adjacent sides of a rectangle is 4 : 5. Find the dimensions of the rectangle if its length is $x + 1$ cm and the width is $(x - 3)$ cm. Hence, determine the ratio of the area of the rectangle to its perimeter.
3. The cost of high grade tea is $(x + 2)$ shillings per kilogram. Find the cost of $\frac{y-5}{3}$ kilograms of the same grade of tea.
4. Find the ratios $x : y : z$ if:
- (a) $x : y = 9 : 10$ and $y : z = 5 : 3$
- (b) $x = 4y$ and $2y = 3z$
5. If $x : y = 9 : 11$, find the ratio $(5x - 3y) : (2x + 3y)$.
6. Two numbers are in the ratio $s : t$. If the first is a , find the second.
7. The ratio of the radii of two spheres is 4 : 5. What is the ratio of:
- (a) their surface areas.
- (b) their volumes.
8. The length of each side of a square is increased by 15%. Calculate the percentage increase in its area.
9. There is a 25% loss when an article is sold at sh. 200. At what price should it be sold in order to make a profit of 5%?
10. A manufacturer sells goods to a shopkeeper at a profit of 15%. The shopkeeper sells them so as to make a profit of 25%. During a sale, the shopkeeper reduced his prices by 10%. Find, to the nearest shilling, the factory price of an article which is marked at sh. 450 during the sale.
11. The area of a trapezium is 20 cm^2 and the lengths of the two parallel sides are 4 cm and 6 cm. Calculate the percentage increase in its area if the perpendicular distance between the parallel sides is increased by 2 cm.

12. Solve each of the following equations:

(a) $\frac{2x}{y-7} = 1$ and $3x : y - 5 = 1 : 2$

(b) $\frac{x+2}{x-1} = 3 = \frac{2}{x-1}$

(c) $\frac{x+2}{3} + \frac{3}{2} \left(\frac{x-4}{5} \right) - 1 = 2\frac{2}{3}$

(d) $\frac{r+1}{2\frac{1}{2}} - \frac{7}{8} \left(\frac{r+3}{3} \right) + \frac{r}{5} = \frac{r}{0.3}$

13. A rectangle which is three times as long as it is wide has the same perimeter as a square of area 64 cm^2 . What is the length of the rectangle?
14. A man hired a car for two weeks for sh. 10 500. How much would it have cost him to hire it for nine days?
15. Mary raised sh. 500 000 for a study course in Britain. She bought an air ticket for sh. 80 000 and converted the balance to Sterling Pounds. Once in Britain, she bought winter clothes worth £ 250 Sterling and paid £ 2 060 Sterling as tuition fees. How much in Sterling Pounds did she remain with?
16. The price of factory equipment to be imported from Britain was quoted as £7 805 000 Sterling. Express this in Kenya Pounds.

Chapter Nineteen

CO-ORDINATES AND GRAPHS

19.1: Co-ordinates

The position of a point in a plane is located using an ordered pair of numbers called co-ordinates and written in the form (x, y) . The first number represents distances along the x-axis and is called the x co-ordinate. The second number represents distance along the y-axis and is called the y co-ordinate. The x and y axes intersect at the point $(0, 0)$, called the origin.

In figure 19.1, the position of the point P is $(3, 2)$. Find the positions of the points Q, R, S, T, U and V.

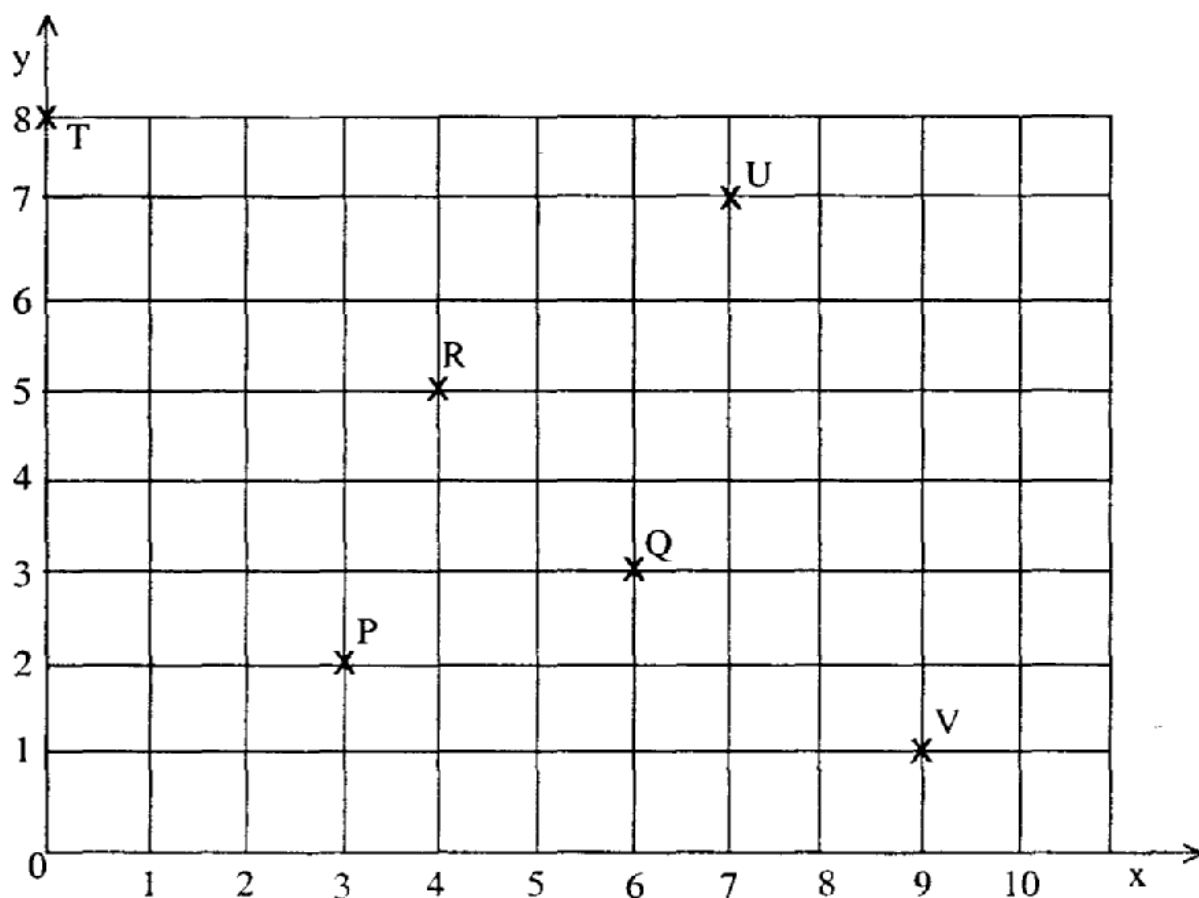


Fig.19.1

What would happen if you tried to plot the following points on the same figure; $A(-2, 3)$, $B(-3, -4)$, $C(3, -1)$?

We would need to extend each of the axes to include negative numbers before we can locate the points A, B and C. Figure 19.2 shows the extended axes with the points $A(-2, 3)$, $B(-3, -4)$ and $C(3, -1)$ marked.

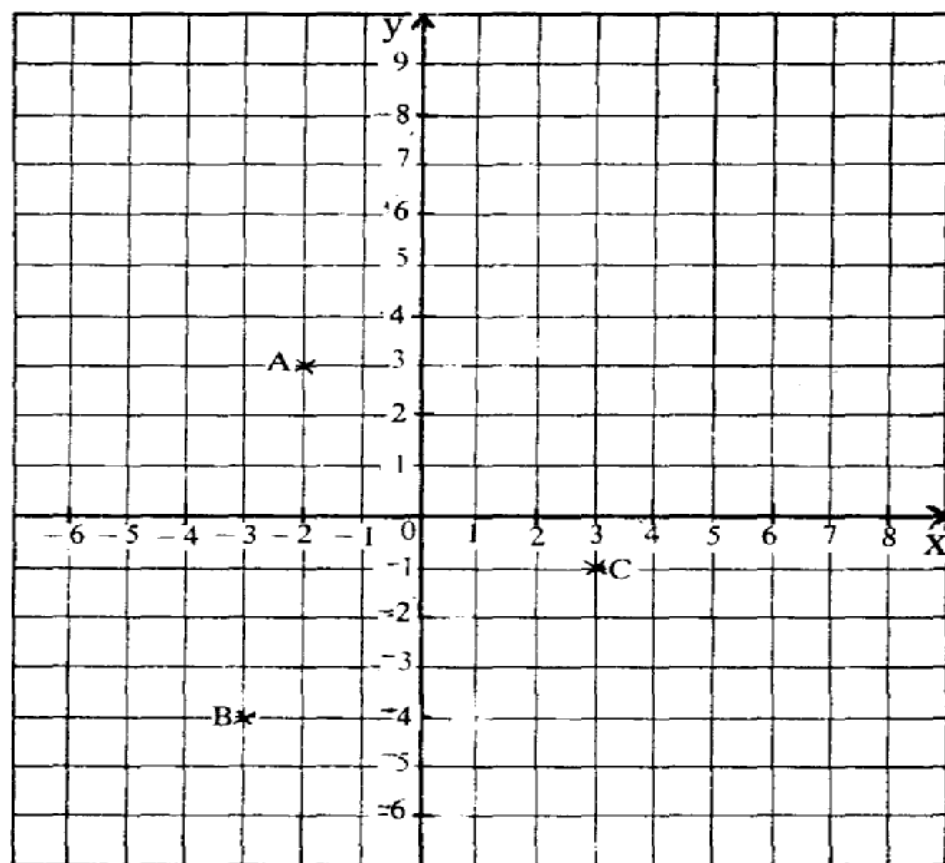


Fig. 19.2

Draw a pair of axes on a squared paper and plot the following points:
 L (4, 2), M(-4, -2), N(-2, 4), S(3, -5), T(5, -3), N(-6, -1), V(1, -6),
 X(-4.25, -0.8), Y(2.75, -3.25).

The x and y axes divide the plane into four regions. Each of these regions is called a **quadrant**. The quadrants are named as first, second, third and fourth, starting with the top right hand quadrant and moving in an anticlockwise direction, as indicated in figure 19.3.

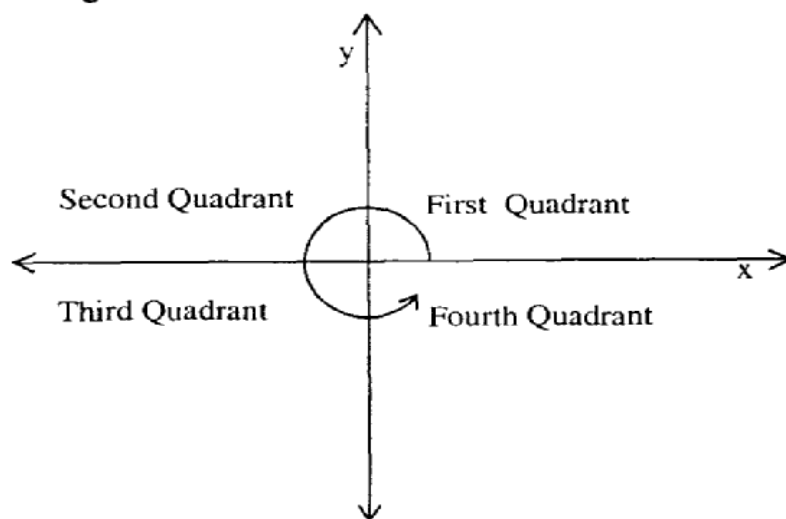


Fig. 19.3

This system of locating points using two axes at right angles is called the rectangular **cartesian co-ordinate system**.

Co-ordinate systems are also used to locate places on the surface of the earth. These are:

- (i) latitude and longitudes.
- (ii) grid references.

More information about these aspects of locating points may be found in Geography titles on map reading.

Exercise 19.1

1. Plot the following points on graph paper and name the quadrant in which each point lies:
(a) A $(-7, -8)$ (b) B $(-8, 7)$ (c) C $(-5, 0)$ (d) D $(1, 5)$
(e) E $(-1, -1)$ (f) F $(-3, 2)$ (g) G $(4, -5)$ (h) H $(-6, 7)$
2. ABCD is a rectangle. If A $(0, 0)$, B $(4, 0)$ and C $(4, 6)$ are three vertices of this rectangle, find:
(a) the co-ordinates of D, the fourth vertex.
(b) the co-ordinates of the point of intersection of the diagonals of the rectangle.
3. The vertices of a triangle are A $(-3, 0)$, B $(3, -3)$ and C $(3, 4)$. Find the area of the triangle.
4. Three vertices of a parallelogram ABCD are A $(-4, -4)$, B $(-3, -5)$ and C $(-3, -2)$. Find the co-ordinates of D, the fourth vertex.
5. Draw a circle with centre C $(-2, -3)$ and passing through R $(-2, 2)$.
(a) Find the co-ordinates of the point on the circle with integer co-ordinates which:
(i) lies in the first quadrant.
(ii) has the smallest value of y.
(iii) has the largest value of x.
(iv) has the smallest value of x.
(v) lies in the third quadrant and has its x-co-ordinate equal to -5 .
(b) Find the distance between the two points on the circle which:
(i) lie on the x-axis.
(ii) have their y - co-ordinate equal to 2.
(iii) have their x - co-ordinate equal to 2.
6. In figure 19.4, the lines intersect at right angles:

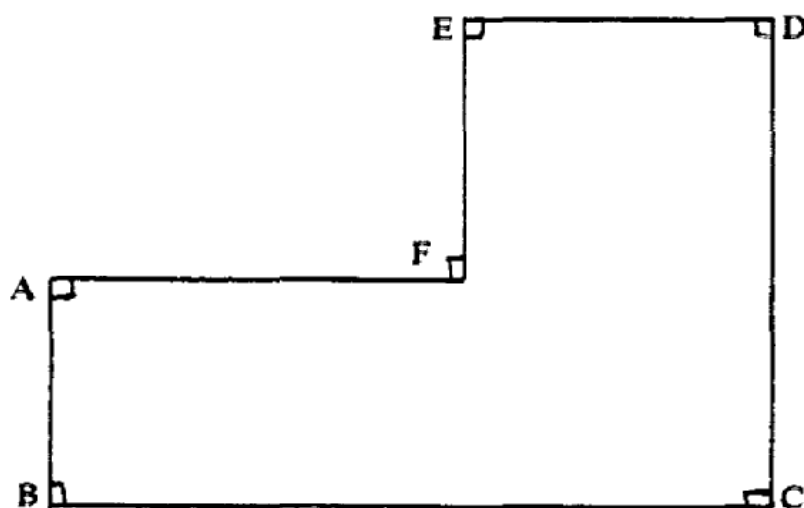


Fig. 19.4

If the co-ordinates of A, C, D and F are $(-3, 1)$, $(3, -3)$, $(3, 4)$ and $(1, 1)$ respectively, find the co-ordinates of B and E.

7. The vertices of a parallelogram are A $(-3, 1)$, B $(3, 0)$, C $(2, -4)$ and D (x, y) . Find x and y.
8. The vertices of a triangle are A $(-0.5, -0.8)$, B $(-0.5, -2.5)$ and C $(2, -0.8)$. Find the area of the triangle.

19.2: The Graph of a Straight Line

Consider the linear equation $y = 3x + 5$. Some corresponding values of x and y are given in the table 19.1:

Table 19.1

x	-2	-1	0	1	2
y	-1	2	5	8	11

If we plot these points, we notice that they all lie on a straight line, as shown in figure 19.5.

Note that the graph of a straight line extends indefinitely beyond the plotted points in either direction.

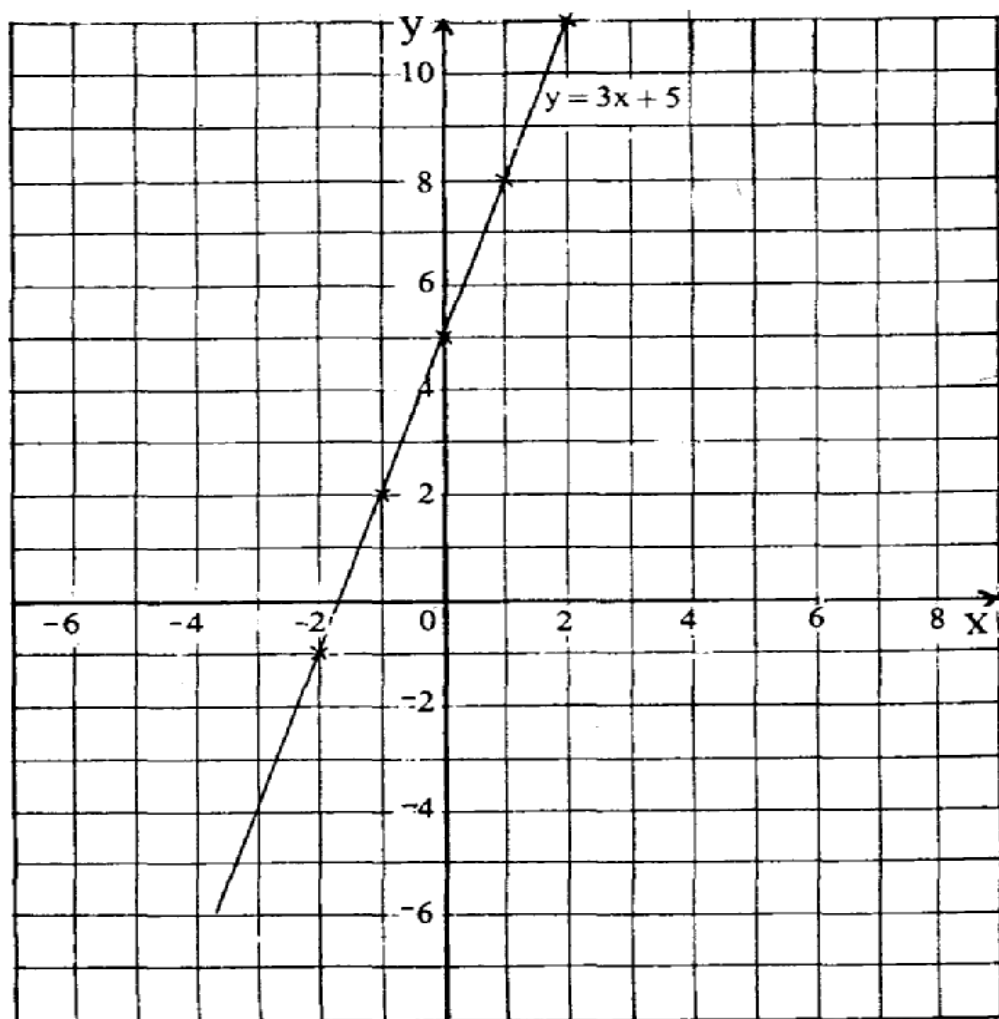


Fig. 19.5

Exercise 19.2

1. For each of the following linear equations, copy and complete the table and hence draw the corresponding graph:

(a) $y = 4x + 3$

x	-2	-1	0	1	2
y					

(b) $y = 6x + \frac{1}{2}$

x	-1	0	1	2	3	4
y						

(c) $3x + 2y = 4$

x	-1	0	1	2	3	4
y						

2. Draw the graph of each of the following:

(a) $y + 2x = 5$

(b) $\frac{y}{2} + 2x = 5$

(c) $\frac{y-x}{4} = \frac{2x-y}{3}$

Generally, two points are sufficient to determine a straight line, but in practice we use a third point as a check. For some equations, it is convenient to choose points where the straight line cuts the axes. For example, the graph of the equation $y = 4x + 8$ cuts the x-axis at (2, 0) and the y-axis at (0, 8).

There are however other equations in which this kind of choice is not suitable. For example, the graph of the equation $y = 7x - 1$ cuts the x-axis at $(\frac{1}{7}, 0)$ and the y-axis at (0, 1). The point $(\frac{1}{7}, 0)$ is not easy to plot. It is advisable to choose two points which can be plotted easily. For example, the points (1, 6), (2, 13) e.t.c, satisfy the equation $y = 7x - 1$ and are easier to plot than $(\frac{1}{7}, 0)$.

Exercise 19.3

1. By choosing any two suitable points, draw the graphs of each of the following. Use a third point as a check:

(a) $x + y = 0$

(b) $3x - y = 3$

(c) $5x + 3y = 6$

(d) $6x - 5y = 10$

(e) $13y - 6x - 18 = 0$

(f) $17x - 4y - 10 = 0$

2. Write the following equations in the form $y = mx + c$, where m and c are constants. In each case, state the value of m and c:

(a) $2x - y + 3 = 0$

(b) $4x + 3y = -8$

(c) $\frac{1}{5}x + 2y = 7$

(d) $\frac{2y+x}{3} = \frac{7x-y}{2}$

(e) $y - 4 = 0$

(f) $3x + 4 = -y$

3. For each of the following pairs of lines:

(i) rewrite the equations in the form $y = mx + c$, and

(ii) draw their graphs on the same axes:

(a) $y = 2x + 3$

(b) $3y + 4x = 6$

$2y = 4x + 5$

$6y = 1 - 8x$

- | | |
|-----------------------|-----------------|
| (c) $3y - 6x + 5 = 0$ | (d) $y = 5$ |
| $3y - 4x - 7 = 0$ | $y + 3 = 0$ |
| (e) $y = 4x + 8$ | (f) $y - 6 = 0$ |
| $2y - 8x = 16$ | $y - 2x = 0$ |

19.3: Graphical solutions of Simultaneous Linear Equations

So far we have seen that equations of the form $ax + by = c$ represent a straight line. When two such linear equations are graphically represented, their graphs may or may not intersect. The co-ordinates of the point of intersection represent the solution to the linear simultaneous equations. For example, in solving the simultaneous equations $x + 3y = 5$ and $5x + 7y = 9$ graphically, the graphs of the two equations are drawn, as in the figure 19.6.

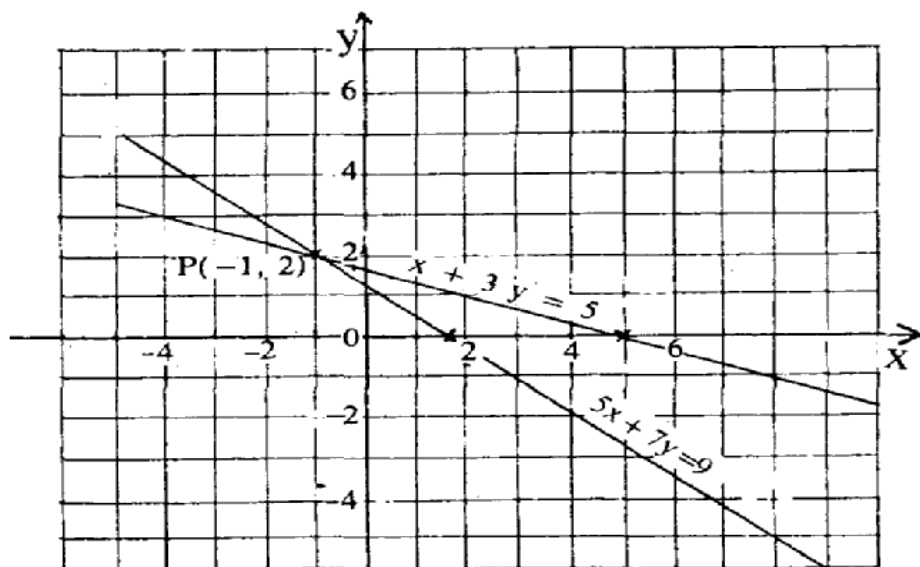


Fig. 19.6

The two lines intersect at $P(-1, 2)$. The solution to the simultaneous equations is, therefore, $x = -1$ and $y = 2$.

Exercise 19.4

Solve each of the following pairs of simultaneous equations graphically:

- | | |
|---------------------|----------------------|
| 1. (a) $y = 3x - 1$ | (b) $2x - y = 3$ |
| $2y + 2x = 3$ | $7x + 2y = 16$ |
| 2. (a) $2x - y = 3$ | (b) $5x + y = 7$ |
| $x + 2y = 14$ | $3x + 2y = 0$ |
| 3. (a) $y = 2x - 1$ | (b) $3y - x - 4 = 0$ |
| $2y + x + 7 = 0$ | $2x - 5y + 7 = 0$ |

4. (a) $3x + 4y = 3.5$
 $7x - 6y = 0.5$ (b) $2y + 3x + 7 = 0$
 $3y - x + 2 = 0$
5. (a) $4x - y = 2$
 $6x + 4y = 25$ (b) $4x - 2y = 4$
 $2x - 3y = 0$
6. (a) $x + y + 1 = 0$
 $4x - 8y = 5$ (b) $4x + 2y = \frac{8}{3}$
 $2x - 3y = \frac{4}{3}$
7. (a) $x - y = 0.3$
 $2x + 3y = 3.1$ (b) $9x + 9y = 5$
 $x - 3y = 1$
8. $\frac{y-1}{2} + \frac{x+3}{3} = \frac{x-3}{2}$

$$\frac{y+1}{3} = \frac{x-2}{2}$$

19.4: General Graphs

Graphs find a wide application in science and many other fields. It is therefore important to master the techniques of drawing graphs that convey information easily and accurately. Of these techniques, one of the most important is the choice of appropriate scales. We illustrate this by considering the following situations:

- (i) A man walks for four hours at an average speed of 5 km/h. Table 19.2 (a) shows the distance covered at given times.
- (ii) A motorist drives for four hours at an average speed of 80 km/h. Table 19.2 (b) illustrates the situation.

Table 19.2

(a)

<i>Time in hours (t)</i>	1	2	3	4
<i>Distance in km (S)</i>	5	10	15	20

(b)

<i>Time in hours (t)</i>	1	2	3	4
<i>Distance in km (S)</i>	80	160	240	320

The corresponding graph for table 19.2 (a) is given in figure 19.7.

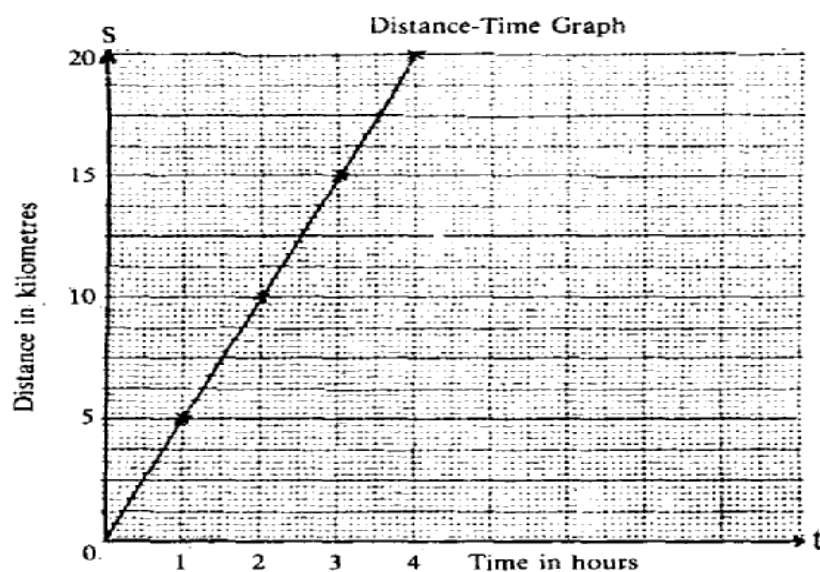


Fig. 19.7

Similarly, the corresponding graph for table 19.2 (b) is given in figure 19.8.

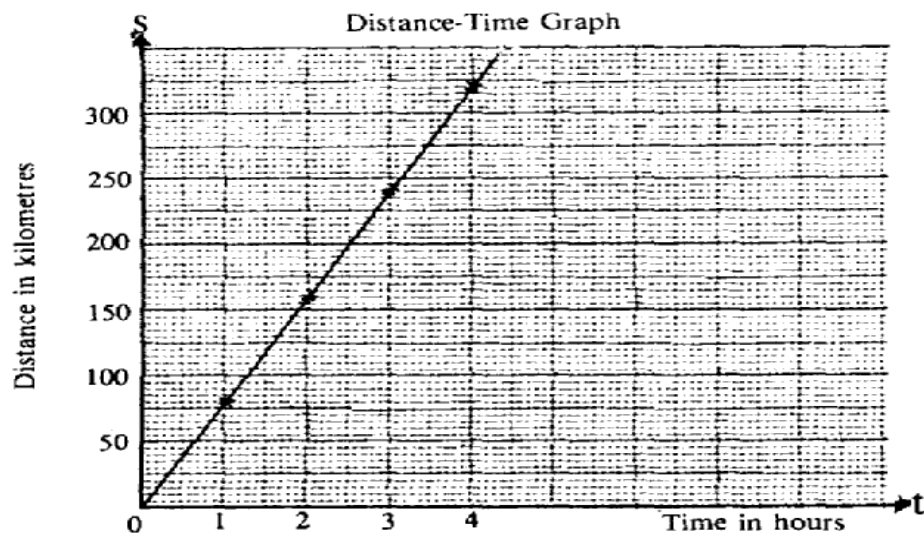


Fig. 19.8

In both graphs, the scales on the horizontal axes are the same.

- Why do you think the vertical scale in figure 19.7 is different from that in figure 19.8?
- What would be the effect on figure 19.7 if we used the vertical scale of figure 19.8?

(iii) What would be the effect on figure 19.8 if we used the vertical scale of figure 19.7?

A good scale is one which uses most of the graph page and enables us to plot points and read off values easily and accurately. Avoid scales which:

(i) give tiny graphs.

(ii) cannot accommodate all the data in the table.

It is also good practice to:

(i) label the axes clearly, and

(ii) give the title of the graph.

Exercise 19.5

1. Telephone bills consist of a fixed standing charge and an amount which depends on the number of calls made. The table below shows the total amount payable by a subscriber for different numbers of calls:

Number of calls (n)	10	20	30	40	50	60
Amount payable in shillings (c)	90	110	130	150	170	190

Choose a suitable scale for each axis and draw the graph of the amount of money payable, C , against the number of calls made, n . From your graph, answer the following questions:

(a) What would be the charges for:

(i) 6 calls?

(ii) 15 calls?

(iii) 53 calls?

(b) How many calls did a subscriber make if he paid:

(i) sh. 72?

(ii) sh. 166?

(iii) sh. 195?

(c) What is the standing charge?

2. The relationship between the temperature in degrees Fahrenheit ($^{\circ}\text{F}$) and degrees Celcius ($^{\circ}\text{C}$) is given in the table below:

Degrees Celcius ($^{\circ}\text{C}$)	-40	-20	0	20	40	60	80	100
Degrees Farenheit ($^{\circ}\text{F}$)	-40	-4	32	68	104	140	176	212

Use a suitable scale to draw the graph of F (Fahrenheit) against C (Celcius).

(a) Use your graph to convert the following temperatures in degrees Fahrenheit to degrees Celcius:

- (i) 0°F (ii) 20°F (iii) 98°F (iv) 25°F

(b) Use your graph to convert the following temperatures in degree Celcius to degrees Fahrenheit:

- (i) 25°C (ii) 4°C (iii) 37°C (iv) 30°C

3. A certain quantity of gas is heated from 0°C and the volume measured at different temperatures. The table gives the corresponding values:

Temperature ($^{\circ}\text{C}$)	20	40	60	80	100
Volume (litres)	1.82	1.95	2.07	2.20	2.32

(a) Draw a graph of volume against temperature using a suitable scale.

(b) Use your graph to find:

- (i) the initial volume of gas.
 (ii) the volume of the gas when the temperature is 50°C and 64°C .
 (iii) the temperature of the gas when the volume is 2.3 and 2 litres.

4. A man deposited a certain amount of money in a bank. The following table shows the amount of money due to him at the end of every year:

Time (years)	1	2	3	4	5
Amount (shillings)	45 000	50 000	55 000	60 000	65 000

(a) Using a suitable scale, plot the graph of the amount of money in the bank against time.

(b) From your graph, estimate his initial deposit in the bank.

(c) Supposing at the end of $3\frac{1}{2}$ years he withdrew some amount of money such that the balance was sh. 40 000, how much did he withdraw?

(d) If he had not withdrawn the money, what would be the amount in the bank after 66 months?

5. The period of swing of a pendulum, is directly proportional to the square root of the length of the pendulum, see figure 19.9:

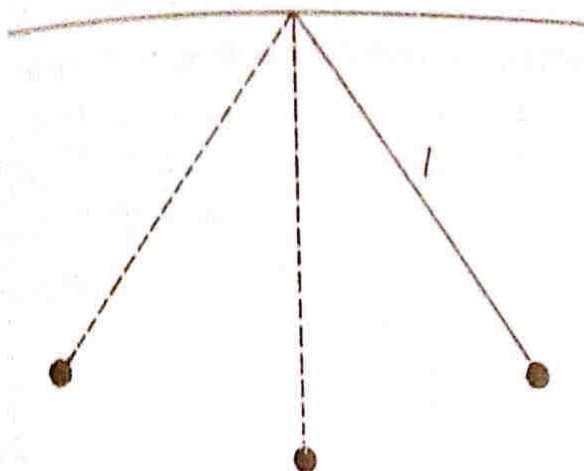


Fig. 19.9

In an experiment, the square roots of the lengths and the corresponding periods were determined. The table below gives the result:

Square root of length ($\sqrt{l}$)	0.32	0.45	0.55	0.63	0.71	0.77	0.84	0.89
Period of swing (T seconds)	0.63	0.89	1.09	1.26	1.40	1.54	1.66	1.78

- Choose a suitable scale for each axis to accommodate values from zero to the highest value given.
- In your scale, what does one small square represent?
- Draw the graph of T against the square root of l .
- From your graph:
 - find the period of swing if $\sqrt{l}$ is: 0.30, 0.52, 0.73.
 - find the square root of the length of the pendulum if the period of swing is 0.40 seconds, 0.92 seconds and 1.65 seconds.
 - find the length of the pendulum if the period of swing is: 1 s, 1.5 s, 1.8 s.

Not all graphs that occur in real life situations are linear. Consider the following example:

A biology teacher records the number of insects in a growing colony every two days. The following table shows his results:

Days (T)	0	2	4	6	8	10	12	14	16	18	20	22	24
Number of insects	150	160	220	300	500	680	770	840	880	900	910	920	930

The graph of the number of insects against time is shown in figure 19.10.

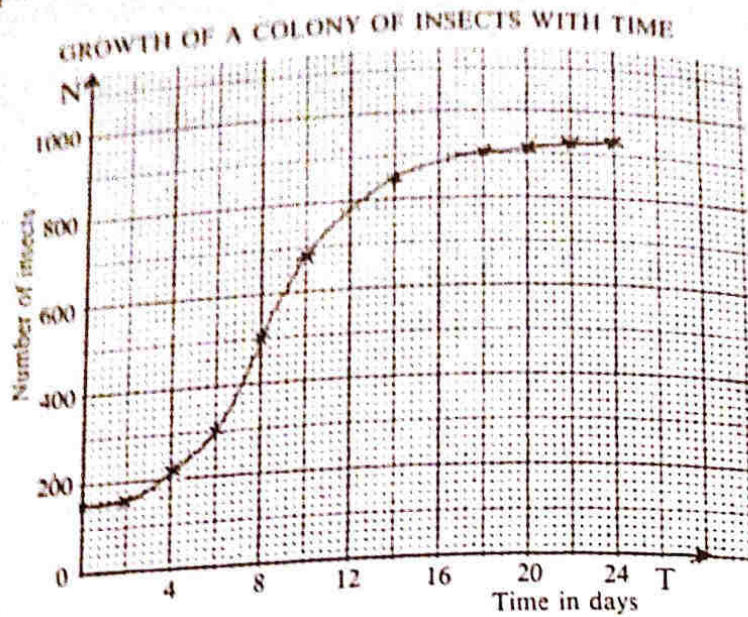


Fig 19.10

You should note that the graph is not a straight line, but a smooth curve.

Exercise 19.6

- Draw the graph in figure 19.10 in your graph book and use it to answer the following questions:
 - State the scale along the x-axis.
 - Estimate the size of the colony:
 - on the fifth day.
 - after three weeks.
 - After how many days was the number of insects:
 - equal to 800?
 - three times the original number?
- If $y = x^2$, make a table of values of y against values of x from $x = -4$ to $x = 4$. Draw a curve passing through the points. From the graph find:
 - $(3.1)^2$
 - $(2.7)^2$
 - 0^2
 - $\sqrt{13.5}$
 - $\sqrt{8.4}$
 - $(2.9)^2$
- The following table gives the expectation of life at different ages, e.g., a person aged 10 years is expected to live 49 more years:

Age in years (x)	0	1	5	10	15	20	25	30	35	40	45	50	55	60
Expectation in years (e)	52	56	53	49	45	41	37	33	29	25	23	21	18	14

Draw the graph of e against x . Use your graph to answer the following questions:

- (a) What is the expectation of life at the age of:
- 7 years?
 - 34 years?
 - 58 years?
- (b) At what age is the expectation:
- 20 years?
 - 25 years?
 - 26 years?
- (c) At what age is the expectation highest?
4. The surface area of an animal may be obtained from the mass of the animal. The following table gives the corresponding values of mass and surface area:

Mass in kg	50	100	150	200	250	300	350	400
Area in m^2	1.4	2.1	2.8	3.4	4.0	4.5	4.9	5.4

Draw the graph of surface area against mass and use it to answer the following questions.

- (a) Find the surface area of an animal weighing:
- 155 kg
 - 215 kg
 - 370 kg
- (b) A butcher **A** slaughters two animals weighing 155 kg and 215 kg. Another butcher **B** slaughters an animal weighing 370 kg. Which butcher gets a larger area of hides?
- (c) What is the mass of an animal whose surface area is one square metre?
5. The following table gives the population of a town at different times:

Year	1955	1960	1965	1970	1975	1980	1985
Population	5 000	6 100	7 500	9 100	11 100	13 600	16 600

Draw a graph to represent the information and use it to answer the following questions:

- (a) Estimate the population of the town in:
- 1963
 - 1974
 - 1982
- (b) At what time was the population:
- double the 1955 figure?
 - triple the 1955 figure?

6. A stone is thrown from the top of a cliff towards the sea, see figure 19.11. Its vertical distance above the top of the cliff at different times is given in the table below:

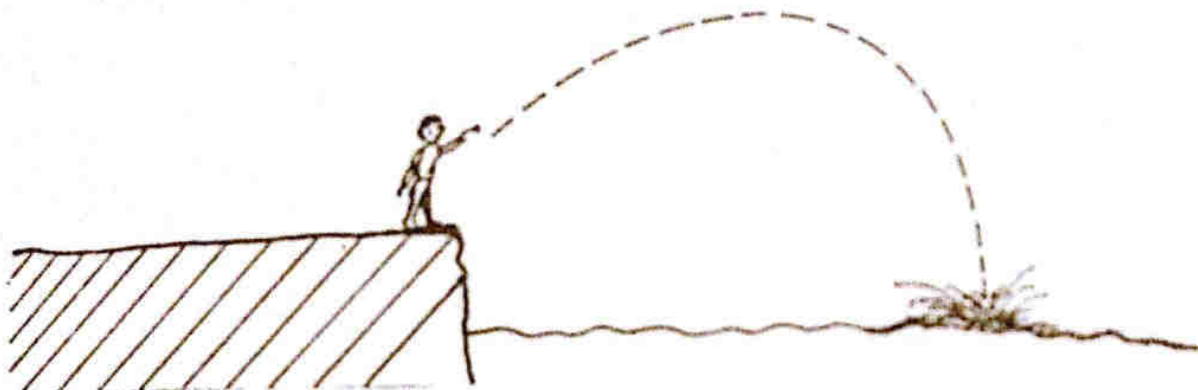


Fig 19.11

Time seconds	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Height (metres)	8.75	15	18.75	20	18.75	15	8.75	0	-11.25	-25	-40	-60

Using a scale of 2 cm to 1 unit on the horizontal axis (time) and 2 cm to 10 units on the vertical axis (height), draw the graph. From your graph, answer the following questions:

- What is the maximum height of the stone above the cliff?
- How long does the stone take to attain this maximum height?
- After how long is the stone:
 - at the same level as the cliff?
 - 15 metres below the cliff?
- Give the two times during which the stone is 15, 10, 5 and 2 metres above the cliff.

ANGLES AND PLANE FIGURES

20.1: Introduction

A flat surface such as the top of a table is called a **plane**. The walls and the floor of a classroom are also planes. The intersection between the wall and the floor is a straight line. Two planes always intersect in a straight line. The intersection of any two straight lines is a point.

Give other examples of planes, straight lines and points.

Representing Points and Lines on a Plane

A point is represented on a plane by a mark labelled by a capital letter. Through any two given points on a plane, only one straight line can be drawn. Consider the following:



Fig. 20.1

The line passes through points A and B and hence can be labelled line AB.

20.2: Types of Angles

When two lines meet, they form an angle at a point. The point where the angle is formed is called the **vertex** of the angle. The symbol $\angle$ is used to denote an angle. Consider again the following:

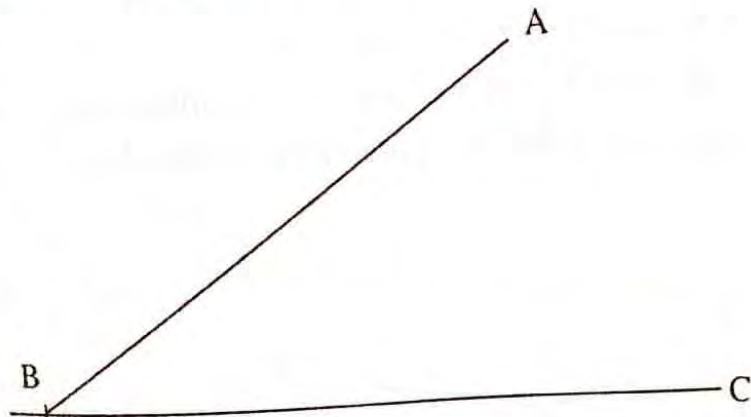


Fig. 20.2

Point B is the vertex of the angle between the lines AB and BC. The angle is referred to as either $\angle ABC$ or $\angle B$. The angle can also be written as $\widehat{ABC}$. Figure 20.3 represents different the types of angles.

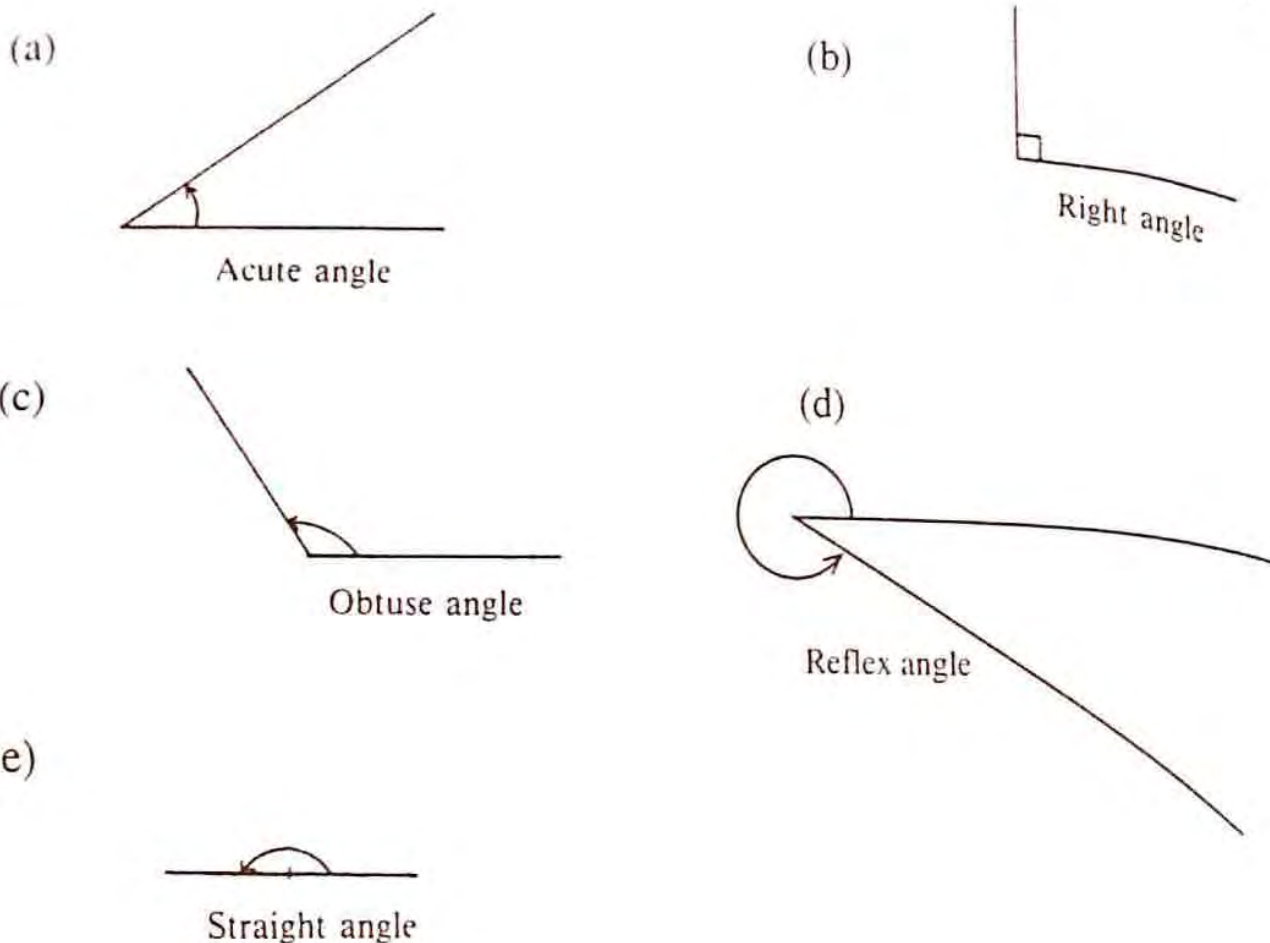


Fig. 20.3

To obtain the size of a reflex angle which cannot be read directly from a protractor, the corresponding acute or obtuse angle is subtracted from 360° . If any two angles X and Y are such that:

(i) $\angle X + \angle Y = 90^\circ$, the angles are said to be **complementary angles**.

Each angle is then said to be the complement of the other.

(ii) $\angle X + \angle Y = 180^\circ$, the angles are said to be **supplementary angles**.

Each angle is then said to be the supplement of the other.

In figure 20.4, $\angle POQ$ and $\angle ROQ$ are a pair complementary angles.

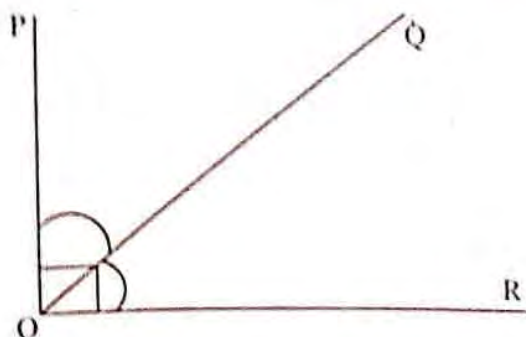


Fig. 20.4

In figure 20.5, $\angle DOF$ and $\angle FOE$ are a pair of supplementary angles.

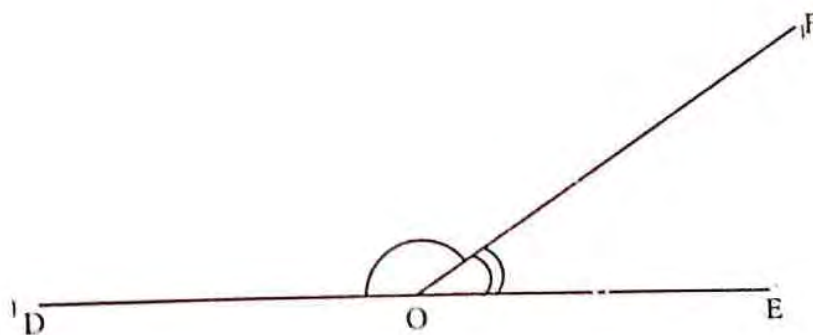


Fig. 20.5

Exercise 20.1

1. Study figure 20.6 and name:
 - (a) an acute angle.
 - (b) a right angle.
 - (c) a straight angle.
 - (d) a pair of complementary angles.
 - (e) a reflex angle.

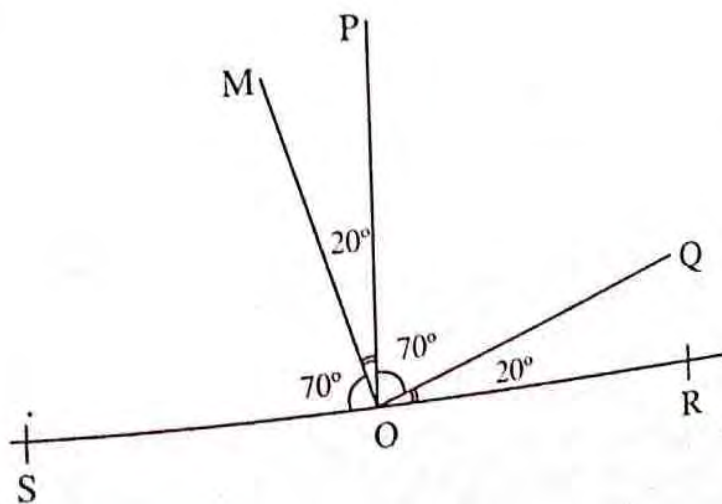


Fig. 20.6

2. Figure 20.7 shows a square ABCD with its diagonals intersecting at point O:

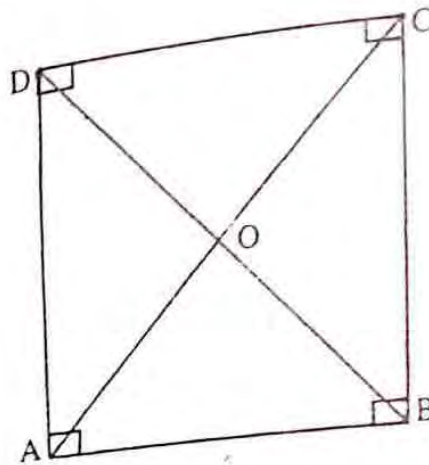


Fig. 20.7

Name:

- (a) two acute angles.
 - (b) eight right angles.
 - (c) two obtuse angles.
 - (d) two pairs of supplementary angles.
 - (e) a pair of complementary angles.
 - (f) two pairs of parallel lines.
3. (a) Give the complement of each of the following angles:
- | | | |
|-----------------|-----------------|--------------------|
| (i) 57° | (ii) 38° | (iii) 56.5° |
| (iv) 89° | (v) 0° | (vi) 78.6° |
- (b) Give the supplements of each of the following angles:
- | | | |
|-----------------|-----------------|-------------------|
| (i) 57° | (ii) 90° | (iii) 139° |
| (iv) 45° | (v) 120° | (vi) 180° |
4. Use figure 20.8 to answer the questions that follow:

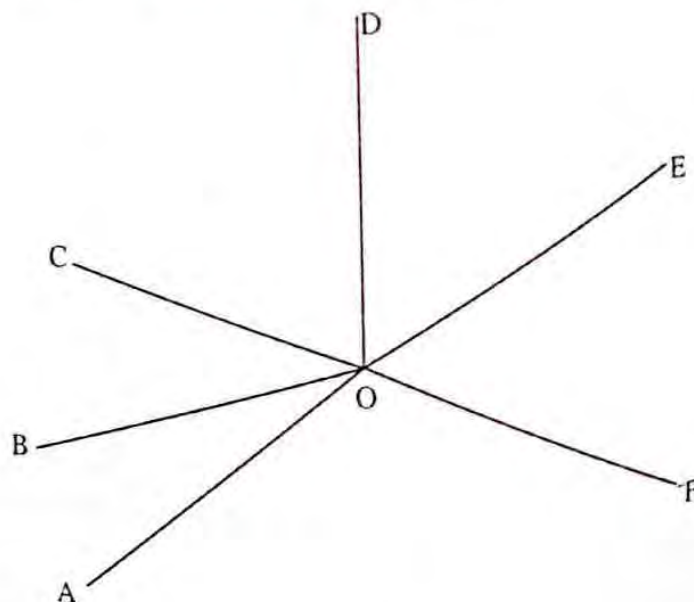


Fig. 20.8

- Measure angles AOB, BOC, COD, DOE and EOF.
- Calculate $\angle AOF$. Check your answer by measurement.
- Name all the acute and obtuse angles.
- Name a pair of supplementary angles.
- Calculate the reflex angles AOB, AOC and FOC.
- Name two pairs of complementary angles.

20.3: Angles on a Straight Line

Figure 20.9 shows a number of angles with a common vertex O. AOE is a straight line.

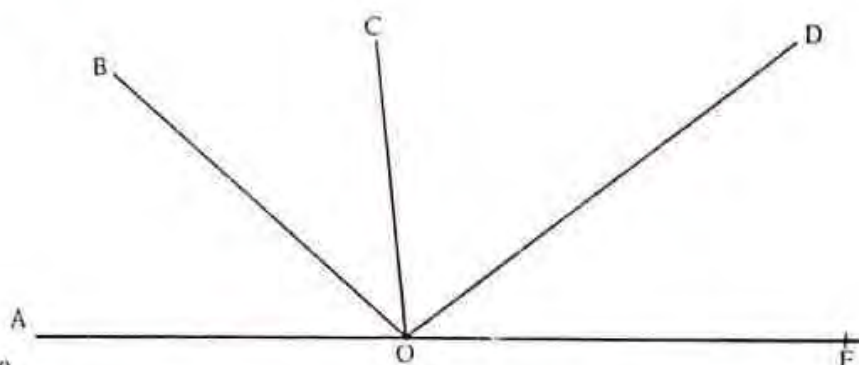


Fig. 20.9

Two angles on either side of a straight line and having a common vertex are referred to as **adjacent angles**. In the figure:

$\angle AOB$ is adjacent to $\angle BOC$.

$\angle BOC$ is adjacent to $\angle COD$.

$\angle COD$ is adjacent to $\angle DOE$.

Give other pairs of adjacent angles in the same figure.

Angles on a straight line add up to 180° .

20.4: Angles at a Point

Two intersecting straight lines form four angles having a common vertex. The angles which are on opposite sides of the vertex are called **vertically opposite angles**. Consider the following:

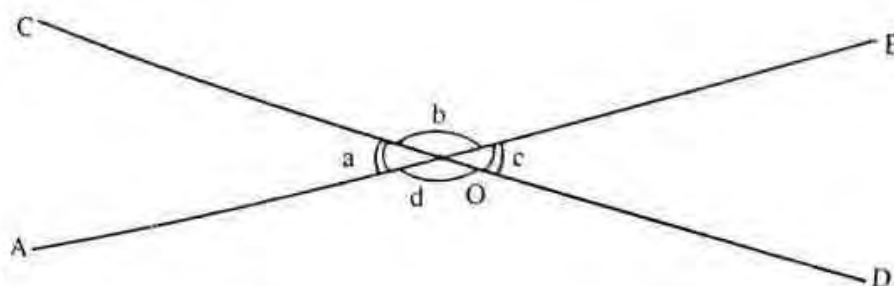


Fig. 20.10

In figure 20.10, $\angle COB$ and $\angle AOC$ are adjacent angles on a straight line. Name other pairs of adjacent angles.

We can show that $a = c$ as follows:

$$a + b = 180^\circ \text{ (angles on a straight line)}$$

$$b + c = 180^\circ \text{ (angles on a straight line)}$$

$$\text{So, } a + b = b + c$$

$$\therefore a = c.$$

Note that a and c are vertically opposite angles. Similarly, show that $b = d$.

We can also show that $a + b + c + d = 360^\circ$ as follows:

$$a + b = 180^\circ \text{ (angles on a straight line)}$$

$$c + d = 180^\circ \text{ (angles on a straight line)}$$

$$\begin{aligned} \text{So, } a + b + c + d &= 180^\circ + 180^\circ \\ &= 360^\circ \end{aligned}$$

This shows that angles at a point add up to 360° :

Exercise 20.2

1. In figure 20.11, find each of the angles marked by a letter:

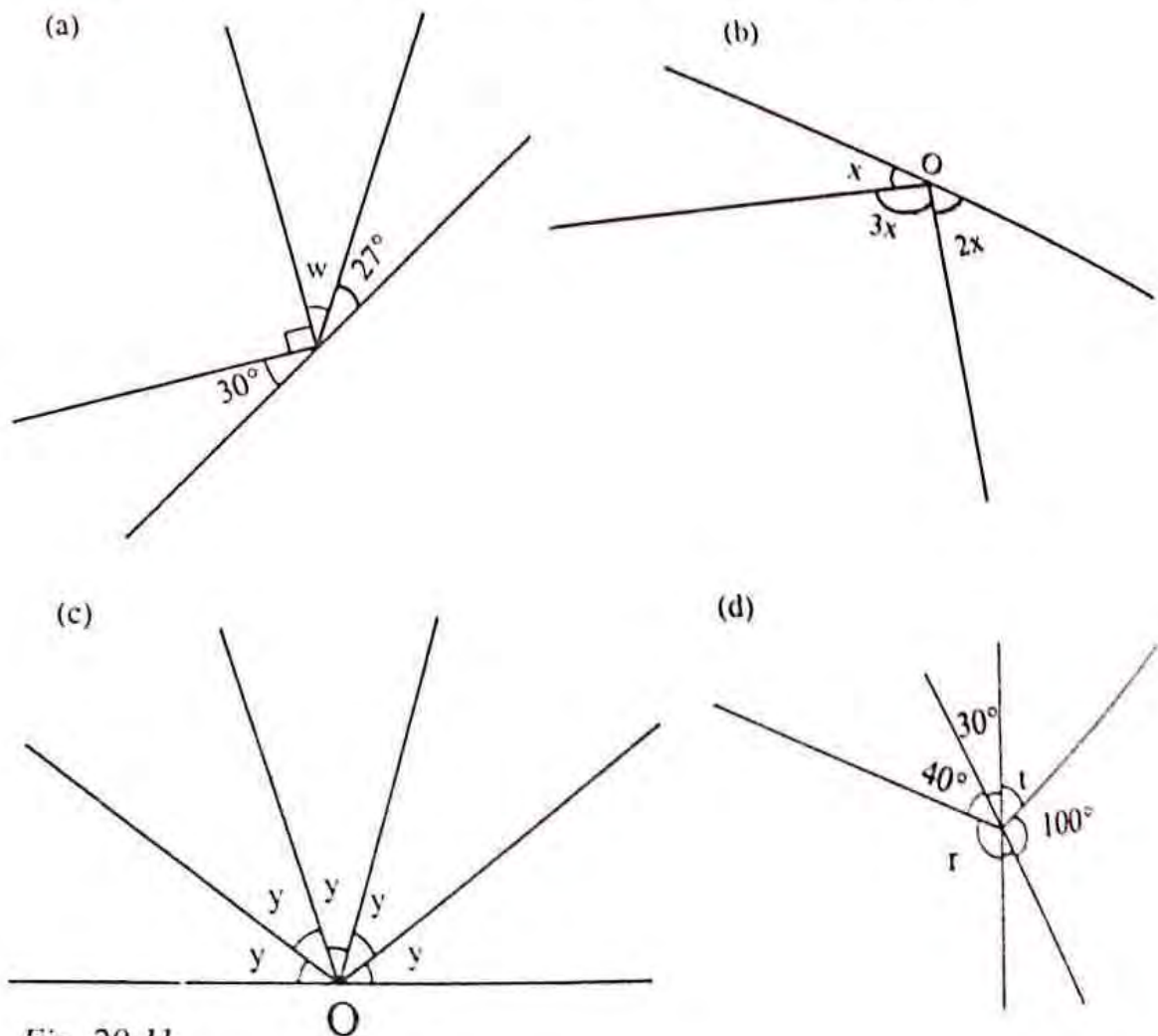
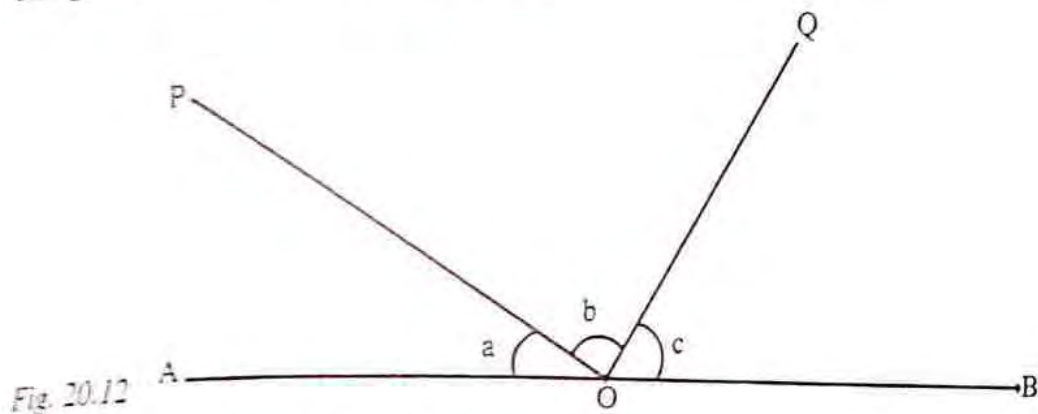


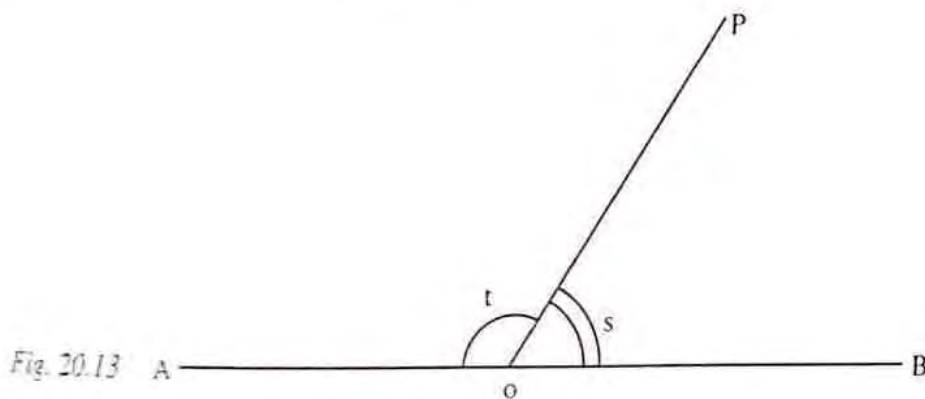
Fig. 20.11

2. In figure 20.12, AOB is a straight line:



Find angles marked a , b , c , if $c = 2a$ and $b = 3a$.

3. Find the size of an angle q if it is five times its supplement.
4. Three angles, $x = (3p + 25)^\circ$, $y = (2p - 20)^\circ$ and $z = (2p + 35)^\circ$ are on a straight line. Find:
- the value of p .
 - the value of x , y and z .
5. In figure 20.13, AOB is a straight line. The angles marked s and t are such that t exceeds s by $\frac{1}{5}$ of a right angle. Find s and t .



6. In figure 20.14, $a = \frac{3b}{2}$. Find a and b :

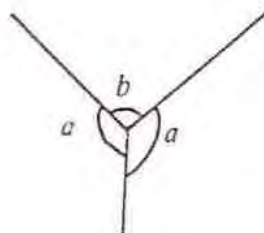


Fig. 20.14

7. In figure 20.15, find the values of p , q , r and s , given that $q = 2p$, $r = \frac{3}{2}q$ and $s = 2r$.

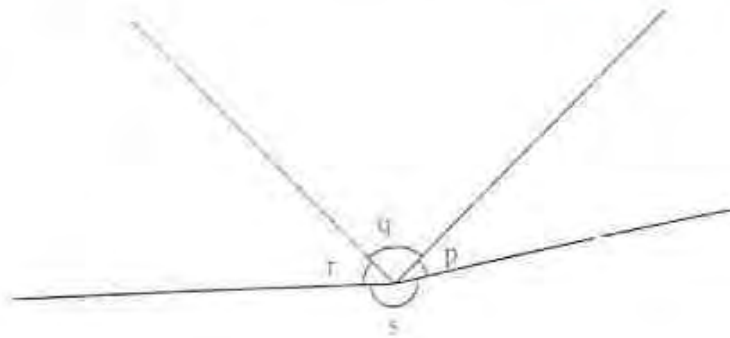


Fig. 20.15

8. In figure 20.16, $a = \frac{r}{2} = \frac{c}{3} = \frac{d}{4} = \frac{e}{5}$.

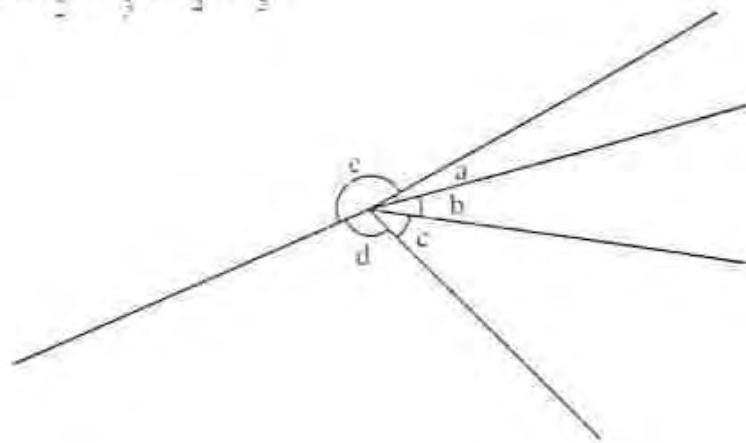


Fig. 20.16

Find the angles a , b , c , d and e .

9. In figure 20.17, find y :

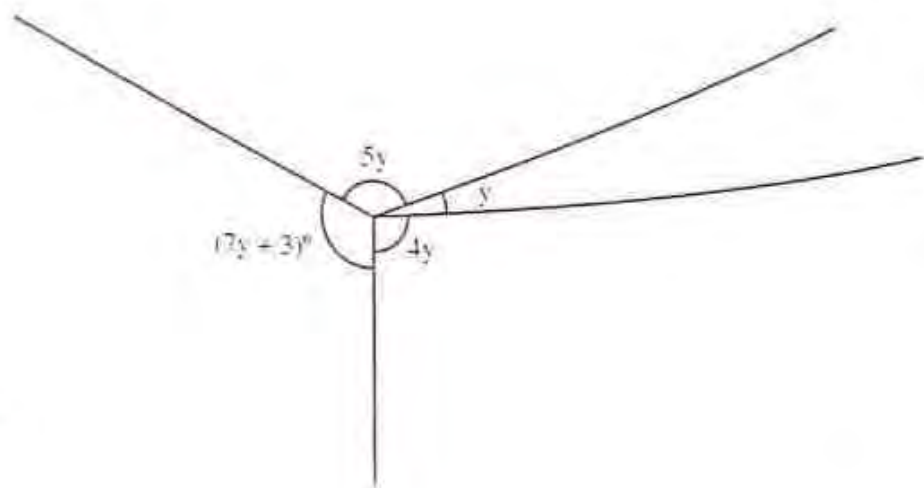


Fig. 20.17

10. In figure 20.18, find x :

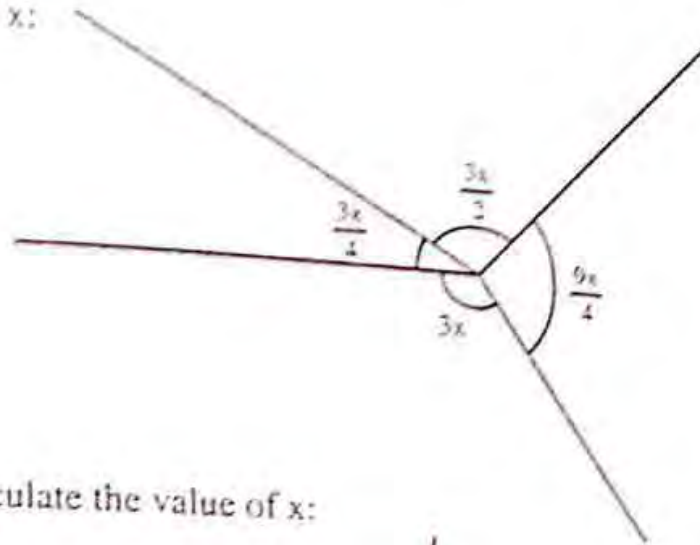


Fig. 20.18

11. In figure 20.19, calculate the value of x :

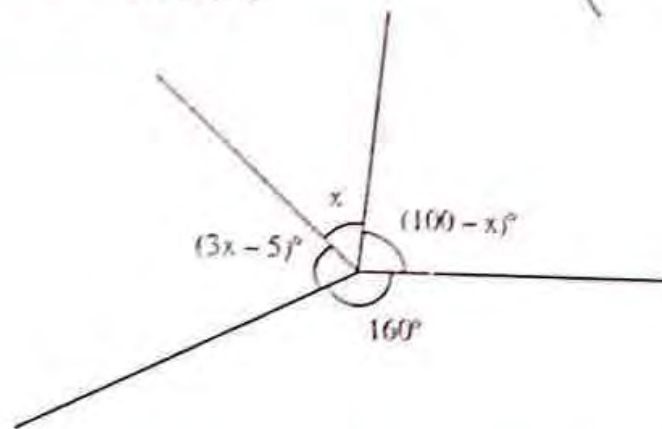


Fig. 20.19

12. Points A, O and B are on a straight line. C is a point such that $\angle AOC = 54^\circ$ and D is a point such that $\angle BOD$ is vertically opposite $\angle AOC$. Calculate the reflex angle $\angle BOC$.
13. In figure 20.20, show that $y = 7(52 - x)^\circ$:

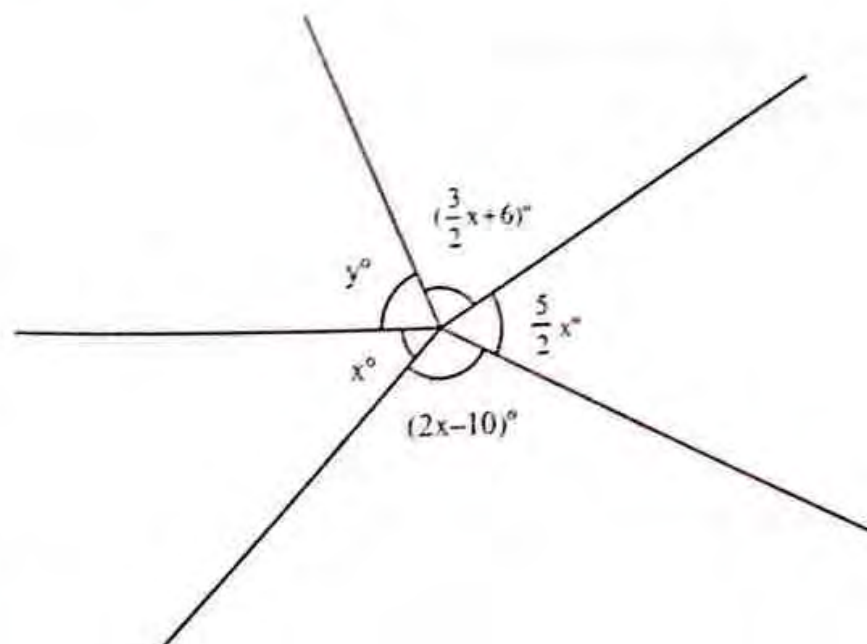


Fig. 20.20

14. In figure 20.21, $x = 3y$ and $z = \frac{3}{2}y$. Calculate the unknown angles:

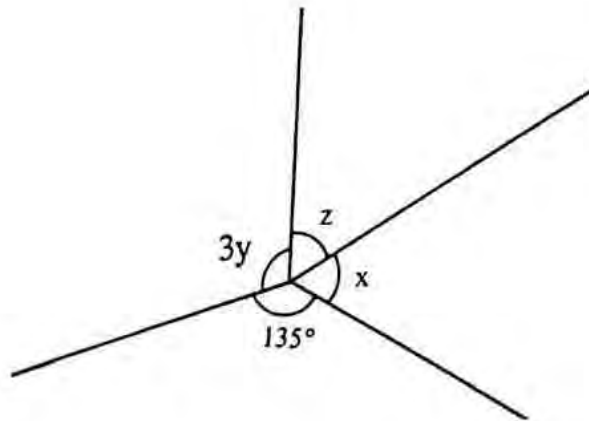


Fig. 20.21

15. In figure 20.22, find x :

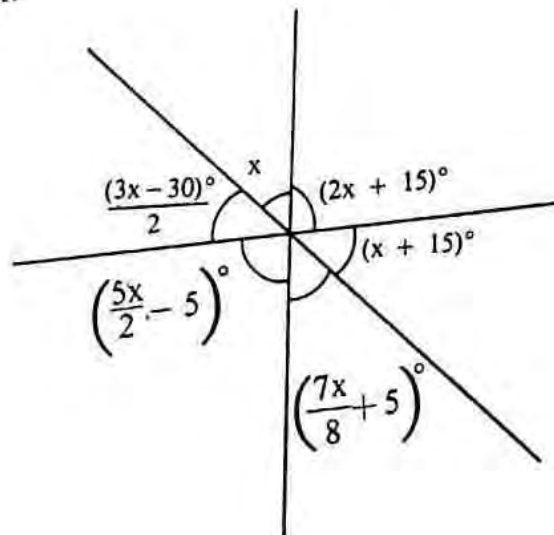


Fig. 20.22

20.5: Angles on a Transversal

A **transversal** is a line that cuts across two parallel lines. Consider figure 20.23.

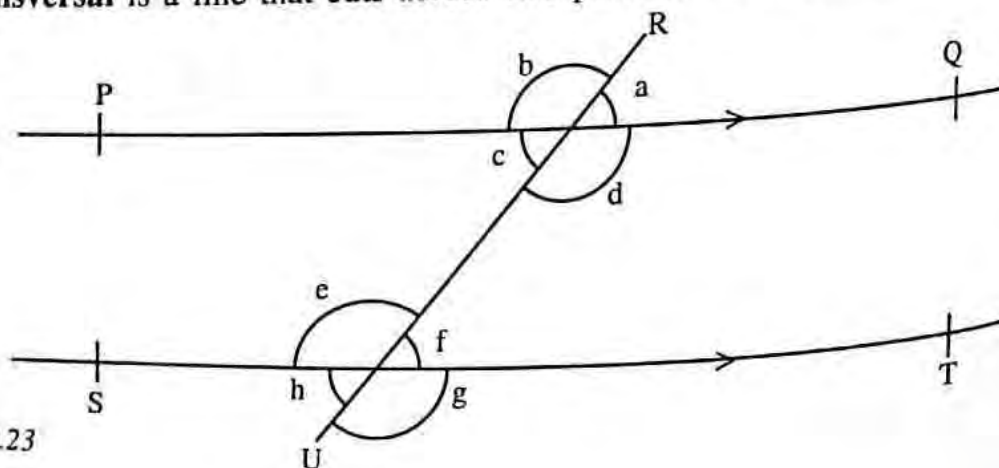


Fig. 20.23

PQ and ST are parallel lines and RU cuts through them. RU is a transversal.

Name:

- (i) corresponding angles.
- (ii) alternate angles.
- (iii) allied angles.

What can you say about each pair of angles?

Note:

The symbol $\rightarrow$ on lines ST and PQ denotes parallel lines.

Exercise 20.3

Calculate the unknown angles marked by letters in each of the following figures:

1.

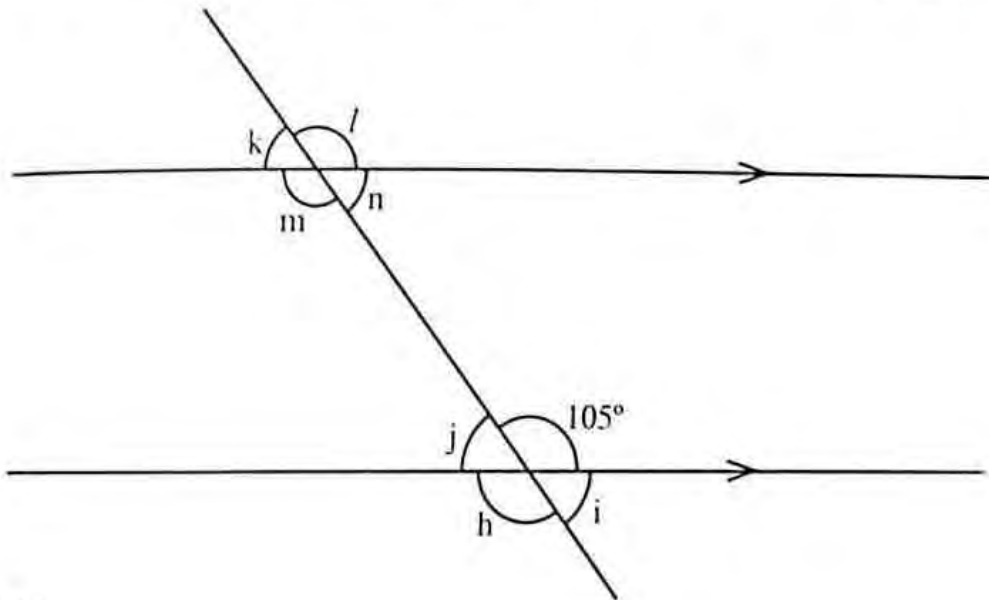


Fig. 20.24

2.

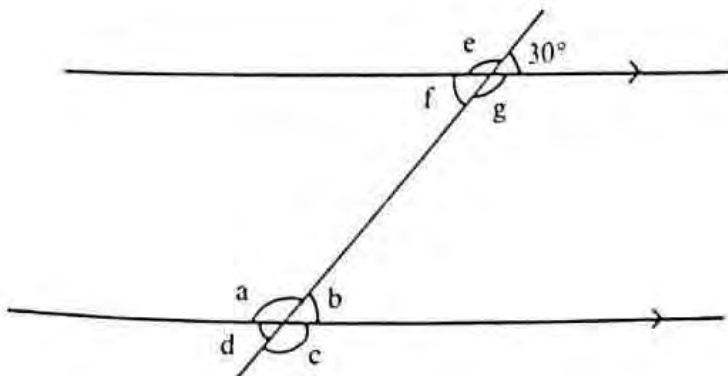


Fig. 20.25

3.

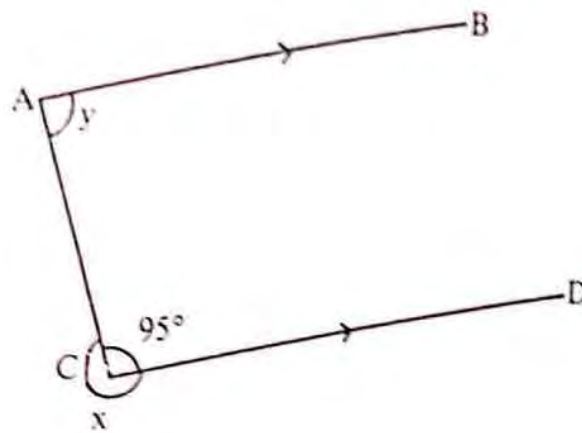


Fig. 20.26

4.

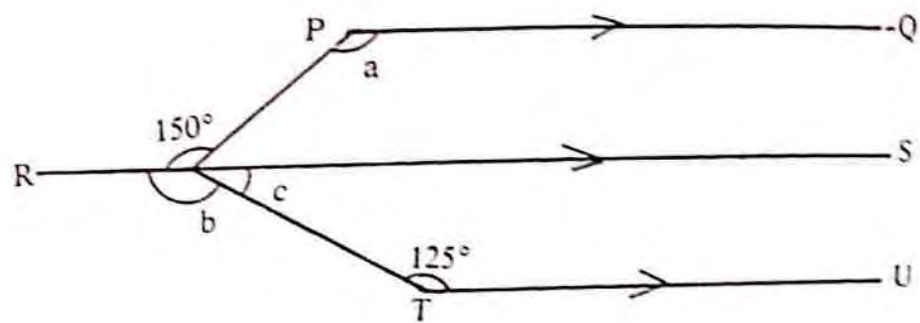


Fig. 20.27

5.

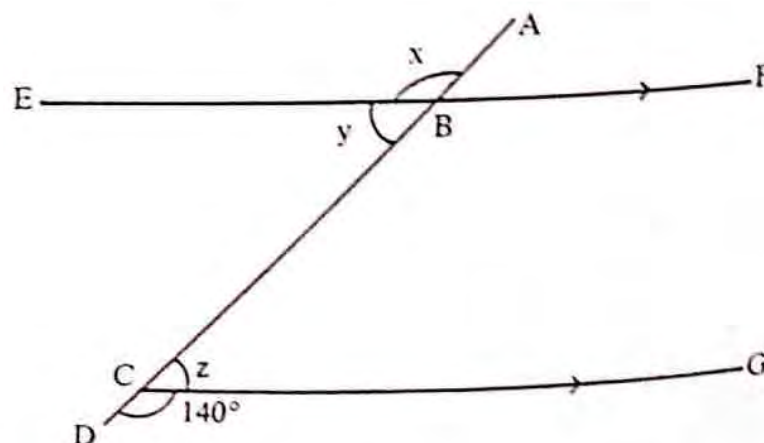


Fig. 20.28

6.

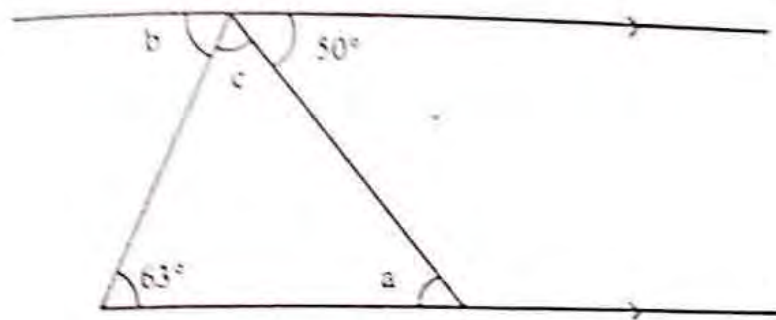


Fig. 20.29

7.

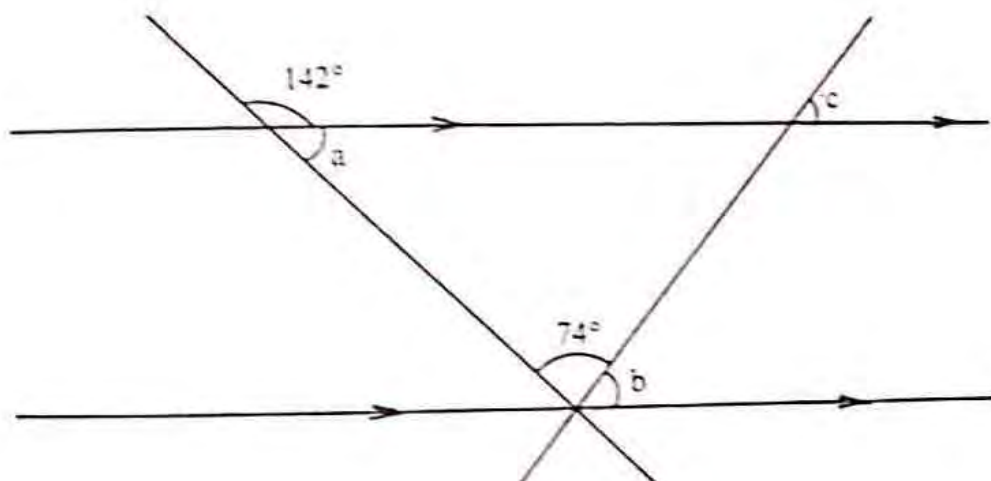


Fig. 20.30

8.

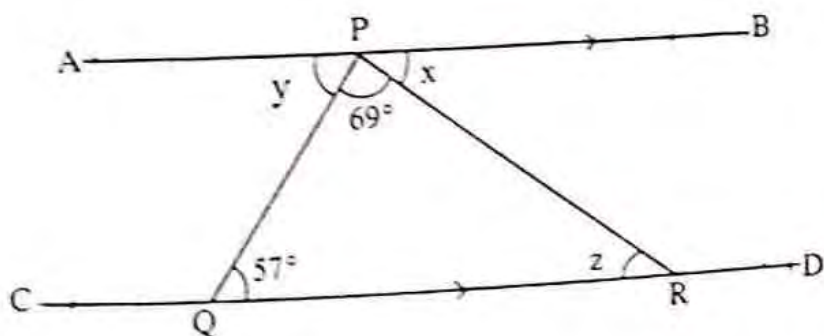


Fig. 20.31

9.

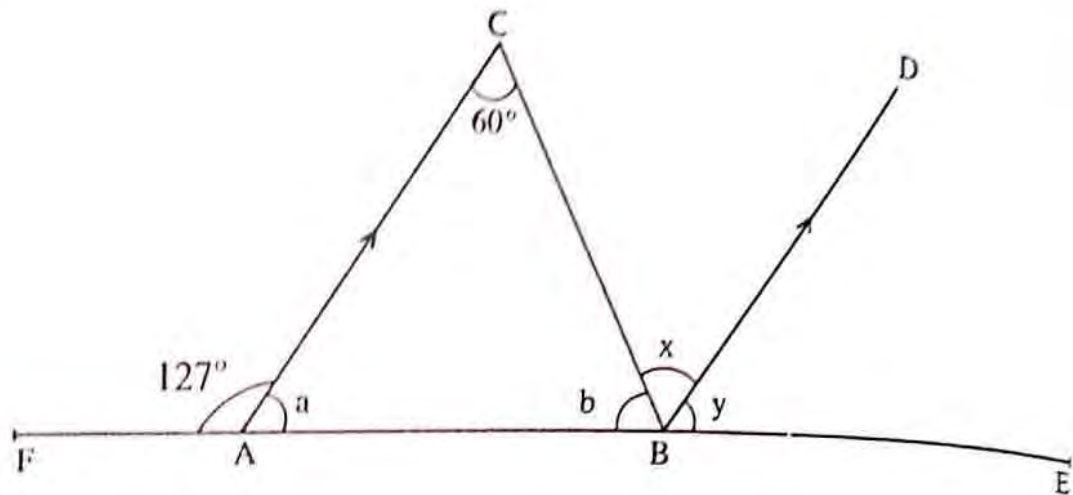


Fig. 20.32

10.

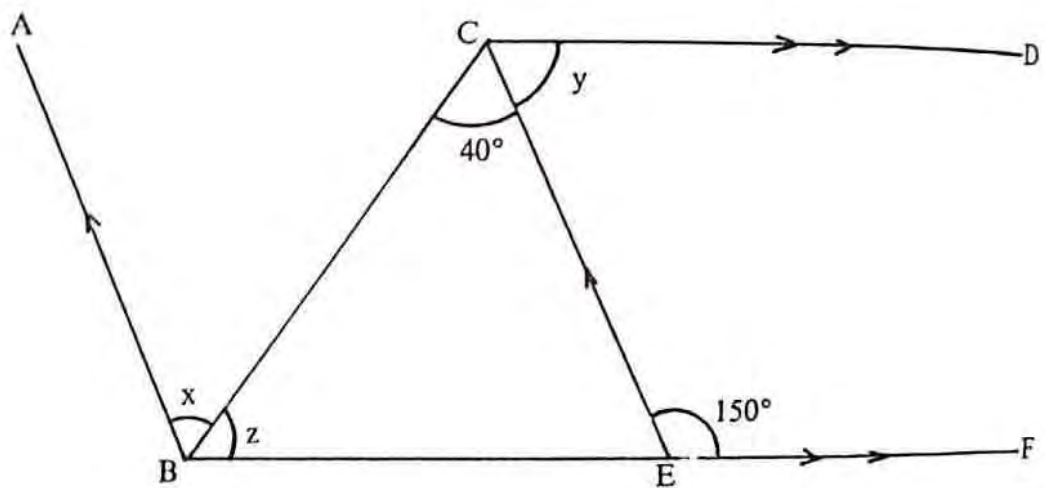


Fig. 20.33

11.

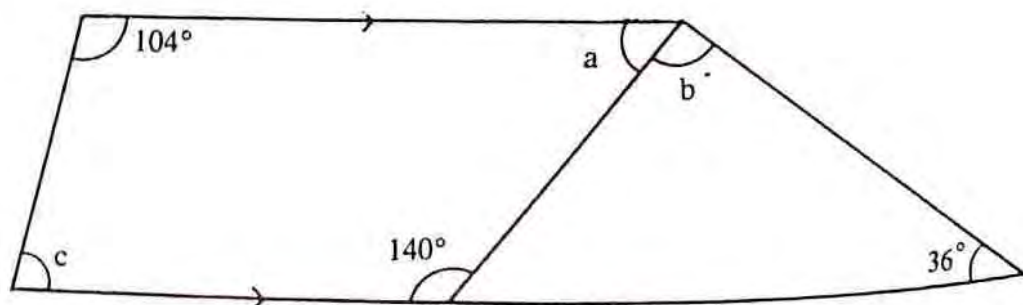


Fig. 20.34

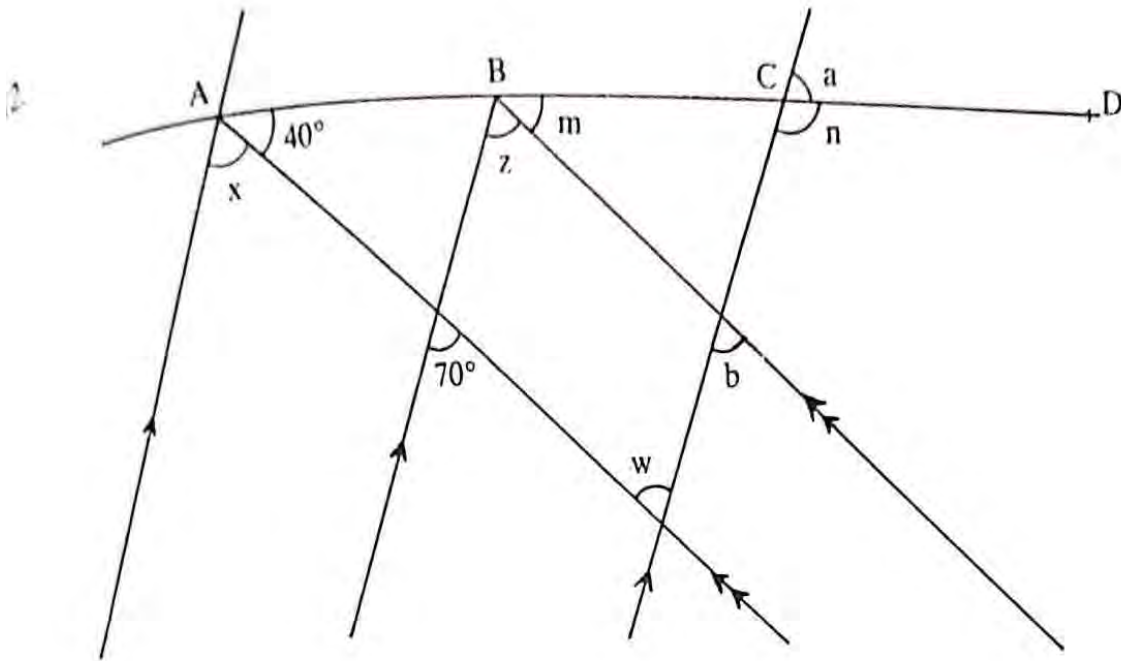


Fig. 20.35

13. In figure 20.36, show that $y + 3x = 90^\circ$:

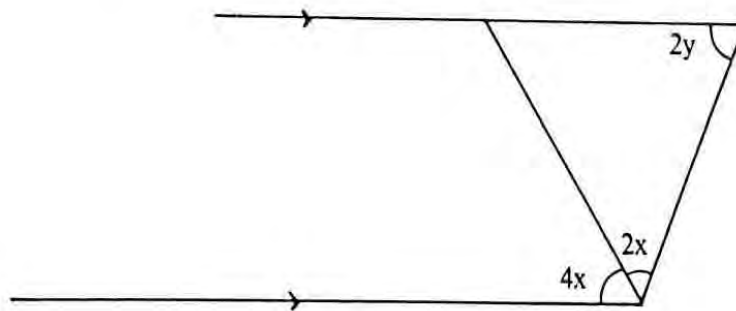


Fig. 20.36

20.6: Angle Properties of Polygons

A polygon is a plane figure bordered by three or more straight lines.

Triangles

A triangle is a three-sided plane figure. The sum of the three angles of a triangle add up to 180° . Triangles are classified on the basis of either angles or sides.

- (i) A triangle in which one of the angles is 90° is called a **right-angled** triangle.
- (ii) A **scalene** triangle is one in which all the sides and angles are not equal.
- (iii) An **isosceles triangle** is one in which two sides are equal and the equal sides make equal angles with the third side.
- (iv) An **equilateral triangle** is one in which all the sides are equal and all the angles are equal.

Use your own measurements to draw scalene, isosceles and equilateral triangles.
Mark equal sides and equal angles in your diagrams.

Exterior Properties of a Triangle

In figure 20.37, AB is parallel to CD:

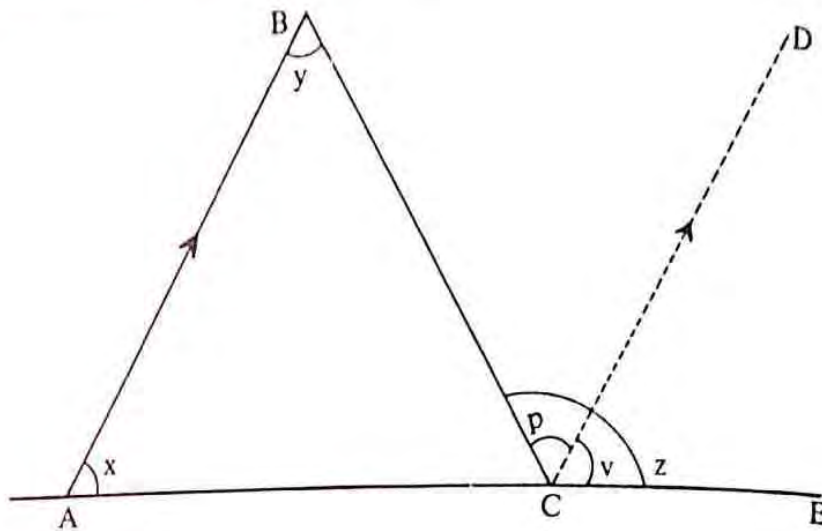


Fig. 20.37

Explain why the angles marked y and p are equal. Explain also why the ones marked x and v are equal. What can you say about the relationship between $\angle BCE$ and the sum of $\angle BAC$ and $\angle ABC$?

Consider the triangle ABC, figure 20.38:

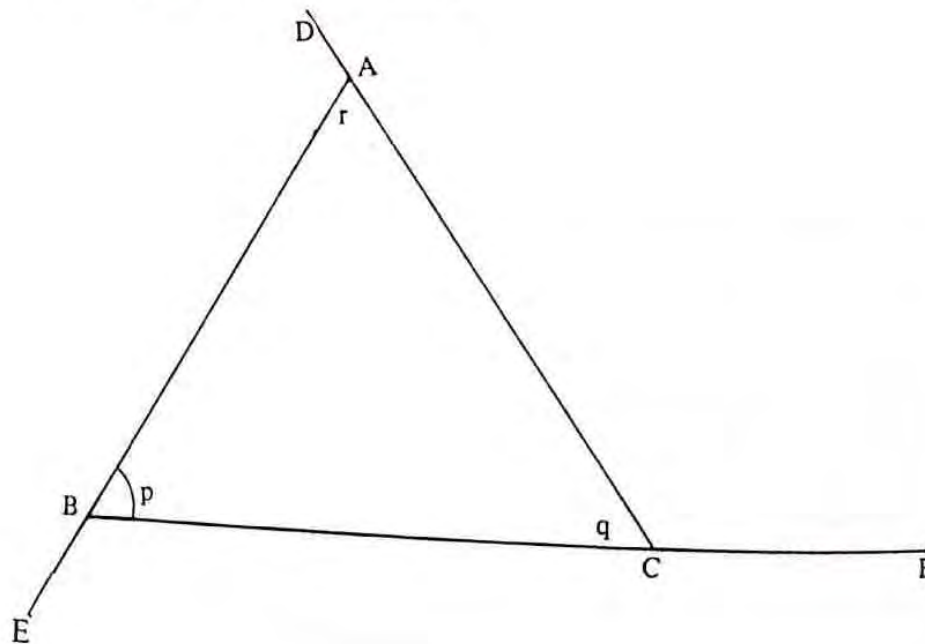


Fig. 20.38

$\angle DAB = p + q$. Why?

Similarly, $\angle EBC = r + q$ and $\angle FCA = r + p$.

But $p + q + r = 180^\circ$

$$\begin{aligned}\therefore \angle DAB + \angle EBC + \angle FCA &= 2p + 2q + 2r \\ &= 2(p + q + r) \\ &= 2 \times 180^\circ \\ &= 360^\circ\end{aligned}$$

Consider the triangle ABC, figure 20.39. What does $x + y + z$ equal to?

$x + r = 180^\circ$. Why?

Similarly, $s + y = 180^\circ$, and $t + z = 180^\circ$

$$\begin{aligned}\therefore x + y + z + (r + t + s) &= 180^\circ + 180^\circ + 180^\circ \\ &= 540^\circ\end{aligned}$$

But $r + t + s = 180^\circ$. Why?

$$\begin{aligned}\text{Therefore, } x + y + z &= 540^\circ - 180^\circ \\ &= 360^\circ\end{aligned}$$

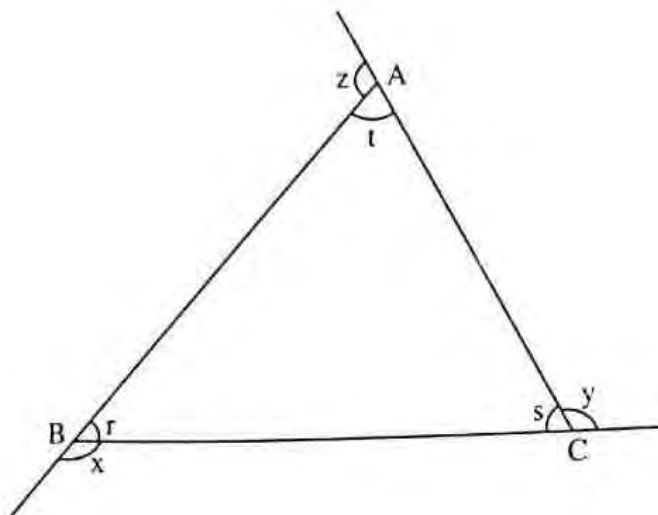


Fig. 20.39

This shows that the sum of the exterior angles of a triangle is 360° .

Exercise 20.4

1. What name do we give to the following triangles?

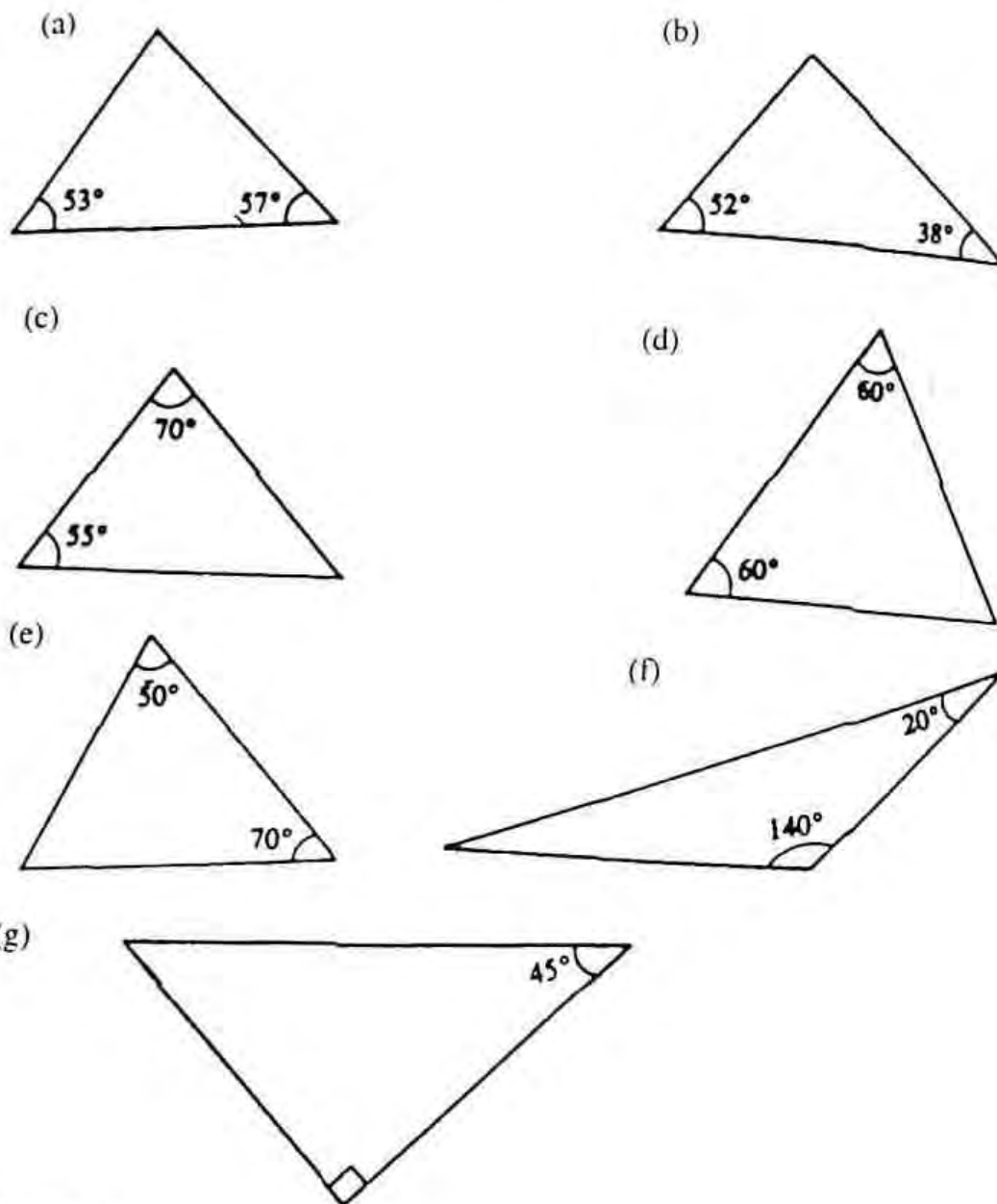
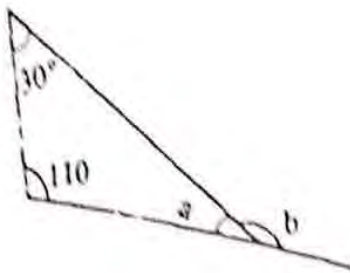


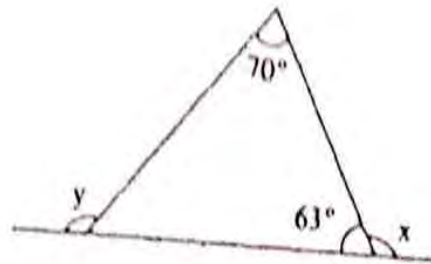
Fig. 20.40

2. Draw straight line $AB = 8$ cm. Draw $\angle BAC = 53^\circ$ and $\angle ABC = 40^\circ$. Measure angle ACB and check your answer by calculation.
3. Draw a triangle ABC such that $AB = BC = CA$. Measure all the angles of the triangle. What do you notice? What name is given to such a triangle?
4. Calculate each of the angles given by a letter:

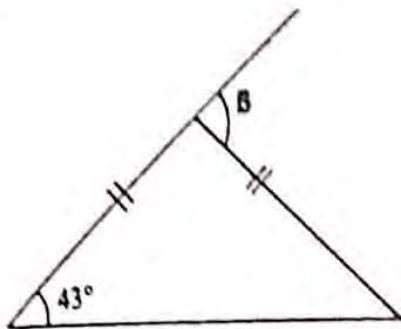
(a)



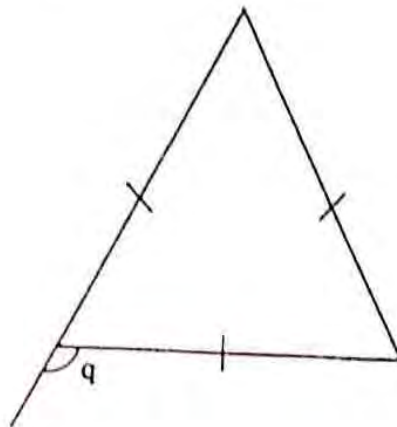
(b)



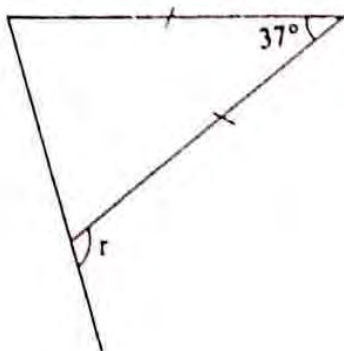
(c)



(d)



(e)



(f)

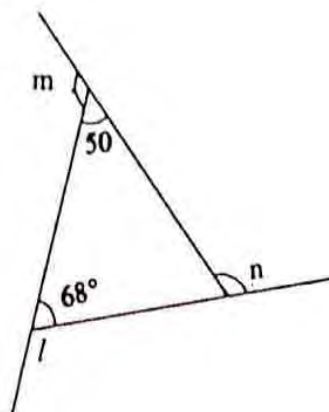


Fig. 20.41

5. Draw a triangle PQR such that $PQ = 5$ cm, $QR = 3$ cm and $RP = 4$ cm. What name do we give to such a triangle?
6. The exterior angles of a triangle LMN are a , b and c . Show that the sum of the angles a , b and c is 4 right angles.
7. Find the value of each of the angles marked by a letter in figure 20.42:

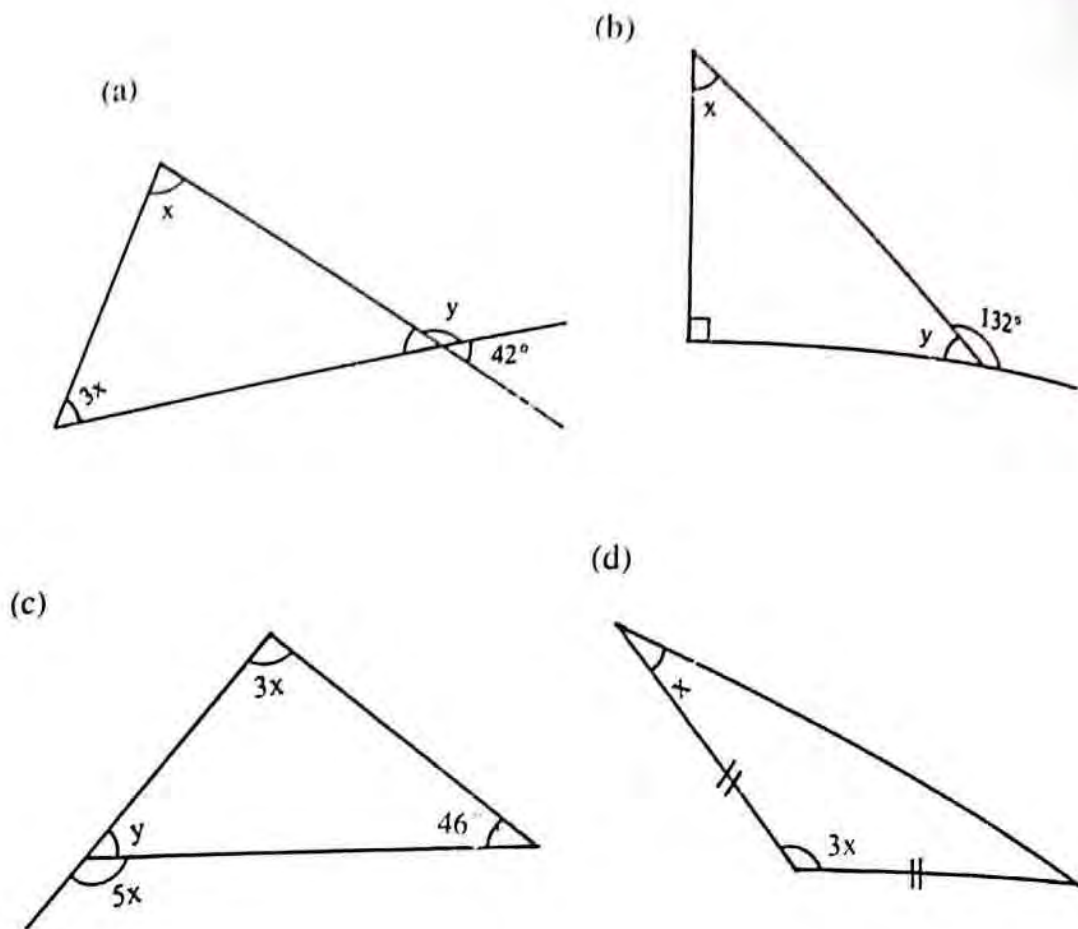


Fig. 20.42

8. In a triangle ABC, $\angle ABC = b$ and $\angle BAC = a$. Line BC is produced to D so that $\angle ACD = x$. Show that $a + b = x$.
9. Figure 20.43 shows a triangle RST in which $\angle RSL = \angle TSL$ and $\angle SML$ is a right angle. Find $\angle MSL$ in terms of r and t .

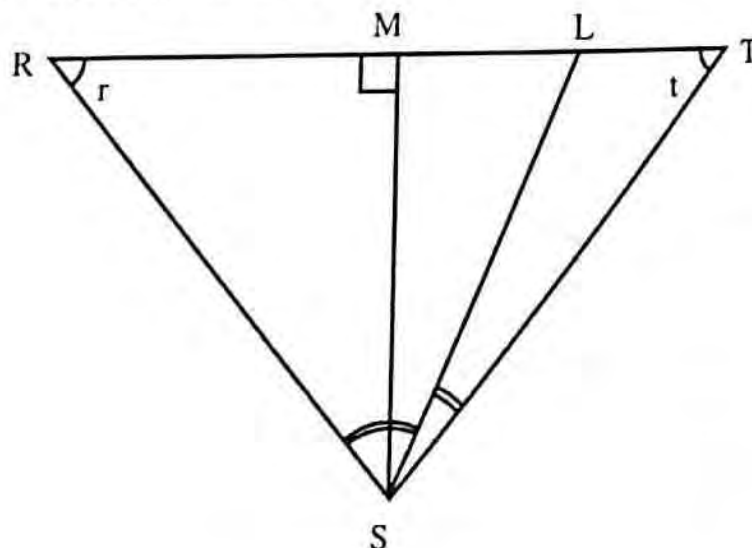


Fig. 20.43

10. Find the size of the obtuse angle FJH in figure 20.44, if lines IJ and EJ are extended to F and H respectively:

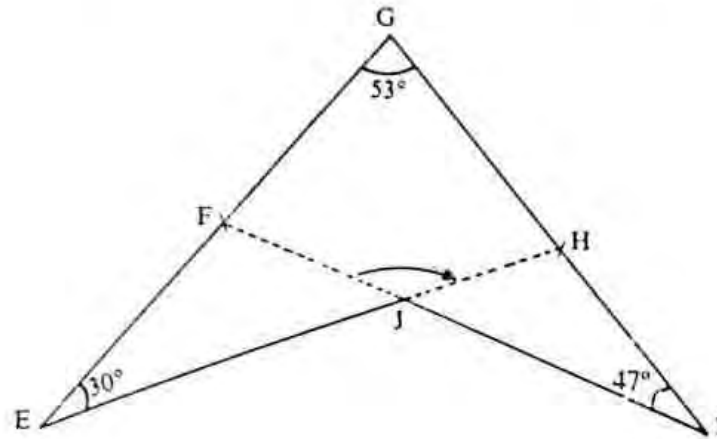


Fig. 20.44

11. In figure 20.45, show that $y + z - 2x + 180^\circ = 2$ right angles:

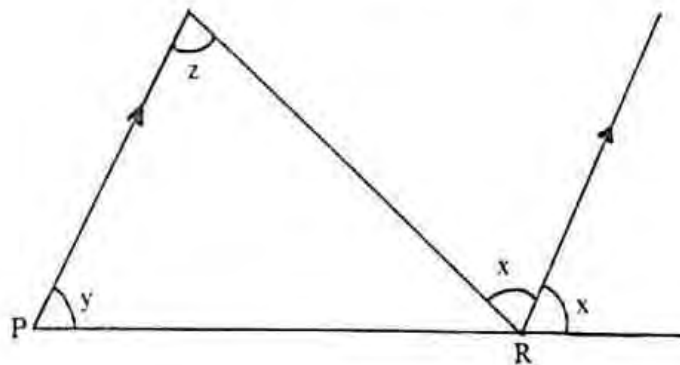


Fig. 20.45

12. In figure 20.46, show that $\angle ABD = x$:

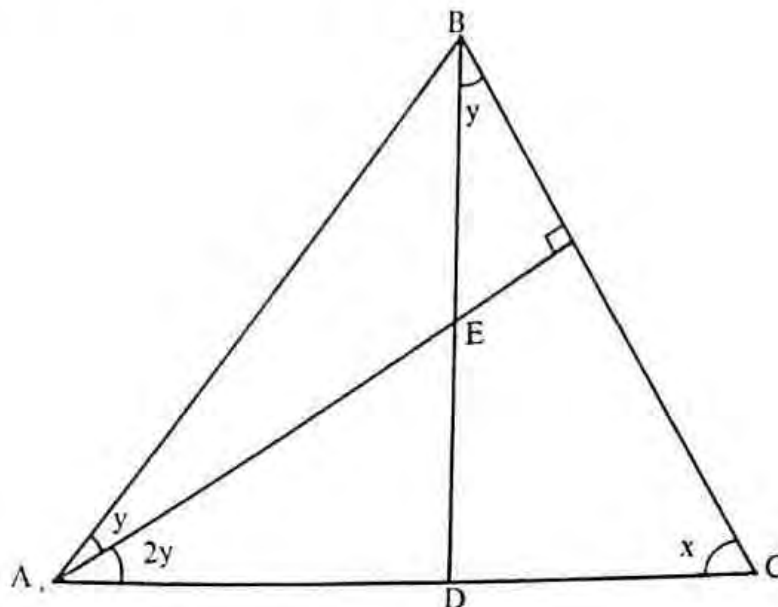


Fig. 20.46

13. A triangle ABC is such that $\angle ABC = 2x$. A perpendicular is dropped from A to meet BC at D. $\angle BAD = x$ and $\angle DAC = \frac{x}{2}$. If BC is extended to E, calculate the value of $\angle ACE$.
14. Draw a line XY of any length. Take M as the midpoint of XY. Draw a circle using XM as the radius and centre M. Take any point Z on the circumference. Measure $\angle XZY$. What name is given to such a triangle?
15. In figure 20.47, calculate the value of $\angle LPM$:

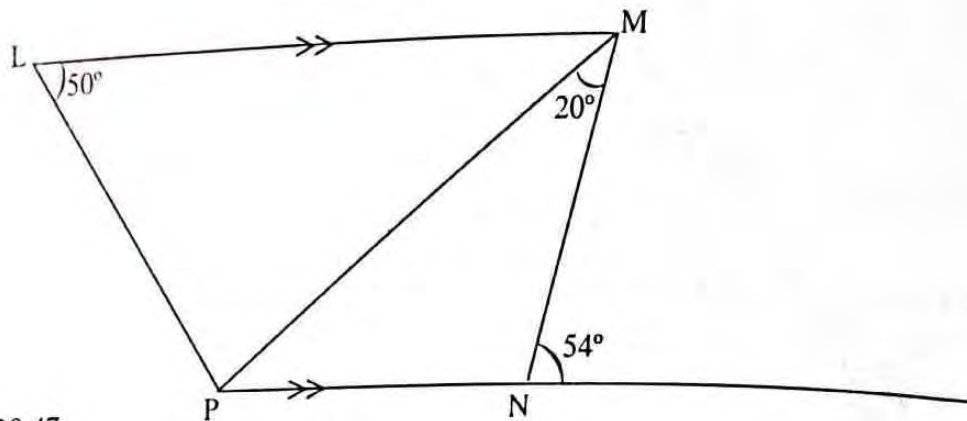


Fig. 20.47

16. In figure 20.48, lines AC and BD are perpendicular to each other and intersect at X. $\angle BCD = 90^\circ$ and $\angle BDC = 60^\circ$. If $AB = BC$, show that $\triangle ABC$ is equilateral.

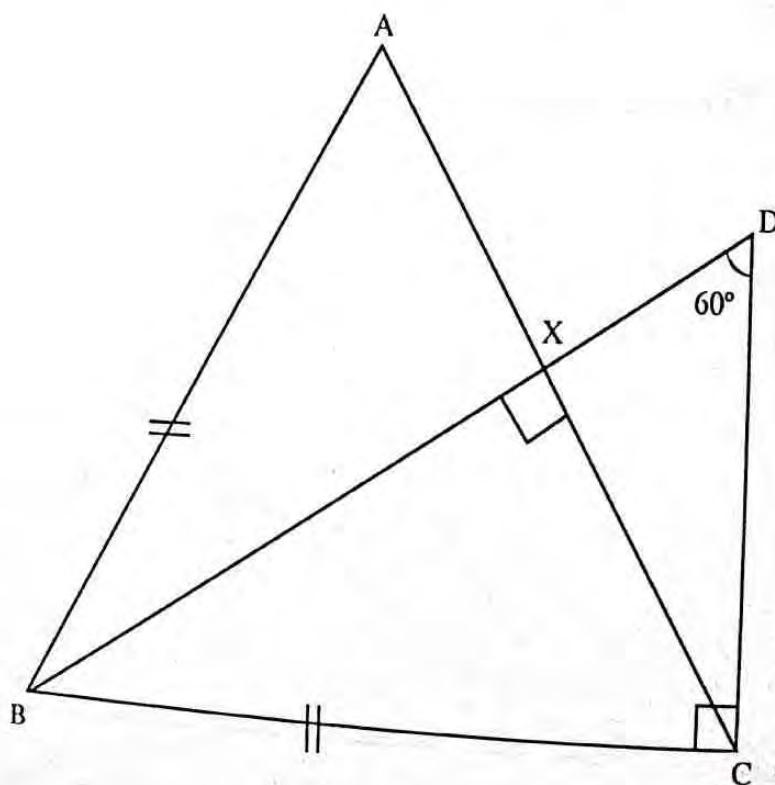


Fig. 20.48

17. In figure 20.49, $\angle PQX = \frac{1}{2}\angle RSX$ and $\angle QPX = 120^\circ$. Show that $\triangle RXS$ is isosceles:

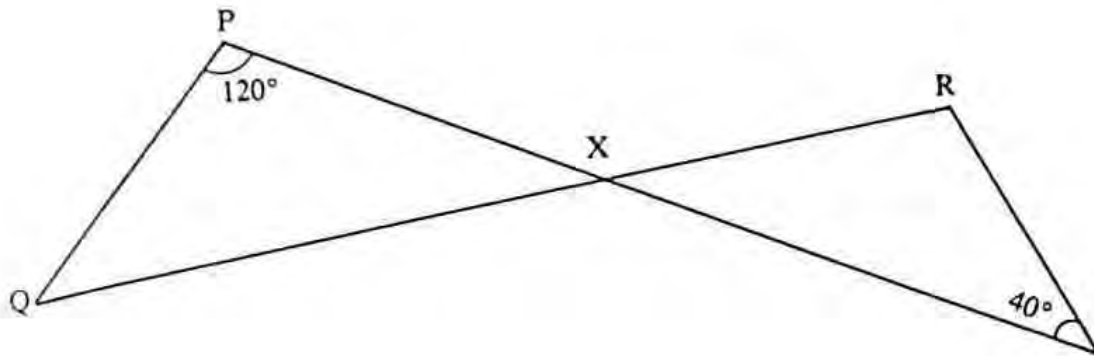


Fig. 20.49

18. In figure 20.50, PQR is a circle with centre O. Show that $\triangle PQR$ is isosceles:

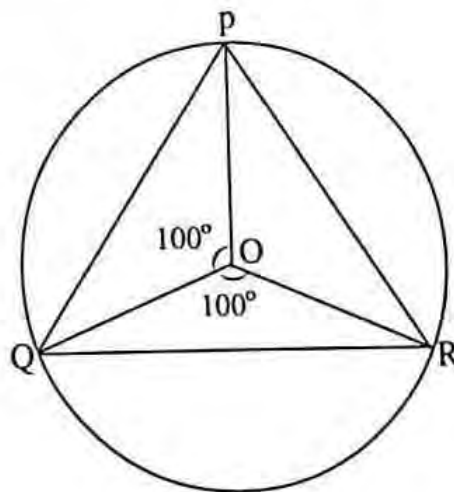


Fig. 20.50

Quadrilaterals

A quadrilateral is a four-sided plane figure. The interior angles of a quadrilateral add up to 360° . Just as in the case of triangles, quadrilaterals are classified in terms of sides and angles. Some quadrilaterals have some pairs of parallel sides. These, together with angles, form the basis by which common quadrilaterals are classified.

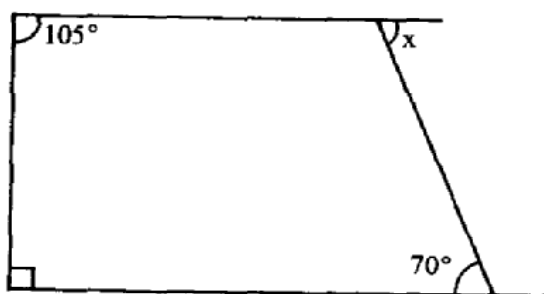
- (i) A **rectangle** is a quadrilateral in which opposite sides are equal and parallel and all angles are right angles.
- (ii) A **square** is a quadrilateral in which:
- opposite sides are equal and parallel.
 - all angles are right angles.
 - adjacent sides are equal.
- (iii) A **parallelogram** is a quadrilateral.
- whose pairs of opposite sides are equal and parallel:
 - a pair of opposite angles are equal.
- (iv) A **rhombus** is a quadrilateral which has:
- opposite sides equal and parallel.
 - adjacent sides equal.
 - pairs of opposite angles equal.
- (v) A **trapezium** is a quadrilateral which has at least one pair of opposite sides parallel.

By taking suitable measurements, draw each of the above quadrilaterals. If none of the angles of a quadrilateral is greater than 180° , the quadrilateral is said to be **convex**. A quadrilateral with one angle greater than 180° is said to be **non-convex** or **re-entrant**.

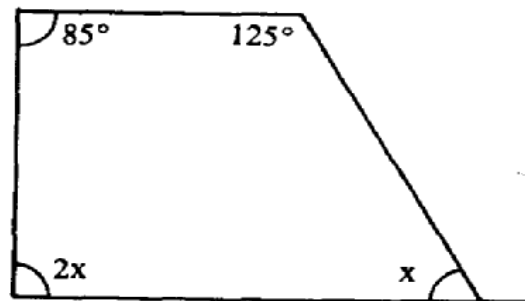
Exercise 20.5

1. Find the value of x in each of the following figures:

(a)



(b)



(c)

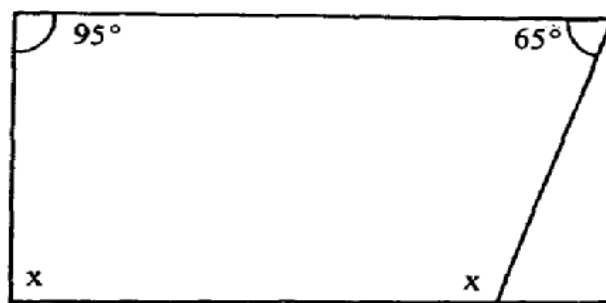


Fig. 20.51

2. If the angles of a quadrilateral are $2p$, $3p$, $7p$ and $8p$ in that order, what are their sizes? Sketch the figure and indicate the lines that are parallel, if any.
3. In quadrilateral PQRS, $\angle SPQ = \angle PQR = \angle QRS$ and $\angle PSR = 150^\circ$. Find $\angle PQR$.
4. Find the fourth angle of each of the following quadrilaterals:

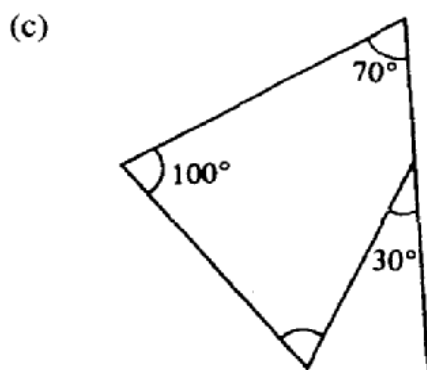
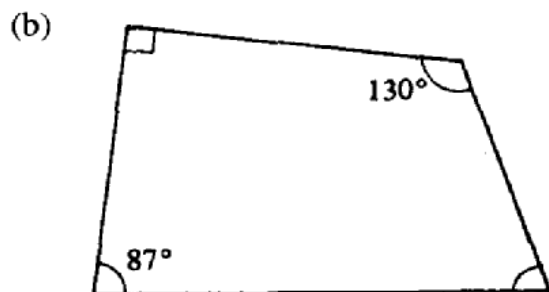
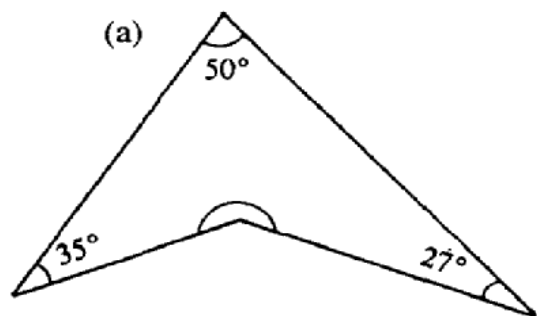


Fig. 20.52

5. In figure 20.53, $x = \frac{1}{3}y$ and $QO = RO = OS$:

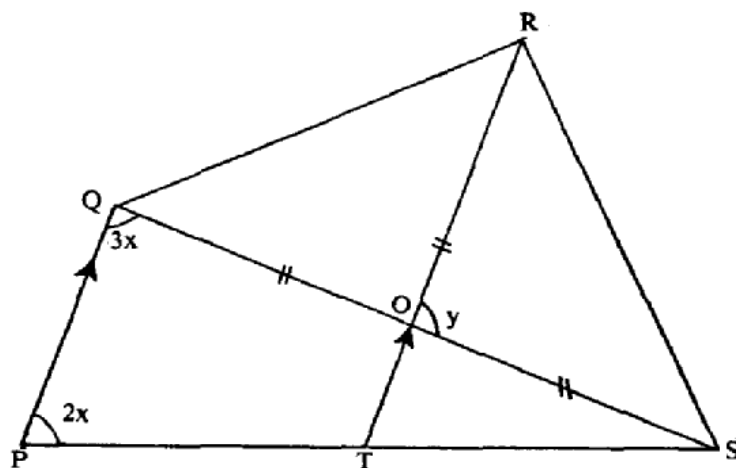


Fig. 20.53

Calculate $\angle QRS$. Show that the sum of the exterior angles of the quadrilateral PQRS is four right angles.

6. In figure 20.54, find the value of $\angle LPM$:

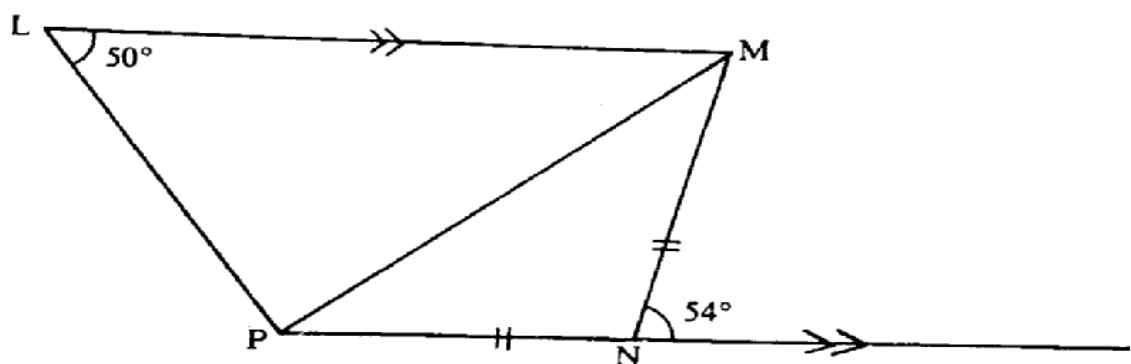


Fig. 20.54

7. BD is a diagonal of the rectangle ABCD. Line AE is drawn parallel to BD to meet CD produced at E. If $\angle AED = 42^\circ$, find $\angle DBC$.
8. In figure 20.55, the side AD of the rectangle ABCD is produced to E such that $AE = AC = CE$. Calculate $\angle BCE$.

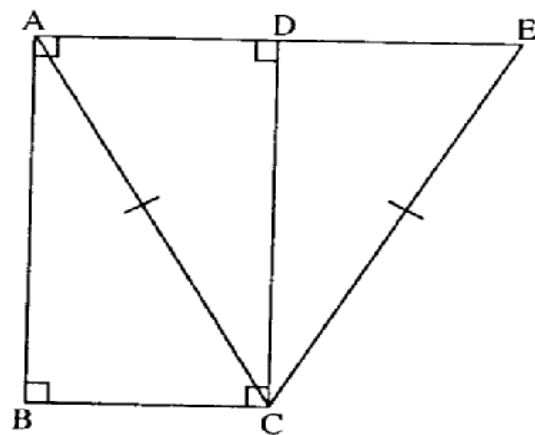


Fig. 20.55

9. PQRS is a rectangle. If $\angle PRS = 64^\circ$, find $\angle PSQ$.
10. The diagonals of rectangle ABCD intersect at X. If $\angle BXC = 128^\circ$, find:
- $\angle ABD$.
 - $\angle ACD$.
11. The diagonals AC and BD of a rectangle ABCD intersect at X. AC is produced to P such that $BC = CP$. If $\angle APB = 28^\circ$, show that:
- $\angle ABD = \frac{1}{2} \angle BXC$.
 - $\angle PAD = 2 \angle PBC$.

12. M is the point of intersection of the diagonals of a rectangle PQRS. An equilateral triangle PMT is constructed such that T and M are on the opposite sides of PQ. If $\angle PQS = 25^\circ$, find $\angle PTQ$.
13. In a square KLMN, the line LN is produced to P such that the parallelogram NPQM is a rhombus. If LQ cuts MP at O:
 - (a) find the angles of $\triangle MOQ$.
 - (b) prove that $PO = OL$.
14. O is the centre of a rectangle WXYZ. M is the midpoint of line WX and YM cuts ZX at P. Calculate $\angle YPO$ if $MX = XY$ and $\angle ZWY = 53^\circ$.
15. Calculate all the angles in figure 20.56. What do you notice about the opposite angles of the figure?

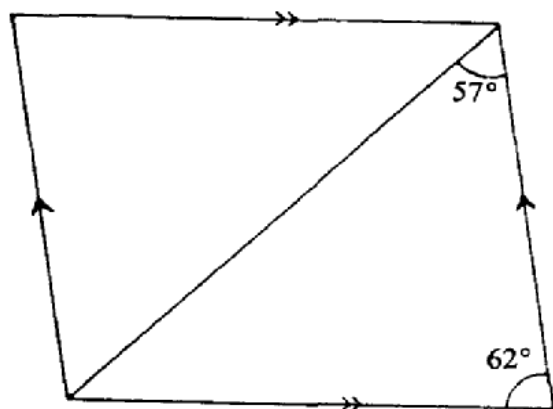


Fig. 20.56

16. A parallelogram ABCD is such that all sides are equal. If $\angle CAB = 32^\circ$, calculate all the angles of the parallelogram. What name is given to such a figure? List its properties.
17. In figure 20.57, EFGH is a rhombus and triangle DEF is equilateral. Calculate $\angle HDG$, given that $\angle HED = 18^\circ$:

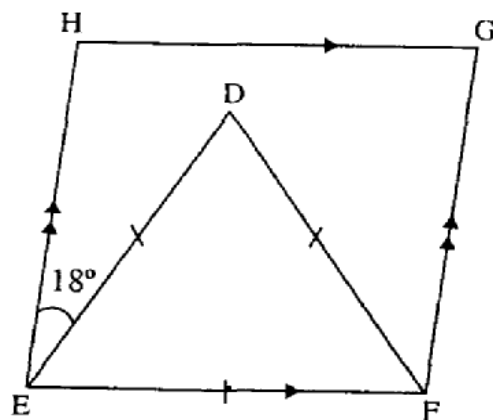


Fig. 20.57

18. The diagonals of a rhombus ABCD intersect at O. $\angle ADO = x$ and $\angle DAO = y$. Show that the diagonals intersect at right angles.

Other Polygons

Polygons are named on the basis of the number of sides they have, e.g., triangle (three sides), quadrilateral (four sides), pentagon (five sides) and hexagon (six sides).

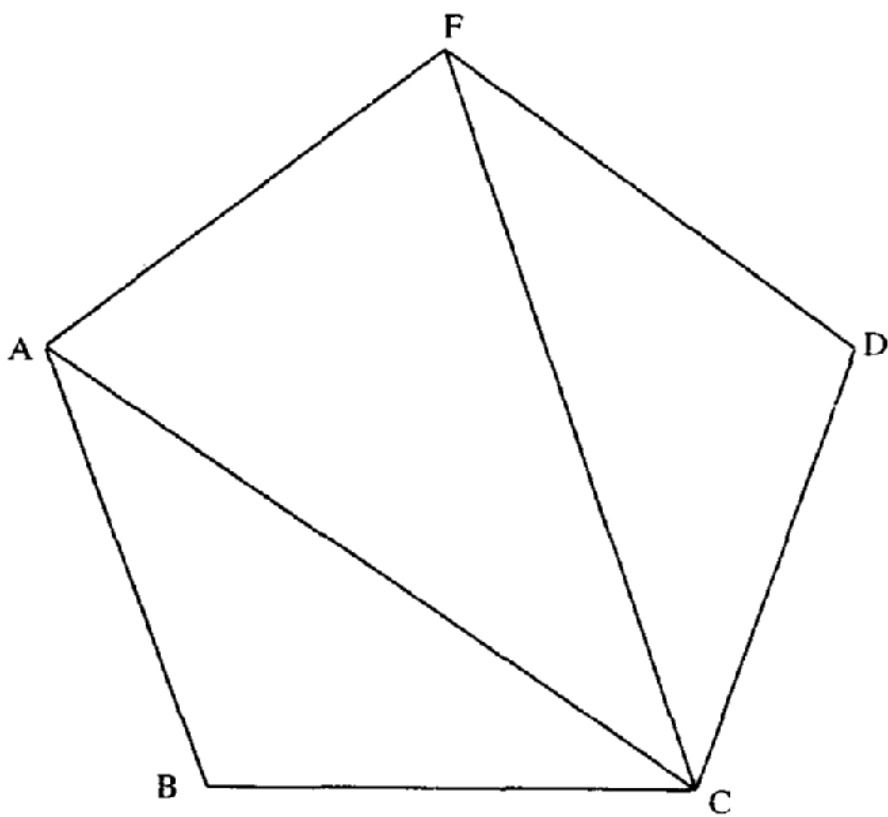


Fig. 20.58

The above polygon has five sides and has been subdivided into three triangles. What is the total sum of the angles of the three triangles in degrees? What is this in right angles? Find the sum of the interior angles of the pentagon in:

- (i) degrees.
- (ii) right angles.

Use the same method of sub-division to determine the sum of the interior angles of a hexagon.

Copy and complete the table below:

<i>Number of sides of polygon</i>	<i>Number of triangles to make the polygon</i>	<i>Sum of interior angles in degrees</i>	<i>Sum of interior angles in right angles</i>
3	1	180^0	2
4	2	360^0	4
5	3	540^0	6
6	—	—	—
7	—	—	—
8	—	—	—
•	•	•	•
•	•	•	•
•	•	•	•
n	—	—	—

Note:

If a polygon has n sides, then the sum of interior angles is $(2n - 4)$ right angles.

The sum of Exterior Angles of a Polygon

Figure 20.59 shows a hexagon with interior angles g, h, i, k and l and exterior angles a, b, c, d, e and f .

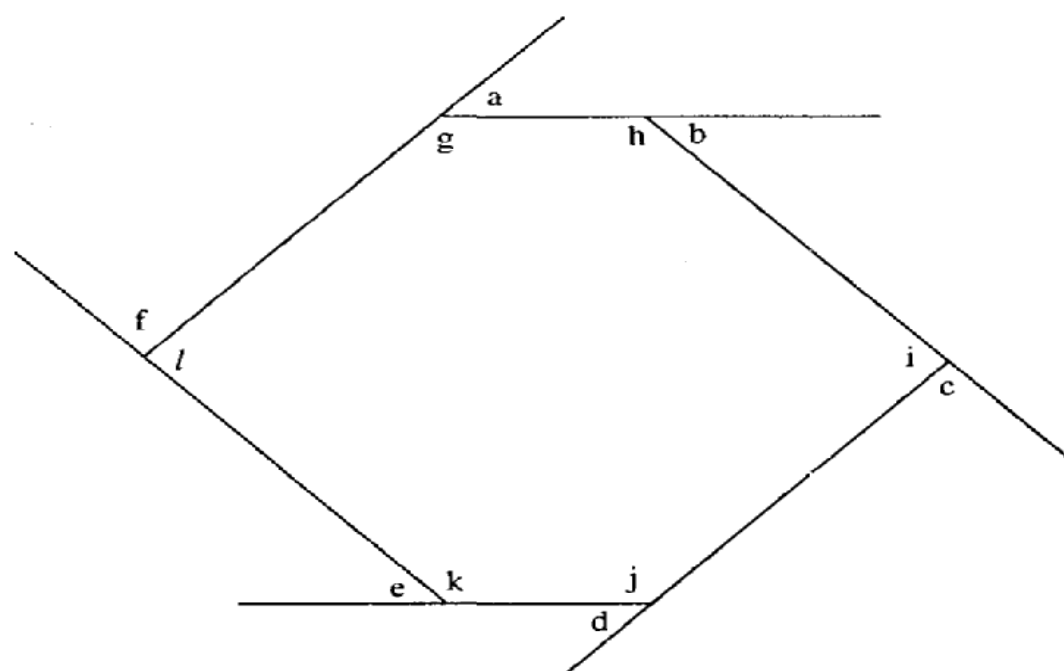


Fig. 20.59

$$a + g + h + b + i + c + j + d + e + k + l + f = 180 \times 6$$

$$= 1\,080^\circ$$

$$(g + h + i + j + k + l) + (a + b + c + d + e + f) = 1\,080^\circ$$

The sum of interior angles of a hexagon is 720°

$$\therefore 720^\circ + a + b + c + d + e + f = 1\,080^\circ$$

$$\text{so, } a + b + c + d + e + f = 360^\circ.$$

$\therefore$ The sum of exterior angles of a hexagon is equal to 360° .

It can also be shown that the sum of exterior angle of any polygon is 360° .

A polygon is said to be **regular** if all its sides and all its interior angles are equal. Those that do not satisfy these conditions are said to be **irregular**.

Exercise 20.6

1. In the figures shown below, determine the value of x :

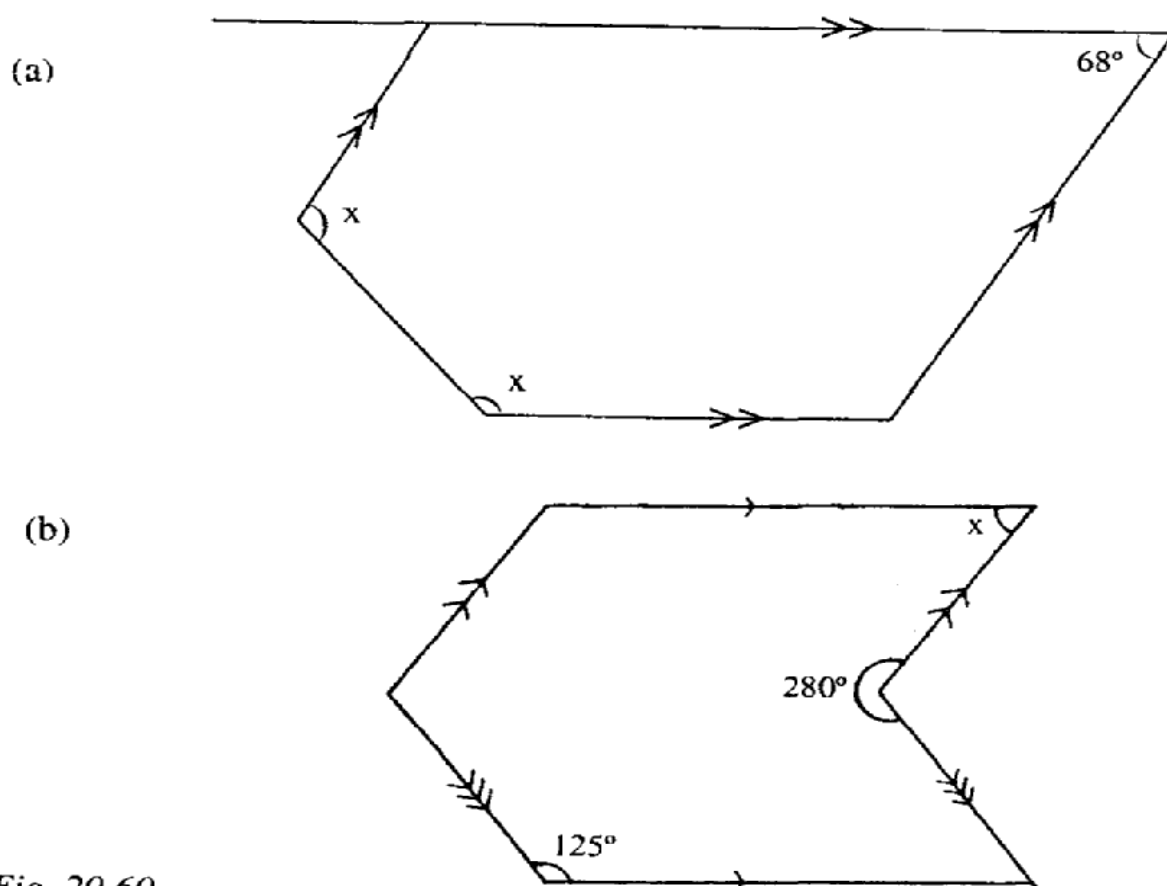


Fig. 20.60

- What do we call a regular polygon, each of whose interior angles is:

(a) 60° ?	(b) 90° ?	(c) 120° ?
------------------	------------------	-------------------
- Find the number of sides of a polygon each of whose exterior angles is:

(a) 30°	(b) 36°	(c) 45°
(d) 60°	(e) 72°	(f) 90°

4. Sketch a hexagon $ABCDEF$. Mark a point O inside the hexagon. Join O to all the vertices of the hexagon. How many triangles do you get? What is the sum of all the angles of these triangles? What is the sum of angles at point O ? Hence, show that the sum of the interior angles of a hexagon is eight right angles.
5. Figure 20.61 shows a pentagon $ABCDE$ divided into four triangles. Using the idea of question 4, show that the sum of the interior angles of the pentagon is six right angles.

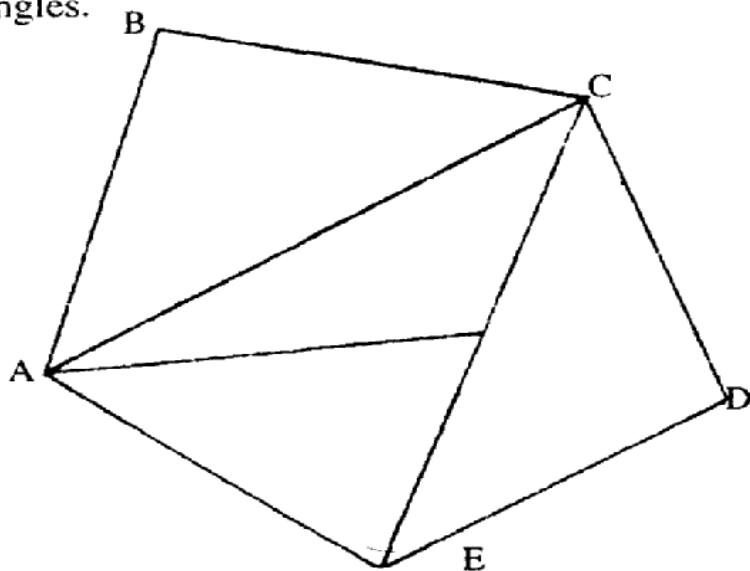


Fig. 20.61

6. Figure 20.62 shows a six-sided re-entrant polygon $ABCDEF$ divided into five triangles. Using the idea of question 4 and 5, find the sum of the interior angles of the hexagon.

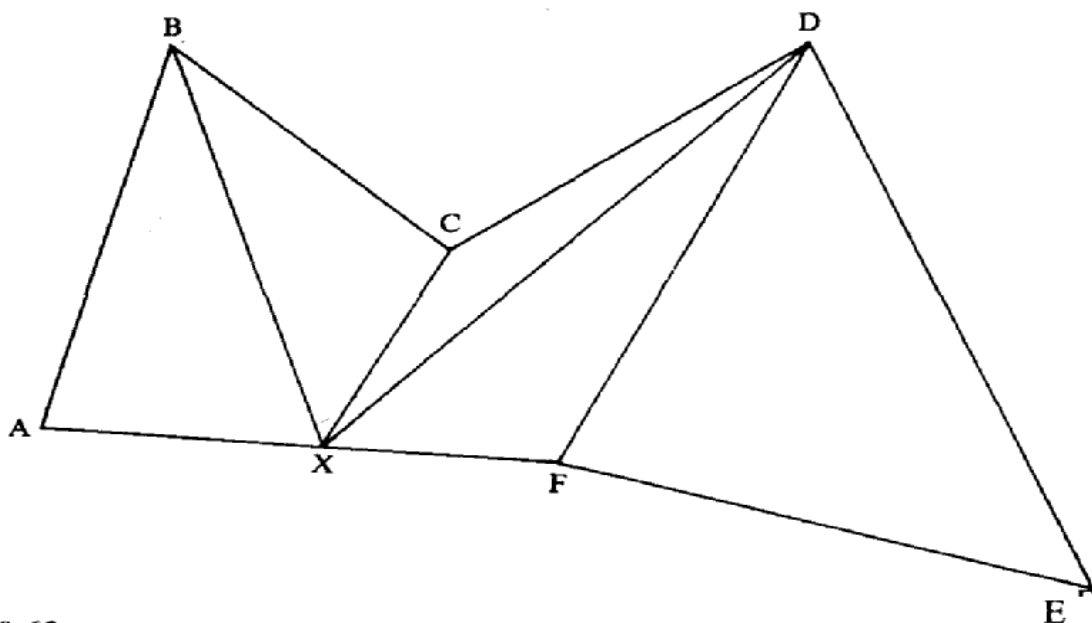


Fig. 20.62

7. In figure 20.63, $ABCDE$ is a regular pentagon. Show that the sum of the exterior angles is four right angles.

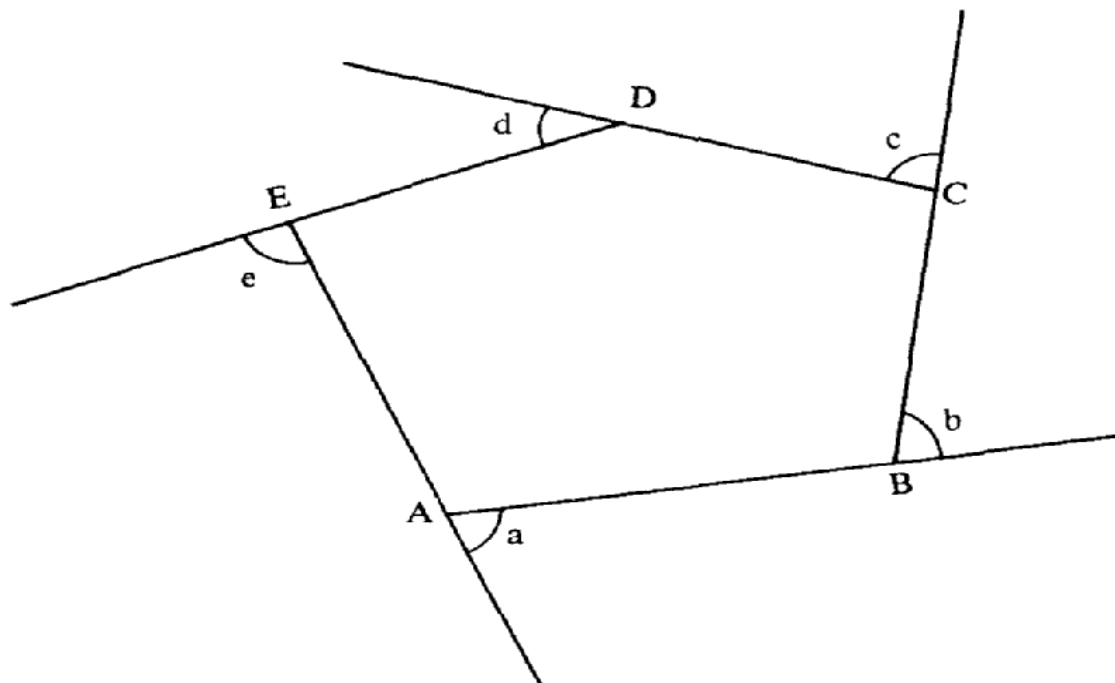


Fig. 20.63

8. A regular pentagon $ABCDE$ is such that BD and CE intersect at X . Show that the triangles BXC , DXE and CXD are isosceles.
9. Find the sum of the interior angles of:
- an octagon.
 - a decagon.
10. The sum of the interior angles of a polygon is $1\ 980^\circ$. Find:
- the number of triangles the polygon can be subdivided into.
 - the number of sides the polygon has.
11. Sketch a regular polygon $ABCDEFGHIJKL$ of twelve sides. Join every alternative points, i.e., A to C , C to E , etc. What regular polygon do you obtain?
12. Find the size of each exterior angle a polygon with:
- 12 sides.
 - 14 sides.
 - 20 sides.

Chapter Twenty One

GEOMETRICAL CONSTRUCTIONS

Geometrical construction is the drawing of accurate figures.

21.1: Construction of a Line

To construct a line AB of length 6 cm

- (i) Draw a line using a ruler.
- (ii) Mark point A close to one end of the line.
- (iii) Using A as centre and a radius of 6 cm, mark an arc on the line to get B.
- (iv) AB is the required line.

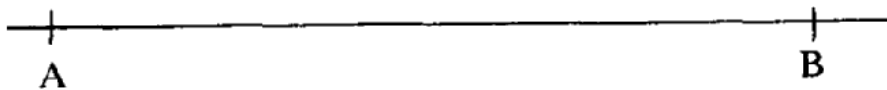


Fig. 21.1

21.2: Construction of Perpendicular and Parallel Lines

Perpendicular Lines

Figure 21.2 shows PQ as a perpendicular bisector of a given line AB.

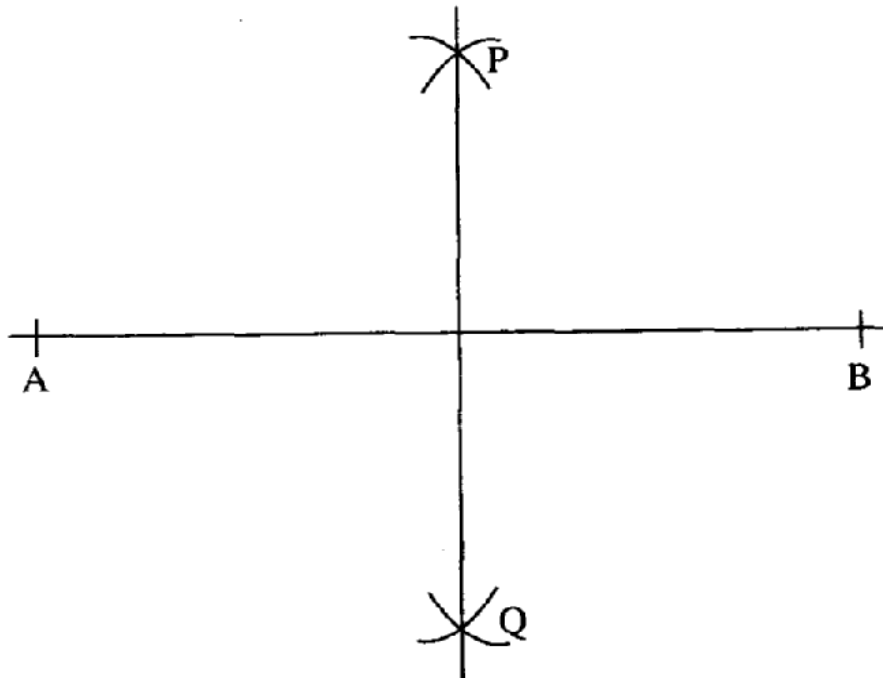


Fig. 21.2

To obtain the perpendicular bisector PQ

- (i) With A and B as centres, and using the same radius, draw arcs on either side of AB to intersect at P and Q.
- (ii) Join P to Q.

Figure 21.3 shows XR, a perpendicular from a point X to a given line AB. Show how this is constructed using a pair of compass and a ruler only.

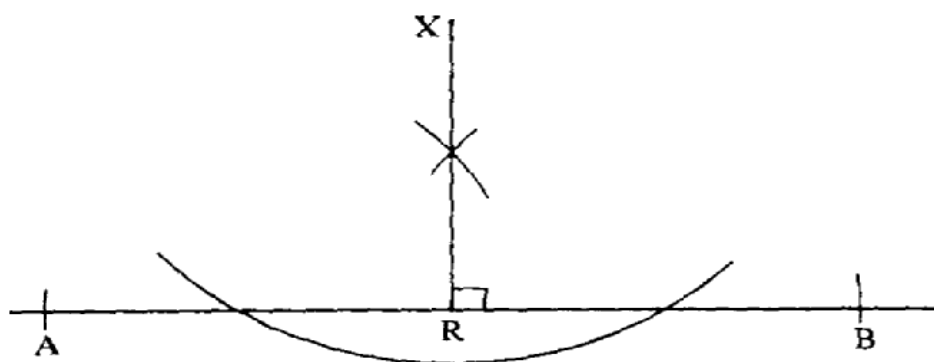


Fig. 21.3

Construction of a perpendicular line from a given point to a given line is useful in finding the shortest distance from a point to a given line

To construct a perpendicular through point P on a given line.

- (i) Using P as centre and any convenient radius, draw arcs to intersect the line at A and B.

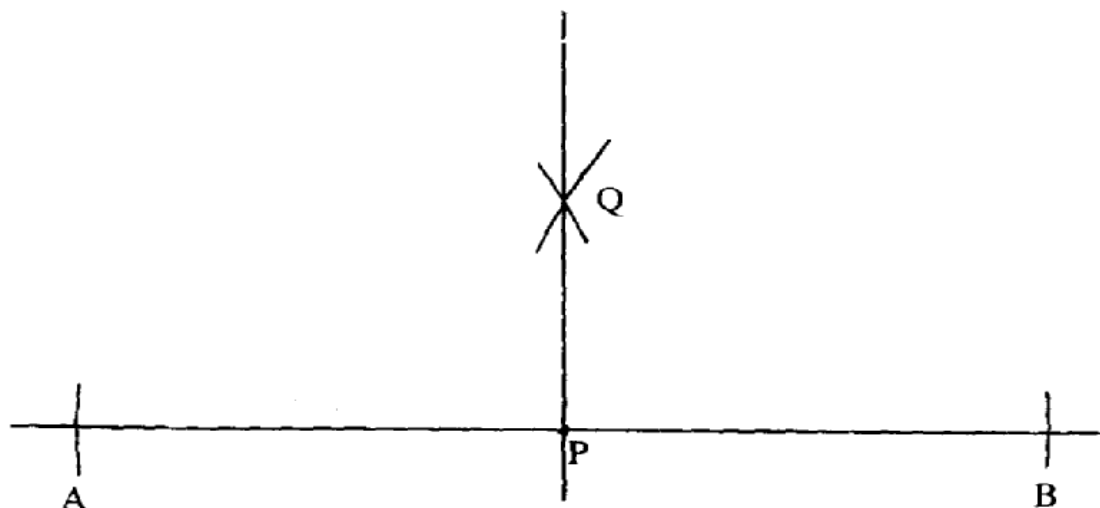


Fig. 21.4

- (ii) Using A as centre and a radius whose measure is greater than AP, draw an arc above the line.
- (iii) Using B as centre and the same radius, draw an arc to intersect the one in (ii) at point Q.
- (iv) Using a ruler, draw PQ.

Construction of Perpendicular Lines using a Set Square

Two edges of a set square are perpendicular. They can be used to draw perpendicular lines. When one of the edges is put along a line, a line drawn along the other one is perpendicular to the given line.

To construct a perpendicular from a point p to a line

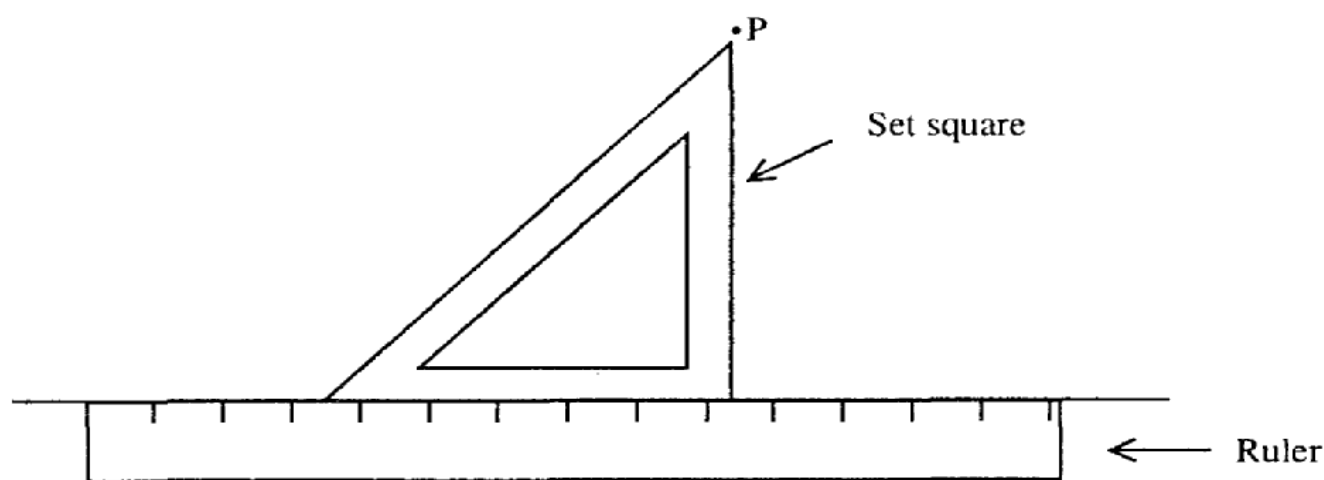


Fig. 21.5

- (i) Place a ruler along the line.
- (ii) Place one of the edges of a set square which form a right angle along the ruler.
- (iii) Slide the set square along the ruler until the other edge reaches P.
- (iv) Hold the set square firmly and draw the line through P to meet the line perpendicularly.

Construction of Angles using a Ruler and Pair of Compasses only

To construct an angle which is equal to a given angle

Suppose we want to construct an angle at a point A on line AB equal to $\angle PQR$, see figure 21.6(a).

Proceed as follows:

- (i) With Q as centre and any radius, draw an arc to cut QR and QP at X and Y respectively.
- (ii) With QX as radius and A as centre, draw an arc to cut AB at X', see figure 21.6(b).

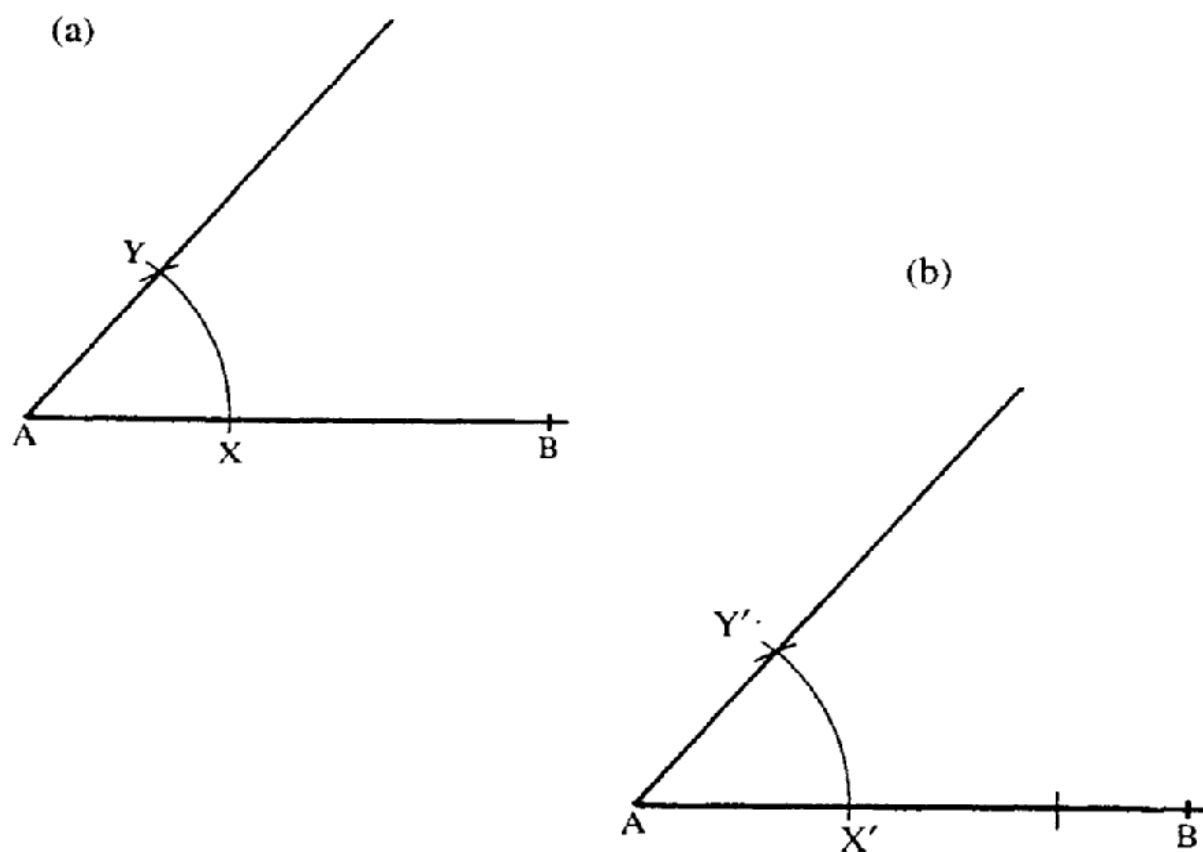


Fig. 21.6

- (iii) With XY as radius and X' as centre, draw another arc to cut the arc in (ii) above at Y'.
- (iv) Join A to Y. The required angle is $Y'AX'$

Bisecting an Angle

Figure 21.7 shows the bisection of angle AOB.

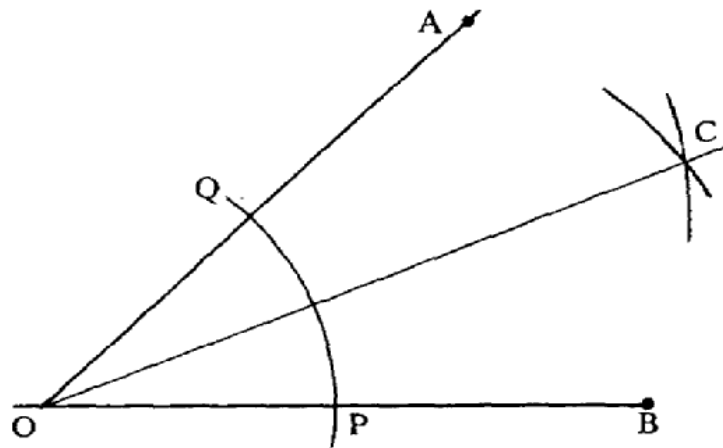


Fig. 21.7

To bisect angle AOB

- (i) With O as centre and a suitable radius, draw an arc to cut OA and OB at Q and P respectively.
- (ii) Taking P and Q as centres and a suitable radius, draw two arcs to intersect at C.

The line OC bisects the angle AOB.

Bisect an angle of 180° .

Construction of an Angle of 60°

Figure 21.8 shows the construction of an angle of 60° :

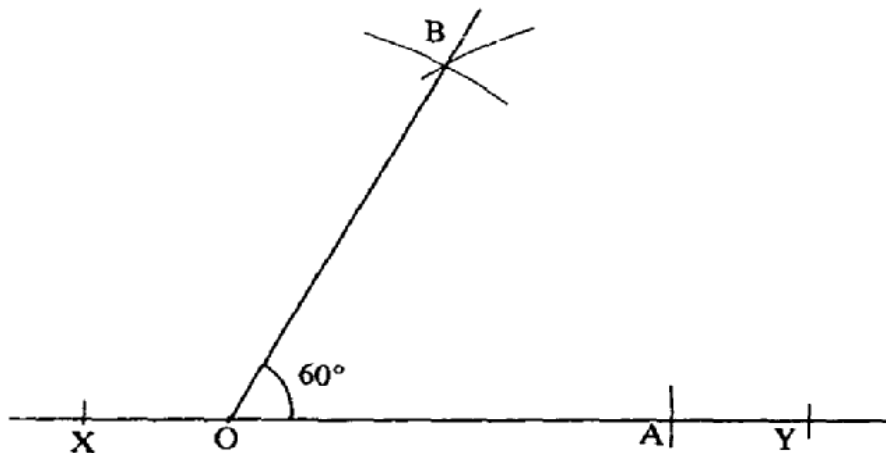


Fig. 21.8

To Construct an Angle of 60°

- (i) Draw a line XY
- (ii) With centre O and a suitable radius, draw an arc to cut XY at A.
- (iii) With centre A and the same radius, draw an arc to intersect the first arc at B. Angle BOA is the required angle.

Using a pair of compasses and a ruler only, construct an angle of 30° .

Exercise 21.1

1. Using a ruler and a pair of compasses only, construct the following angles:
 - (a) 45°
 - (b) 75°
 - (c) $37\frac{1}{2}^\circ$
 - (d) 165°
2. Construct triangle PQR such that $PQ = 7.9$ cm, angle $RPQ = 30^\circ$ and angle $PQR = 120^\circ$. The bisector of angle PQR meets PR at M. Measure PM and RM. What do you notice?
3. Construct a triangle XYZ such that $\angle XYZ = 135^\circ$, $XY = 4.6$ cm and $YZ = 6.1$ cm. Measure XZ and $\angle XZY$.
4. Construct triangle ABC such that $AB = 12$ cm, $BC = 8$ cm and $\angle ACB = 90^\circ$. Measure AC. On the opposite side of AB, a triangle ABD is drawn such that angle $ABD = 60^\circ$ and angle $BAD = 45^\circ$. Measure BD.
5. Draw triangle PQR as shown in figure 21.9. On the opposite side of PR, draw an equilateral triangle PRS. Measure SQ.

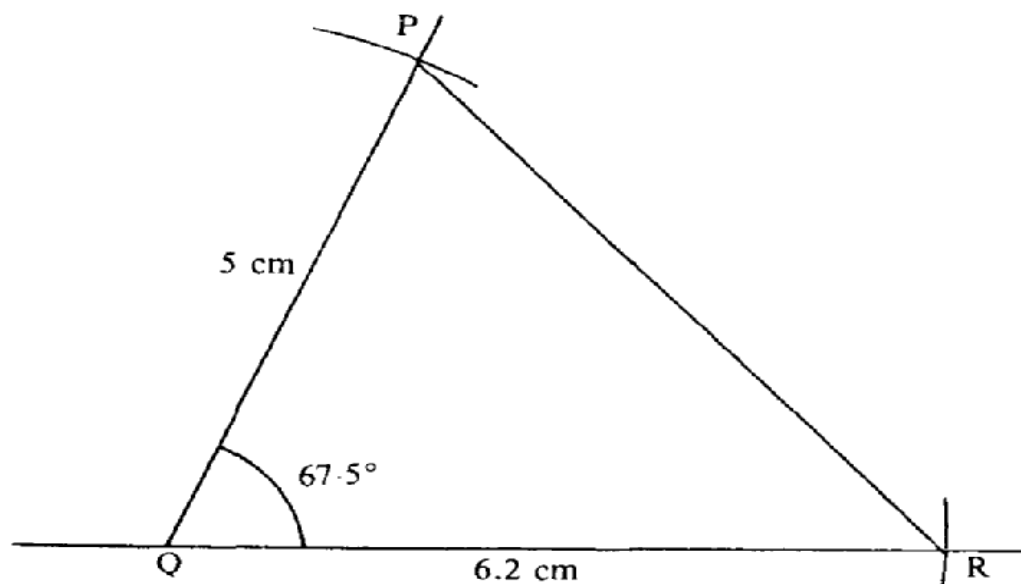


Fig. 21.9

6. Construct triangle LMN such that $LM = 7.3$ cm, $MN = 4.9$ cm and angle $LMN = 75^\circ$. Calculate the area of triangle LMN.

21.4: Construction of Parallel Lines

To construct a line through a given point and parallel to a given line, we may use a ruler and a pair of compasses only, or a ruler and a set square.

Using a Ruler and a Pair of Compasses only

Parallelogram method

Figure 21.10 illustrates the parallelogram method. The line XR parallel to PQ is constructed as follows:

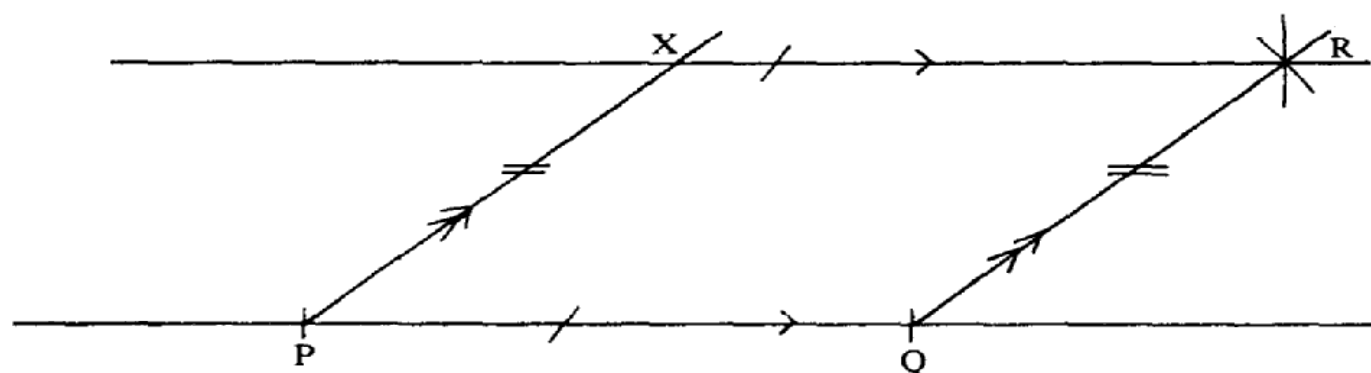


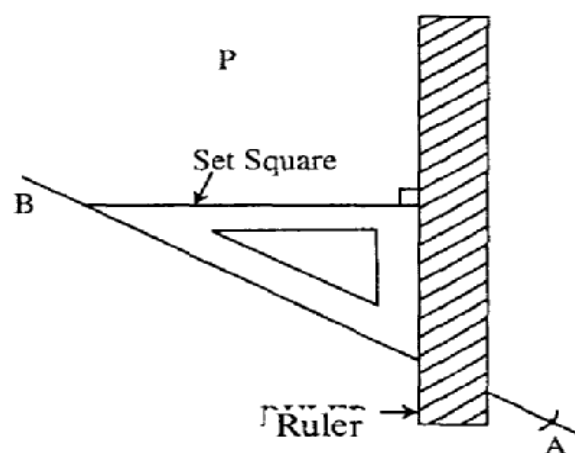
Fig. 21.10

- (i) With X as centre and radius PQ, draw an arc.
- (ii) With Q as centre and radius PX, draw another arc to cut the first arc at R.
- (iii) Join X to R.

Using a set square and ruler

Figure 21.11(a) shows a line AB and a point P through which a line parallel to AB is to be drawn.

(a)



(b)

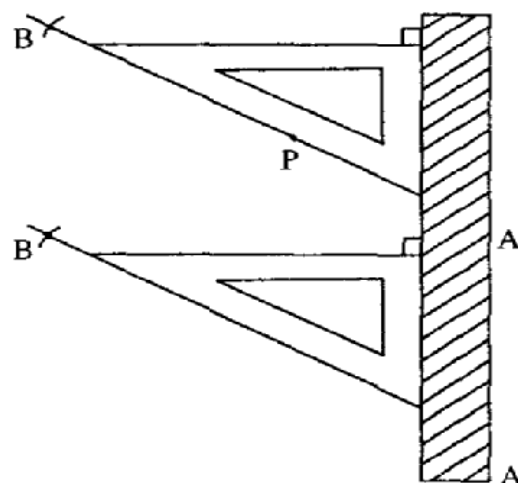


Fig. 21.11

A set square is placed along AB so that one of its edges lies on AB. A ruler is then placed on either of the remaining two edges. Keeping the ruler firmly in place, the set square is slid along the edge of the ruler until the set square edge that is along AB passes through P, as shown in figure 21.11(b). The edge through P is used to draw a straight line.

The straight line so drawn is parallel to AB.

Proportional Division of Lines

Lines can be proportionately divided into a given number of equal parts by use of parallel lines. Consider figure 21.12 which shows proportional division of

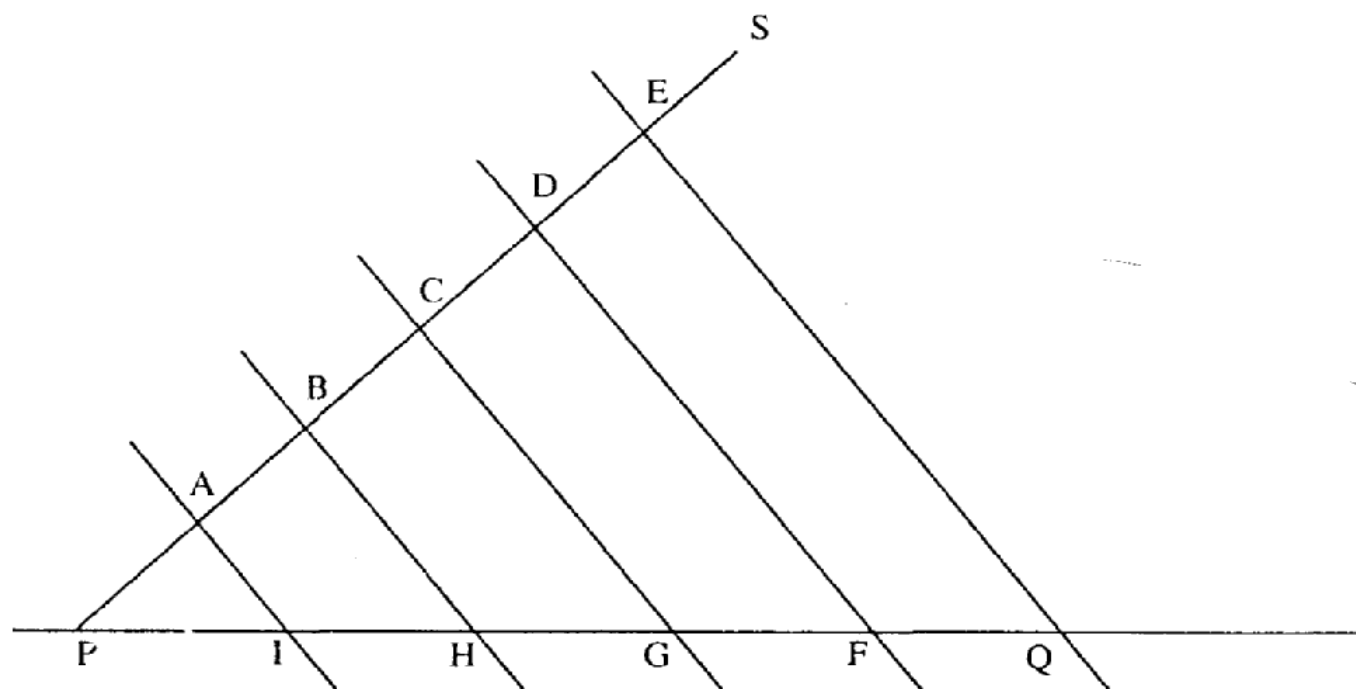


Fig. 21.12

line PQ into 5 equal parts.

This is done as follows:

- (i) Through P, draw a line PS of any convenient length at a suitable angle with PQ.
- (ii) Using pair of compasses, mark off, along PS, five equal intervals PA, AB, BC, CD and DE.
- (iii) Join E to Q. By using a set square and ruler, draw lines DF, CG, BH and AI parallel to EQ.

The line PQ is then divided into PI, IH, HG, GF and FQ which are at equal intervals.

Exercise 21.2

1. Draw a triangle ABC in which angle $ABC = 75^\circ$, $BC = 10$ cm and angle $BCA = 75^\circ$. Drop a perpendicular from A to meet BC at N. Measure BN and AN.
2. Triangle LMN is such that angle $LMN = 82.5^\circ$, $LM = 8.3$ cm and $MN = 5.8$ cm. Draw perpendicular bisectors of the sides of the triangle to meet at a point O. With O as the centre, draw a circle passing through points L, M and N. Measure the radius of the circle.
3. Draw triangle WXY such that angle $XWY = 75^\circ$, $WX = 9.2$ cm and $WY = 5.3$ cm. Construct the bisector of angle XWY to cut XY at M and the perpendicular bisector from W to cut XY at N. Measure angle MWN and the length of MN.
4. Draw triangle PQR such that $PQ = 6.4$ cm, $QR = 3.9$ cm and $RP = 8.2$ cm. Draw perpendicular bisectors of PQ and QR to meet at O. With O as the centre and OP as the radius, draw a circle. Draw a line from Q through the centre to meet the circle at S. Measure PS and find the relationship between angle PR and PQR.
5. Using a ruler and a pair of compasses only, construct a square WXYZ side 5.6 cm. Measure XZ.
6. Using a ruler and a pair of compasses only, construct a rectangle PQRS in which the diagonals are 10 cm and intersect at 45° . Measure PQ and RS.
7. A parallelogram LMNP is such that $LM = 4.8$ cm, $LP = 7.5$ cm and $LN = 7.0$ cm. Measure angle MLP.
8. Line $AB_1 = 7.2$ cm, angle $AB_1 B_2 = 30^\circ$ and $B_1 B_2 = 9.7$ cm. Using line AB_1 , divide line $B_1 B_2$ into six equal intervals and measure the length of three intervals.
9. A line $PQ = 11.7$ cm is perpendicular to line QR. By using line QR, divide line PQ into ten equal parts.
10. A line $LM = 13.9$ cm. By using another line, divide LM into nine equal parts.
11. In a triangle XYZ, $XY = 6$ cm, $YZ = 7.2$ cm and $ZX = 8.5$ cm. M and N are midpoints of XY and XZ respectively. What can you say about the line MN?

21.5: Construction of Regular Polygons

As stated elsewhere in this book, polygon is regular if all its sides and angles are equal, otherwise it is irregular.

Remember that for a polygon of n sides, the sum of interior angles is $(2n - 4)$ right angles. The size of each interior angle of the regular polygon is therefore equal to $\frac{2n-4}{n}$ right angles.

The sum of exterior angles of any polygon is 360° . Each exterior angle of a regular polygon is therefore equal to $\left(\frac{360}{n}\right)^\circ$

Construction of a Regular Triangle

To construct a triangle of side 5 cm

- (i) Construct a line $AB = 5$ cm.
- (ii) Using A as centre and a radius of 5 cm, mark an arc.
- (iii) Using B as centre and a radius of 5 cm, mark another arc to intersect the one in (ii) at C.

ABC is the required triangle, see figure 21.13.

Measure the angles of the triangle. What name is given to a regular triangle?

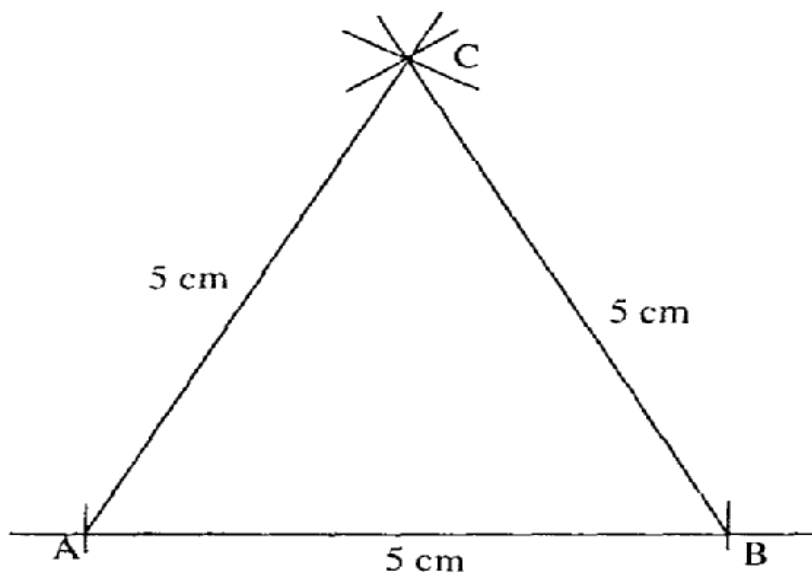


Fig. 21.13

Construction of a Regular Quadrilateral

To construct a quadrilateral ABCD of side 4 cm

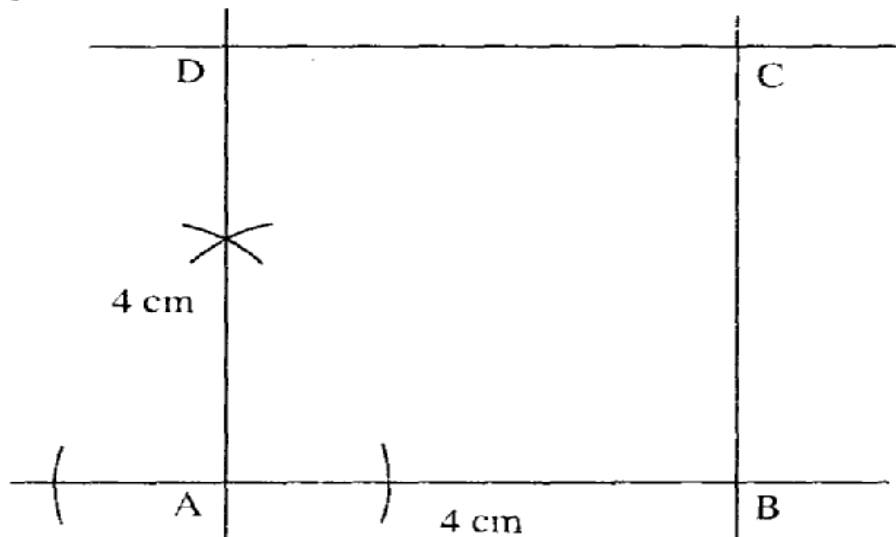


Fig. 21.14

- (i) Draw a line $AB = 4$ cm.
- (ii) Construct a perpendicular at A.
- (iii) Mark point D on the perpendicular, 4 cm from A.
- (iv) Using the same radius, B and D as centres, with mark arcs to intersect at C.
- (v) Join B to C and D to C. What name is given to a regular quadrilateral?

Construction of a Regular Pentagon

To construct a regular pentagon ABCDE of side 4 cm

Each of the interior angles = $\frac{(10-4)}{5}$ right angles
 $= 108^\circ$.

- (i) Draw a line $AB = 4$ cm long.
- (ii) Draw angle $ABC = 108^\circ$ and $BC = 4$ cm.
- (iii) Use the same method to locate points D and E.

Figure 21.15 shows the pentagon.

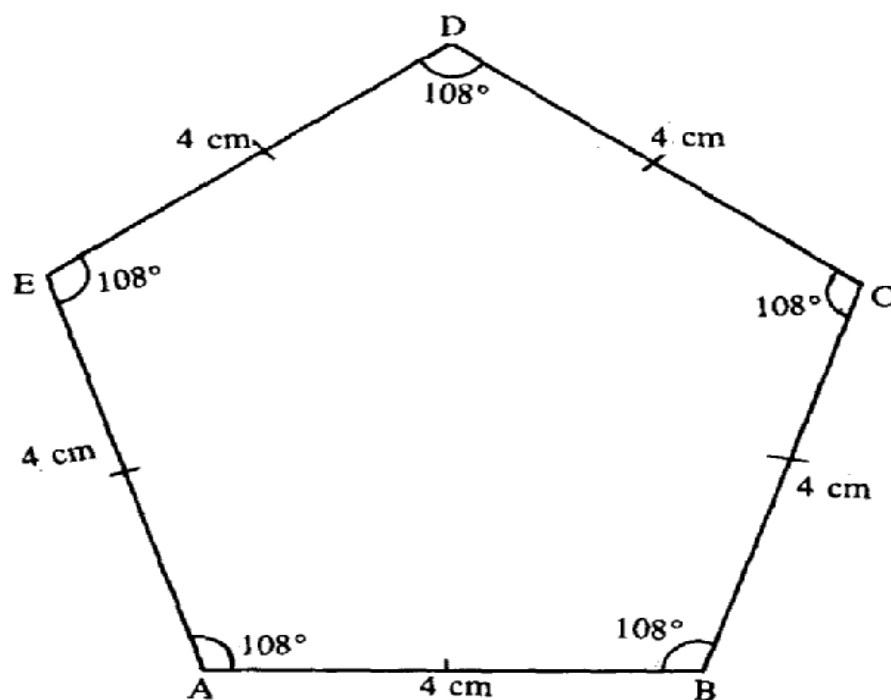


Fig. 21.15

Use the same procedure to construct a regular hexagon of side 5 cm using a ruler and a pair of compasses only.

21.6: Construction of Irregular Polygons

There is no definite method for the construction of irregular polygons. Sufficient angular and linear measurements must be given in order. Examples of irregular polygons are scalene triangles and general quadrilaterals.

Construction of Triangles

Construction 1: To construct a triangle, given the lengths of its sides

Construct a triangle ABC in which $AB = 3$ cm, $BC = 5$ cm and $CA = 7$ cm. Figure 21.16(a) shows a rough sketch of the $\triangle ABC$.

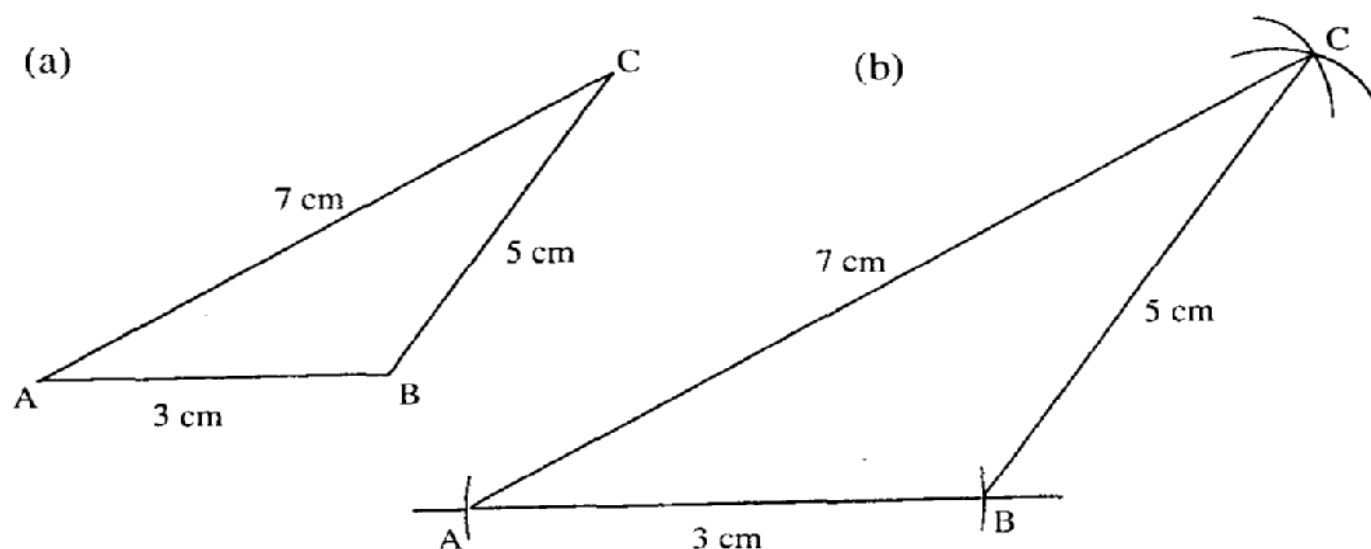


Fig. 21.16

The construction is carried out as follows:

- (i) Draw a line and mark a point A on it.
- (ii) On the line, mark off with a pair of compasses a point B, 3 cm from A.
- (iii) With B as centre and radius 5 cm, draw an arc.
- (iv) With A as the centre and radius 7 cm, draw another arc to intersect the arc in (iii) at C. Join A to C and B to C.

Use the same skill to construct:

- (i) an equilateral triangle ABC of length 5 cm.
- (ii) an isosceles triangle ABC, in which $AB = AC = 8$ cm and $BC = 10$ cm.

Construction 2: To construct a triangle, given the size of two angles and the length of one side

Construct a triangle ABC in which $\angle BAC = 60^\circ$, $\angle ABC = 50^\circ$ and $BC = 4$ cm.

Figure 21.17(a) is a sketch of $\triangle ABC$.

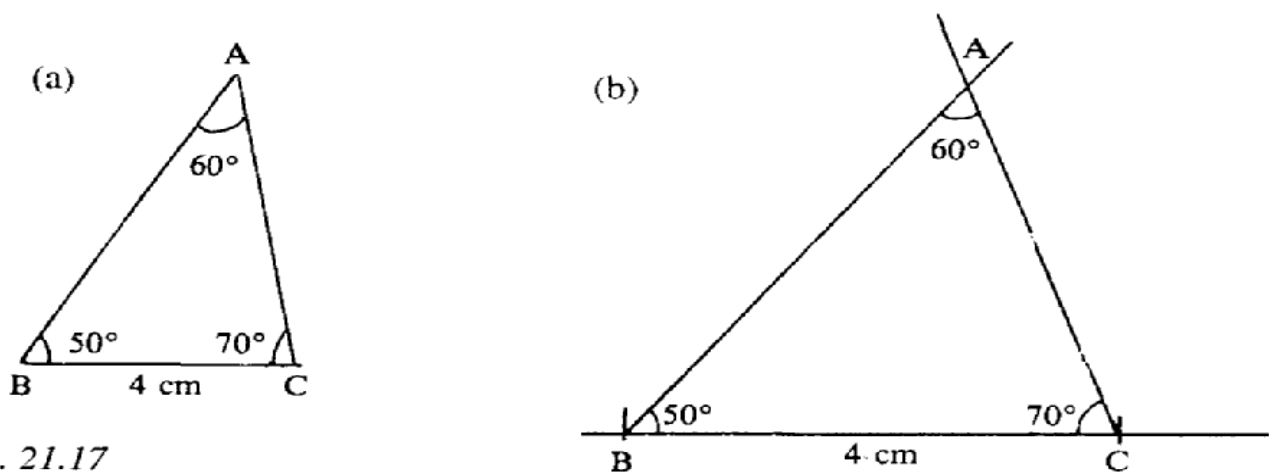


Fig. 21.17

The construction is carried out as follows:

- (i) Draw a line and mark a point B on it.
- (ii) Mark off a point C on the line, 4 cm from B.
- (iii) Using a protractor, measure an angle of 50° and 70° at B and C respectively.
Draw the arms of the two angles to meet at A, as shown in figure 21.17(b).

Construction 3: To construct a triangle given two sides and one angle

Case 1: Given the lengths of two sides and the size of the included angle

Construct a triangle ABC, in which $AB = 4$ cm, $BC = 5$ cm and $\angle ABC = 60^\circ$.

A rough sketch of the triangle ABC is shown in figure 21.18(a).

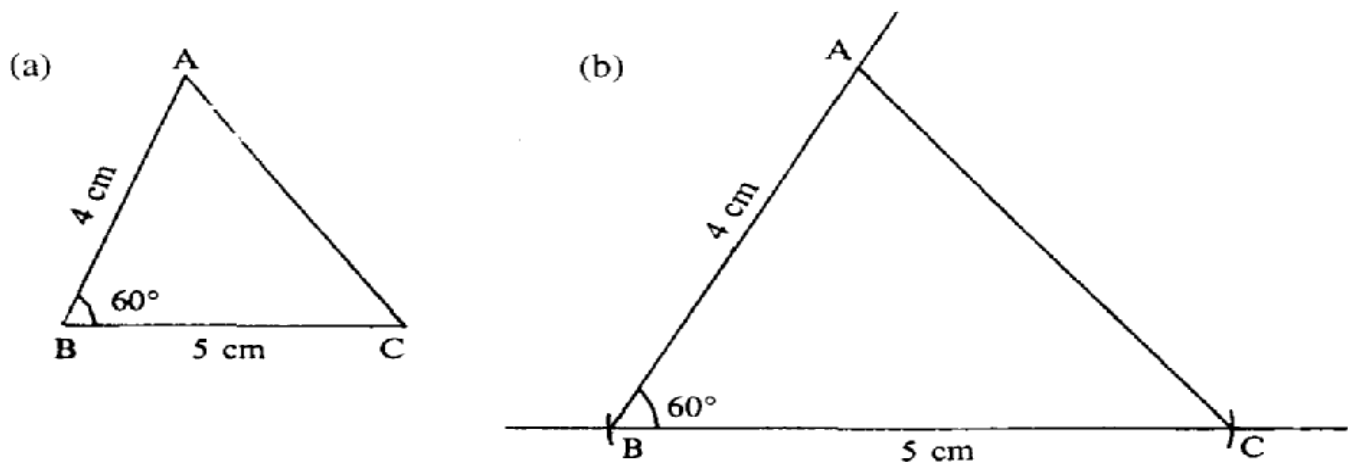


Fig. 21.18

The construction is carried out as follows:

- (i) Draw a line BC, 5 cm long
- (ii) Measure an angle of 60° at B and mark off a point A, 4 cm from B,
- (iii) Join A to C.

Case 2: Given the lengths of two sides and the size of a non-included angle

Construct $\triangle ABC$ in which $AC = 4$ cm, $BC = 5$ cm and $\angle ABC = 30^\circ$.

Figure 21.19 (a) and (b) [rough and accurate drawing, respectively] shows two possible triangles that can be constructed from the information. Note that there are two possible positions of the vertex A.

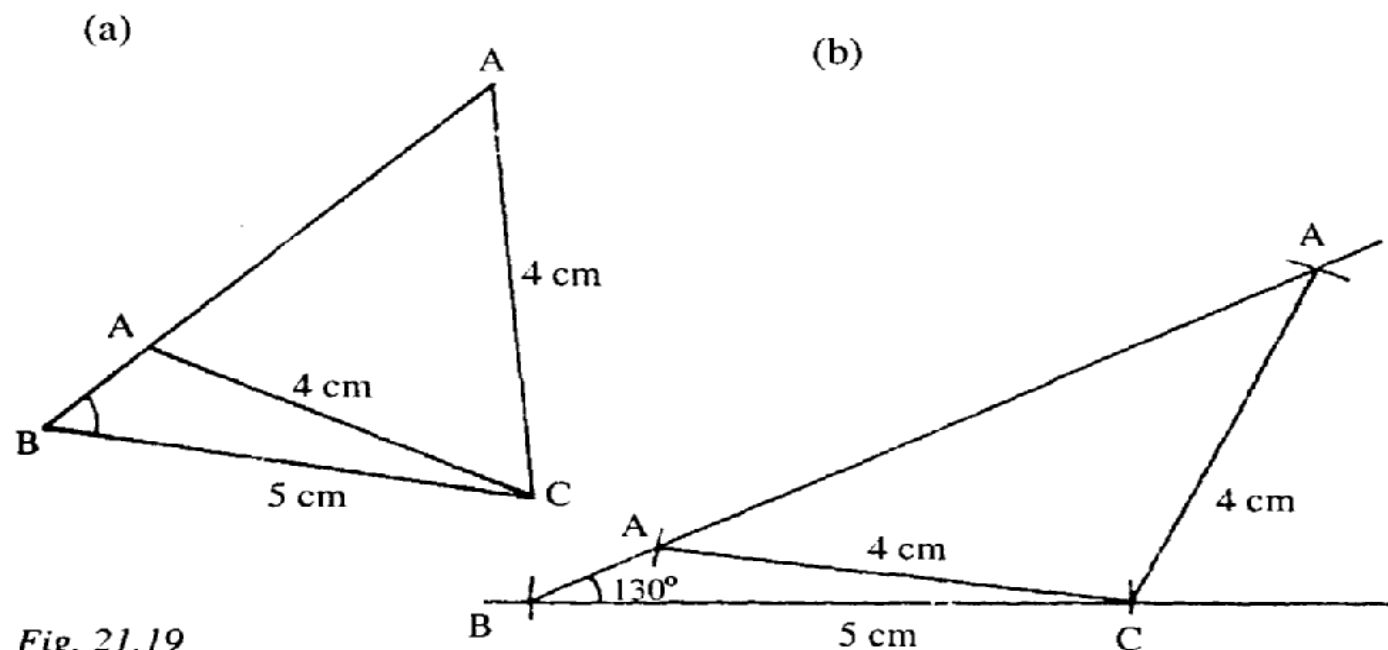


Fig. 21.19

Exercise 21.3

1. Draw a triangle ABC such that $AB = 7.4$ cm, $AC = 8.6$ cm and $BC = 4.5$ cm. Measure the sizes of all the angles.
2. Draw a triangle LMN such that $LM = 4.1$ cm, $MN = 4.9$ cm and $LN = 8.3$ cm. Measure $\angle LMN$.
3. Draw a triangle XYZ , where $XY = 5.5$ cm, $YZ = 5.1$ cm and $ZX = 7.5$ cm. Measure angle XYZ . What type of triangle is XYZ ?
4. Draw a triangle ABC , in which $AB = 6.3$ cm, $BC = 8.3$ cm and $AC = 2.2$ cm. Measure all angles of the triangle ABC .
5. Construct an equilateral triangle DEF of side 5.4 cm. EF is extended to G such that FG is 5.4 cm. Measure $\angle DFG$ and DG . What type of triangle is GDF ?
6. Triangle PQR is such that $PQ = 9.1$ cm, $QR = 6.5$ cm and $RP = 8.5$ cm. S is a point on PQ such that $PS = 6.5$ cm. Measure RS and $\angle SRP$.
7. Triangle STU is such that $ST = 4.5$ cm, $TU = 7.5$ cm and $US = 10.5$ cm. Measure the largest angle of the triangle.
8. Construct triangle ABC in which $AB = 5$ cm, $\angle ABC = 47^\circ$ and $BC = 7.3$ cm. Measure AC and $\angle ACB$.

9. Construct triangle PQR in which $PQ = QR = 4.5$ cm and $\angle PQR = 110^\circ$. Measure PR.
10. PQR is a triangle in which $\angle PQR = 90^\circ$, $PQ = 4$ cm and $QR = 3$ cm. Construct the triangle and measure RP and $\angle PRQ$.
11. Draw triangle LMN, given that $\angle NLM = 34^\circ$, $LM = 4.3$ cm and $MN = 6.5$ cm. Measure LN and $\angle LMN$.
12. Triangle RST is such that $\angle RST = 53^\circ$, $ST = 10$ cm and $TR = 8$ cm. Measure SR and $\angle TRS$.
13. Construct triangle JKL such that $\angle JKL = 30^\circ$, $\angle KLJ = 73^\circ$ and $KL = 5.9$ cm. Measure LJ and JK.
14. Triangle PQR is such that $\angle PQR = 41^\circ$, $QR = 8$ cm and $\angle QRP = 74^\circ$. Measure the remaining sides.
15. Triangles ABC and ABD have the same base $AB = 5$ cm. $\angle BAC = 30^\circ$ and $AC = 7$ cm. $\angle ABD = 50^\circ$ and $BD = 6$ cm. Construct the two triangles and measure:
 - (a) AD and BC.
 - (b) $\angle BAD$ and $\angle ABC$.
16. Construct a triangle ABC in which $AC = BC = 4$ cm and $AB = 6$ cm. A point P is such that P and C lie on the opposite sides of AB, and $PA = PB = 9$ cm. Construct the triangle PAB and join P to C. If PC meets AB and Q, measure $\angle PQB$.
17. Construct triangles ABC and BCD in which $BC = 5$ cm is a common base, $AC = 7.5$ cm, $DC = 4$ cm. $\angle CBD = \angle BCA = 30^\circ$. Find two possible values of $\angle BDC$.

Other Irregular Polygons

Construction 1: Construction of a rectangle

To construct a rectangle ABCD of length 8 cm and width 5 cm.

- (i) Draw a line $AB = 8$ cm.
- (ii) Construct a perpendicular at A and B.
- (iii) Using A as centre and radius of 5 cm, mark an arc to intersect the perpendicular at D.
- (iv) Using B as centre and a radius of 5 cm, mark an arc to intersect the perpendicular at C.
- (v) Join C to D.

Construction 2: Construction of a parallelogram

To construct a parallelogram ABCD with $AB = 8$ cm, $BC = 6$ cm and $\angle DAB = 30^\circ$, using a ruler and a pair of compasses only.

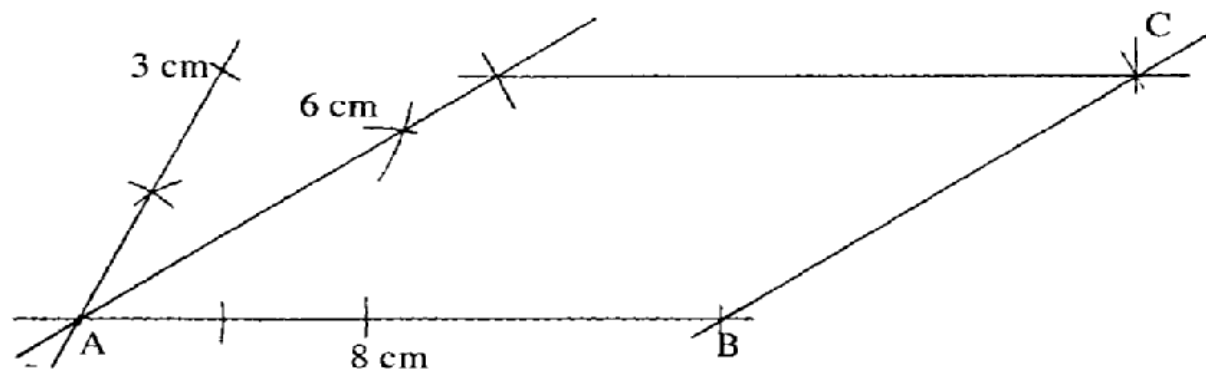


Fig. 21.20

- (i) Draw a line $AB = 8$ cm, as in figure 21.20.
- (ii) Construct an angle 30° at A.
- (iii) Using A as centre and a radius of 6 cm, mark an arc to intersect the line drawn in (ii) at D.
- (iv) Using D as centre and a radius of 8 cm, draw an arc.
- (v) Using B as centre and a radius of 6 cm, draw an arc to intersect the one in (iii).
- (vi) Join B to C and C to D to form the parallelogram.

Construction 3: Construction of a trapezium

The construct a trapezium ABCD with $AB = 8$ cm, $BC = 5$ cm, $CD = 4$ cm $\angle ABC = 60^\circ$ and AB is parallel to DC is done as follows:

- (i) Draw a line $AB = 8$ cm, see figure 21.21.
- (ii) Construct an angle of 60° at B.
- (iii) Using B as centre and a radius of 5 cm, mark an arc to intersect the line in (ii) at C.
- (iv) Through C, draw a line parallel to AB.
- (v) Using C as centre and a radius of 4 cm, mark an arc to intersect the line in (iv) at D.
- (vi) Join D to A to form the trapezium.

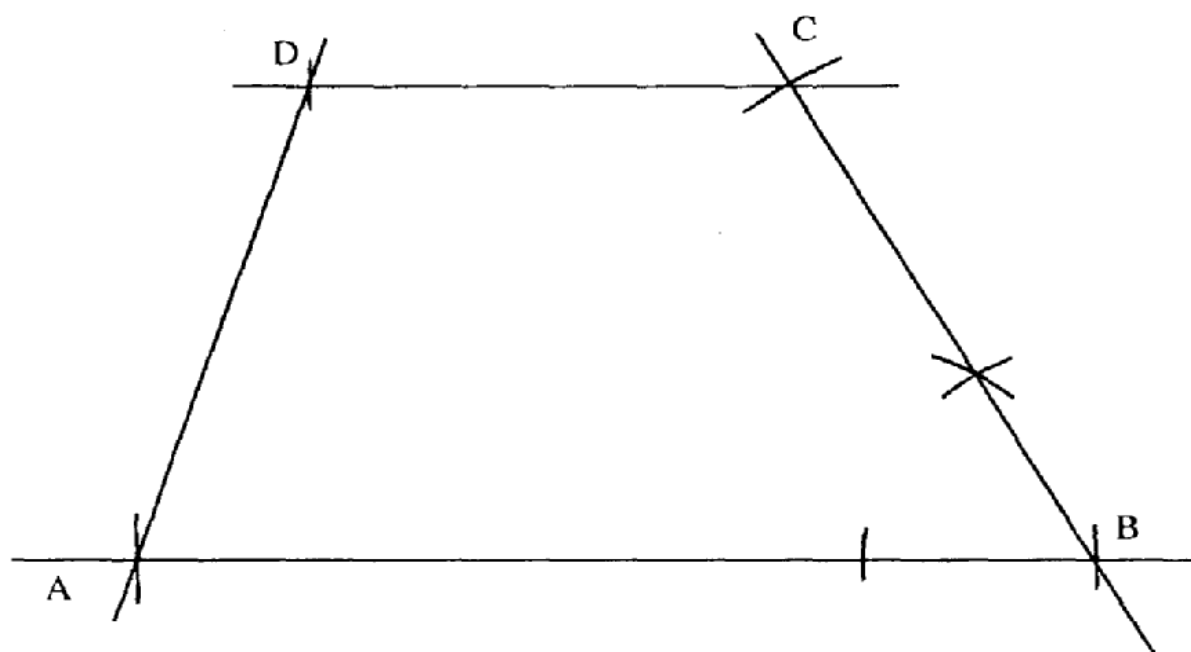


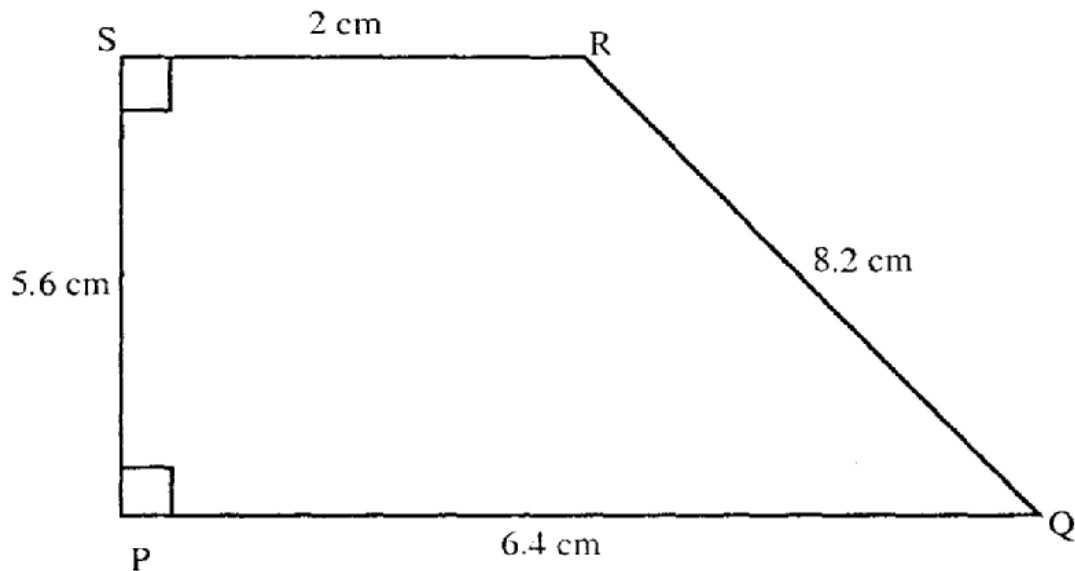
Fig. 21.21

What is the altitude of the trapezium?

Exercise 21.4

- Construct a quadrilateral ABCD in which $AB = BC = CD = 6$ cm, $AD \parallel BC$ and angle $BCD = 47^\circ$.
- Draw a triangle ABD in which $AB = 6$ cm and $\angle BAD = \angle ABD = 72^\circ$. Points C and E lie on AD and BD respectively, such that $DC = DE = 6$ cm. Join A to E and B to C. Measure angles BCD and AED. Measure also CE and AC.
- Draw lines $AE = AB = BC = 5$ cm and $\angle BAE = \angle ABC = 108^\circ$. With C and E as centres and radius 5 cm, draw arcs to intersect at D and F. Join CD, DE, EF and FC. What type of quadrilateral is CDEF? What is the area of CDEF?
- Draw a regular octagon ABCDEFGH of side 5 cm.
 - Measure:
 - AD and BE.
 - angles ADC and BAD.

- (b) Join AD, BC, CF and EH.
- What type of figure is ABGH? Find its area.
 - What is the area of the octagon?
5. ABCDE is an irregular pentagon in which $AB = 8$ cm, $BC = CD = 6$ cm. The exterior angle at B is 130° , $\angle C$ is 40° , $\angle D$ is 30° and $\angle A$ is 37° . Measure EA, ED and angle AED.
6. Draw an irregular pentagon PQRST in which $\angle PQR = 120^\circ$, $\angle TPQ = 100^\circ$, $TP = 6$ cm, $PQ = 5$ cm, $TS = 8$ cm, $RQ = 4$ cm and $SR = 7.2$ cm. Measure $\angle PTS$, $\angle TSR$, TR and PS.
7. Draw a circle of radius 5 cm. Mark off points A, B, C, D, E and F on the circumference of the circle such that $AB = BC = CD = DE = EF = FA = 5$ cm. Measure $\angle BCD$, $\angle ABC$ and $\angle CDE$. What type of polygon is ABCDEF?
8. Draw a triangle ABC, in which angle $BAC = 90^\circ$, $AC = 4$ cm and $BC = 8$ cm. Draw a circle to pass through points A, B and C. Using AC as radius, with centre at:
- A, mark off a point K on the minor arc AB.
 - B and C, mark off points M and N respectively, such that M and N are on the major arc AB. Join AK, KB, BM, MN, NC and CA. Measure the angles BNC and BKC. What type of a triangle is ABN?
9. Draw a line $BC = 8$ cm. Mark the point O as the midpoint of BC. Construct two rhombi, BKOM and CAON, each of side 4 cm, such that M and N are on the same side of line BOC. What is the area of the polygon BMNCAK?
10. $PQ = 7.0$ cm, angle $PQR = 53^\circ$ and $RQ = 5$ cm. Using a ruler and either a set square or a pair of compasses only, complete the parallelogram PQRS and measure its diagonals.
11. Using a ruler and a pair of compasses only, construct a rhombus WXYZ, given that $WX = 6$ cm and $WY = 7.9$ cm. What is the size of angle XYZ?
12. Construct the trapezium shown in figure 21.22.

*Fig. 21.22*

13. Using a ruler and a pair of compasses only, construct a square WXYZ whose side is 5.6 cm. Measure XZ.
14. Using a ruler and a pair of compasses only, construct a rectangle PQRS in which the diagonals are 10 cm long and intersect at 45° . Measure PQ and RS.
15. A parallelogram LMNP is such that $LM = 4.8$ cm, $LP = 7.5$ cm and $LN = 7.0$ cm. Measure angle MLP.
16. Construct a trapezium ABCD with AB parallel to DC. $AB = 10$ cm, $BC = 5$ cm, $CD = 4$ cm and angle $ABC = 45^\circ$. Measure AD and its altitude.

Chapter Twenty Two

SCALE DRAWING

22.1: The Scale

Figure 22.1 shows the relative positions of Mombasa, Nairobi and Nakuru. The distance between Mombasa and Nairobi on a straight line is 450 km and that between Nairobi and Nakuru is 142 km.

Measure in centimetres the distance between:

- (i) Mombasa and Nairobi.
- (ii) Nairobi and Nakuru.

How many kilometres does 1 cm represent between:

- (i) Mombasa and Nairobi?
- (ii) Nairobi and Nakuru?

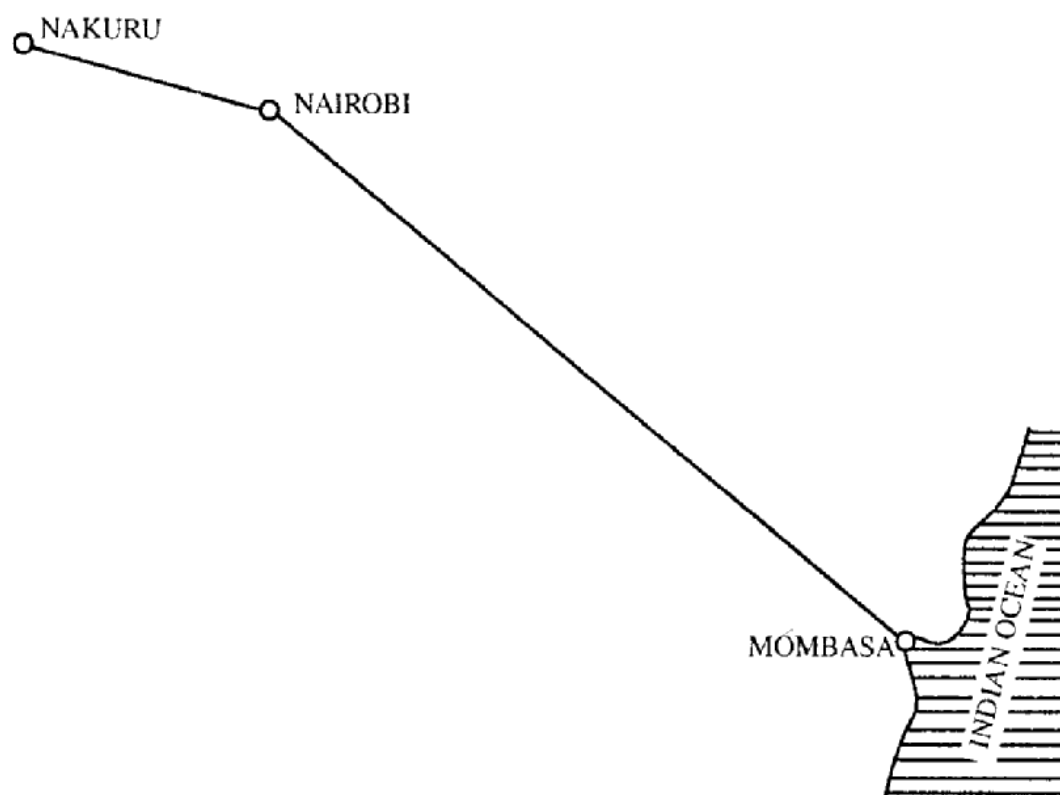


Fig. 22.1

You should notice that 1 cm on the map represents 50 km on the ground. Since 50 km is equal to 5 000 000 cm, this statement can be written in ratio form as 1: 5 000 000.

As a representative fraction (R.F.), 1 : 5 000 000 is written as $\frac{1}{5\,000\,000}$.

The ratio of the distance on a map to the actual distance on the ground is called the **scale** of the map.

Example 1

The scale of a map is given in a statement as '1 cm represents 4 km.' Convert this to a representative fraction (R.F.).

Solution

1 cm represents 4 x 100 000 cm

Therefore, the ratio is 1 : 400 000 and the R.F. is $\frac{1}{400\,000}$.

Example 2

The scale of a map is given as 1 : 250 000. Write this as a statement.

Solution

1 : 250 000 means 1 cm on the map represents 250 000 cm on the ground.

Therefore, 1 cm represents $\frac{250\,000}{100\,000}$ km, i.e., 1 cm represents 2.5 km.

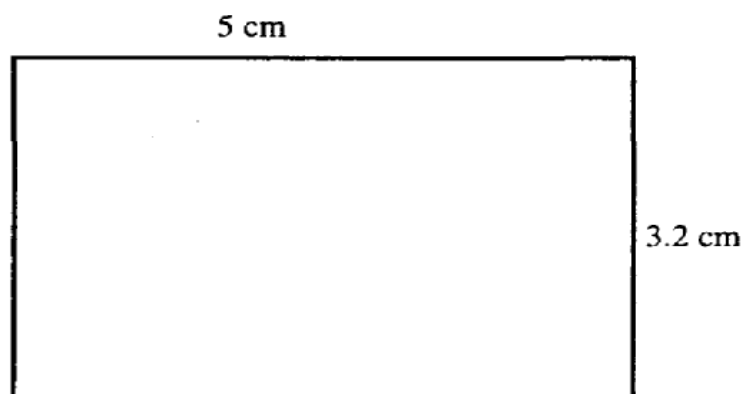
Exercise 22.1

1. On a map, 1 cm represents 4 kilometres.
 - (a) Re-write this scale as R.F.
 - (b) What distance on the ground is represented by 3.7 cm on the map?
 - (c) Two towns A and B are 42.8 km apart on the ground. What is this distance on the map?
2. A map is drawn to a scale of 1:50 000.
 - (a) Write this scale as a statement connecting map distance to ground distance.
 - (b) What is the actual distance if the distance on the map is 12.7 cm?
 - (c) A railway line measures 8.3 km. What is its length on the map?

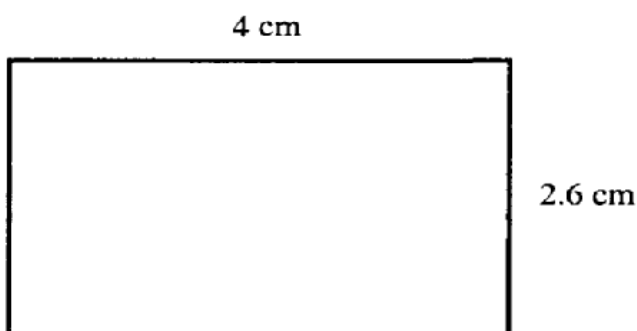
22.2: Scale Diagrams

The length of a classroom is 10 metres and its width 6.4 metres. By scale drawing, represent this on a figure.

You must have noticed that you have to look for a scale. Consider a scale of 1 cm to represent 2 m. Hence, the classroom will be $\frac{10}{2} = 5$ cm by $\frac{6.4}{2} = 3.2$ cm.

*Fig. 22.2*

Suppose instead, 1 cm represented 2.5 metres. The dimensions would be 4 cm by $\frac{6.4}{2.5}$, which gives = 2.56 cm (about 2.6 cm).

*Fig. 22.3*

What do you observe?

You must have noticed that the bigger the scale, the smaller the figure.

Note:

One should be careful in choosing the right scale, so that the drawing fits on the paper without much detail being lost.

Exercise 22.2

1. A plot of land in form of a rectangle has dimensions 120 m by 180 m. Draw this on a paper.
2. A rectangular field measures 40 m by 100 m. The length of the field on the map is 5 cm.
 - (a) Write the scale of the map as:
 - (i) a statement.
 - (ii) a representative fraction.
 - (b) What is the width of the field on the map?

3. Two villages M and N are connected by a straight road 750 m long on a level ground. A third village L is 450 m from M and 650 m from N. Using a suitable scale, draw the diagram and find the shortest distance of L from the road.

22.3: Bearings and Distances

Captains and pilots always need to know the directions which they are sailing or flying. They use a magnetic compass to find direction. Figure 22.4 shows a magnetic compass.

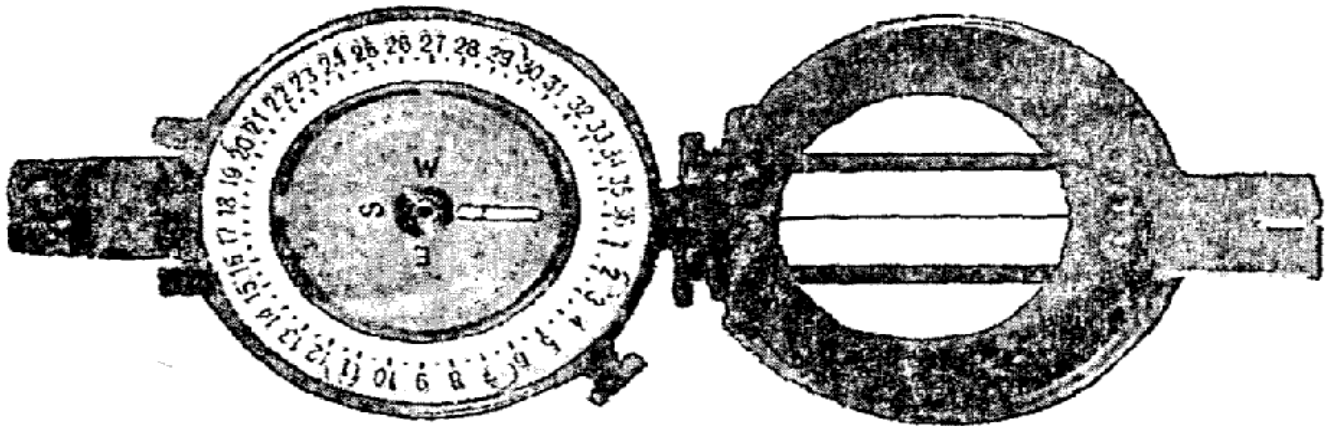


Fig. 22.4

Points of the compass

Figure 22.5 shows the eight points of the compass.

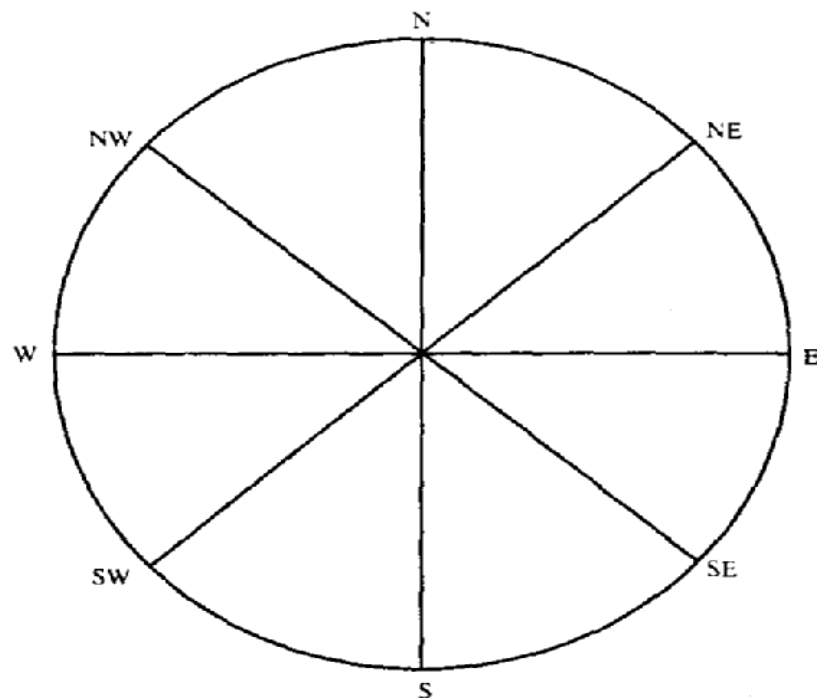


Fig. 22.5

The four main points of the compass are North (N), South (S), East (E) and West (W). The other four points shown are secondary and include the North East (NE), South East (SE), South West (SW) and North West (NW). Each angle formed at the centre of the compass of the eight directions is 45° . The angle between N and E is 90° .

Compass Bearing

Let P and Q be two points. Join the points P and Q as in figure 22.6. Measure angle NPQ clockwise.

Notice that the compass bearing of Q from P is $180^\circ - 138^\circ = 42^\circ$, written as S 42° E.

Similarly, the angle NQP measured **clockwise** is 318° . We say the compass bearing of P from Q is N 42° W. Why?

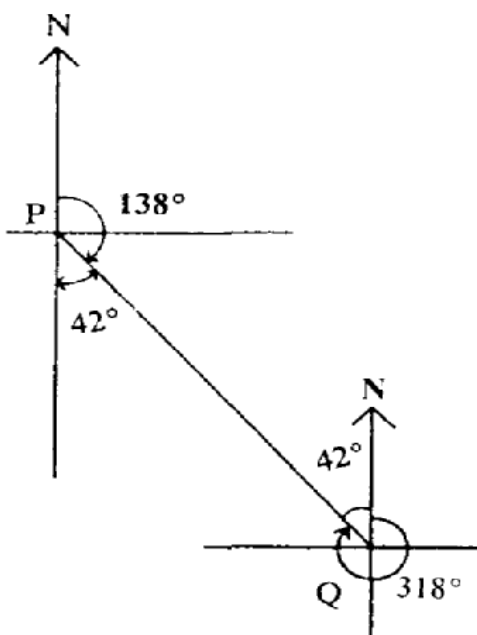


Fig. 22.6

When the direction of a place from another is given in degrees and in terms of four main points of a compass, e.g., N 45° E, then the direction is said to be given in compass bearing. Notice that the compass bearing is measured either clockwise or anticlockwise from North or South and the angle is acute.

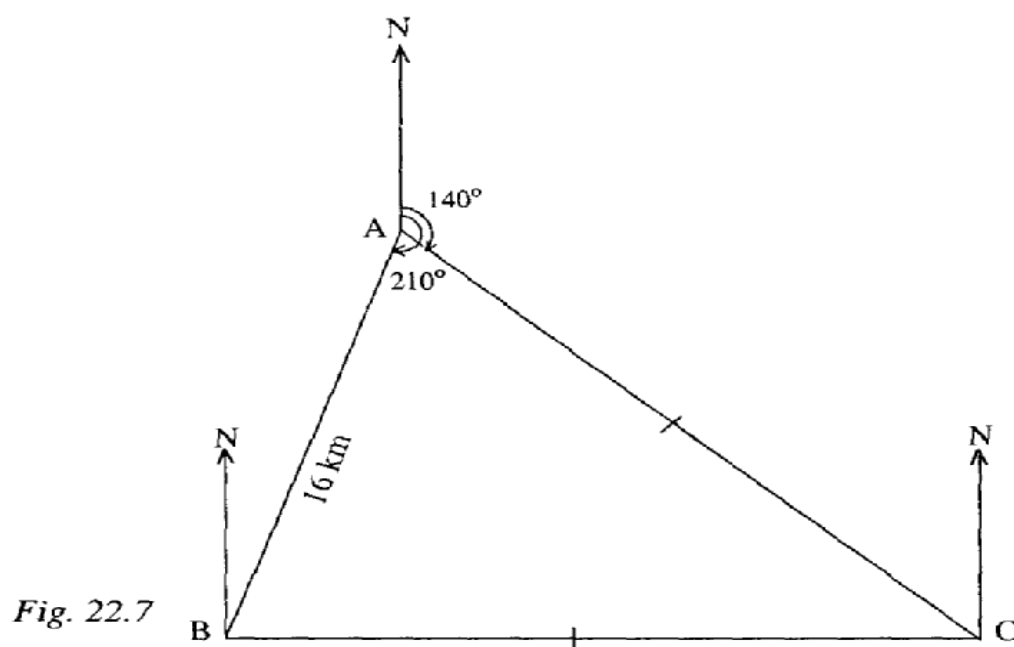
True Bearing

North East direction, written as N 45° E can be given in three figures as 045° measured clockwise from True North. This three-figure bearing is called the **true bearing**.

The true bearings due north is given as 000° . Due South East as 135° and due North West as 315° , etc. In figure 22.7, the true bearing of Q from P is 138° .

Exercise 22.3*Use scale drawing in this exercise*

1. Three boys Isaac, Alex and Ken are standing in different parts of a football field. Isaac is 100 metres north of Alex and Ken is 120 metres east of Alex. Find the compass bearing of Ken from Isaac.
2. Kilo school is 12 kilometres from Sokomoko on a bearing of 320° . Tiba dispensary is 10 kilometres from Kilo on a bearing of 120° . Find the compass bearing of Sokomoko from Tiba.
3. Survey posts R, Q and P are situated such that they form a triangle. If Q is on a bearing of 210° and 12 kilometres away and R is on a bearing of 150° and 8 kilometres away from P, find the compass bearing of Q from R.
4. In figure 22.7, the bearing of C from A is 140° . B is 16 kilometres away on a bearing of 210° from A. If the distances AC and BC are equal, find:
 - (a) the distances BC and AC.
 - (b) the bearing of B from C.



5. Kisumu and Nanyuki are situated in such a way that Nanyuki is on a bearing of 075° from Nakuru and Kisumu on a bearing of 280° from Nakuru. If Kisumu is 190 km and Nanyuki is 160 km from Nakuru, find:
 - (a) the bearing of Kisumu from Nanyuki.
 - (b) the distance of Kisumu from Nanyuki
6. Figure 22.8 shows a fenced rectangular farm ABCD with $AB = 600$ m and $BC = 900$ m. P is a water pump inside the farm. If P is on a bearing of 225° from D and 300° from C, find:

- (a) the distance PD.
- (b) the distance BP.
- (c) the bearing of P from B.

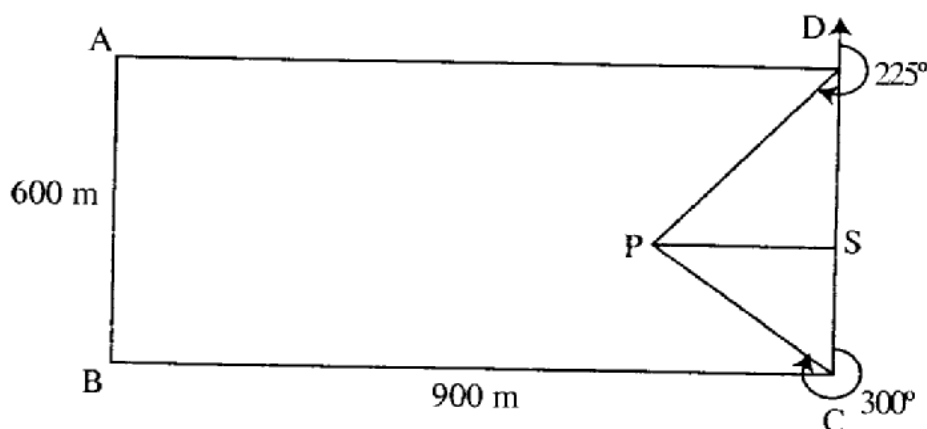


Fig 22.8

7. Town A is on a bearing 050° from town C. Town B is on a bearing 020° from C. If B is 500 km from C and A is 500 km from B, find by scale drawing:
 - (a) the distance of A from C.
 - (b) the bearing of B from A.
8. In figure 22.9, determine:
 - (a) the bearing of C from A.
 - (b) the distance between B and C.
 - (c) the bearing of C from B.

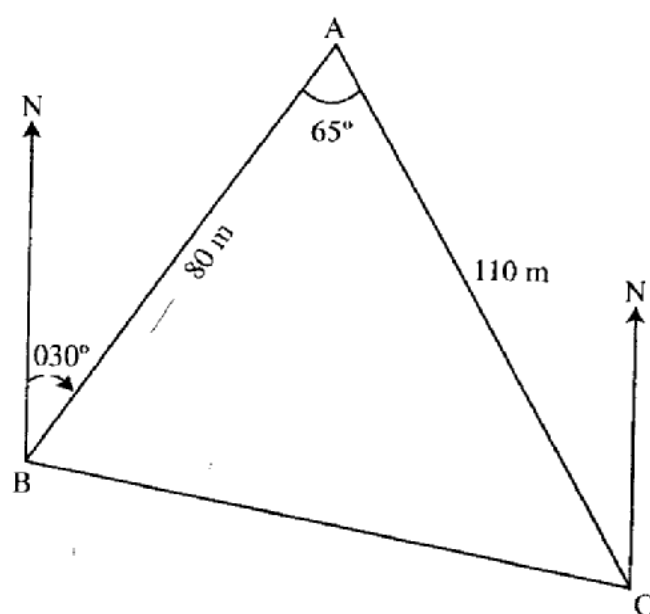


Fig 22.9

9. A coastguard at a port observes two steamships approaching the harbour. The first ship P appears on a bearing 100° and the second ship Q on a bearing 020° . If the guard estimates the distances of the ships to be 120 km and 80 km respectively, find:
- the distance between ships P and Q.
 - the bearing of Q from P.
10. A prison guard on a watchtower sees a bridge 120 m away on a bearing of 230° and a bus stop 80 m away on a bearing of 090° . Use scale drawing to find:
- the bearing of the bridge from the bus stop.
 - the distance between the bus stop and the bridge
11. Figure 22.10 shows two ships X and Y steaming away from a harbour H. X is 20 km away on a bearing 035° , and Y is 50 km away on a bearing 110° .

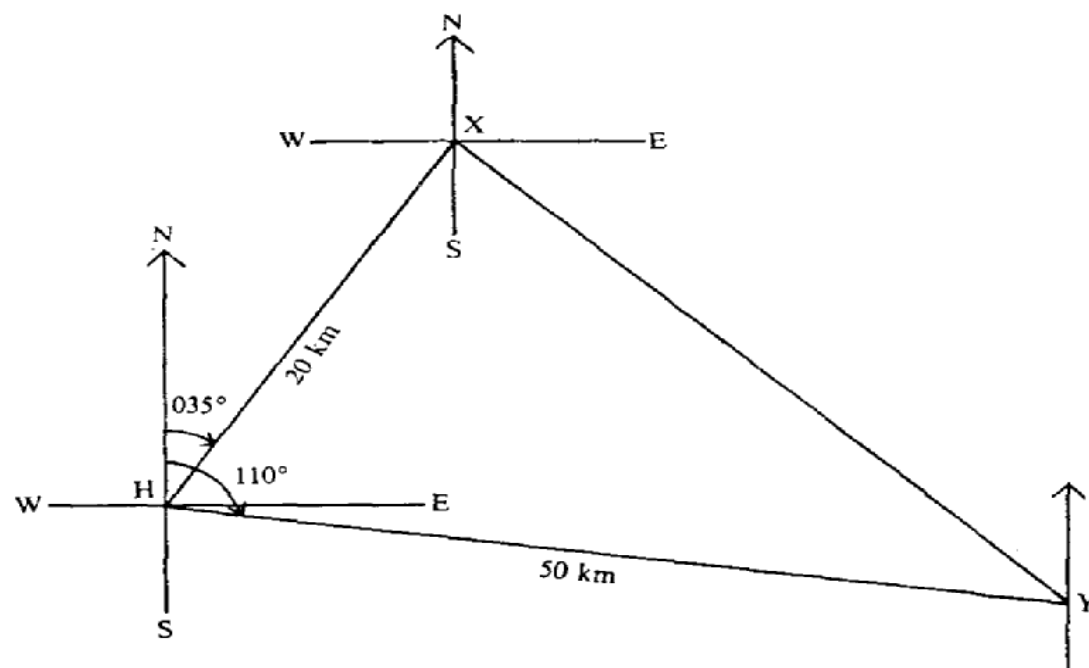


Fig. 22.10

Use scale drawing to find:

- the bearing of X from Y.
 - the distance between X and Y.
12. From a meteorological weather station P on a plateau, a hill Q is 5 km on a bearing 078° and a railway station, R, is 1.5 km away on a bearing 200° . Use scale drawing to find:
- the bearing of Q from the railway station.

- (b) the distance between Q and R.
 (c) the shortest distance between Q and the line RP.
13. The minibuses, m_1 , m_2 and m_3 are approaching a stage P which is on a bearing of 340° from an adjacent stage W. Minibus m_2 is east of stage P and 6 km from W, on a bearing of 040° , while m_1 is on a bearing of 045° from P. Minibuses m_3 and m_1 are due North of W. If m_3 is on a bearing of 250° from m_2 , find by scale drawing:
- the bearing of m_2 from m_1 .
 - the bearing of m_3 from P.
 - the distance between:
 - m_1 and m_2
 - W and m_3
 - m_1 and m_3

22.4: Angles of Elevation and Depression

A boy 1.5 m tall standing at a distance of 10 m from a storey building notices his friend looking at him through the window of the building, as shown in figure 22.11.

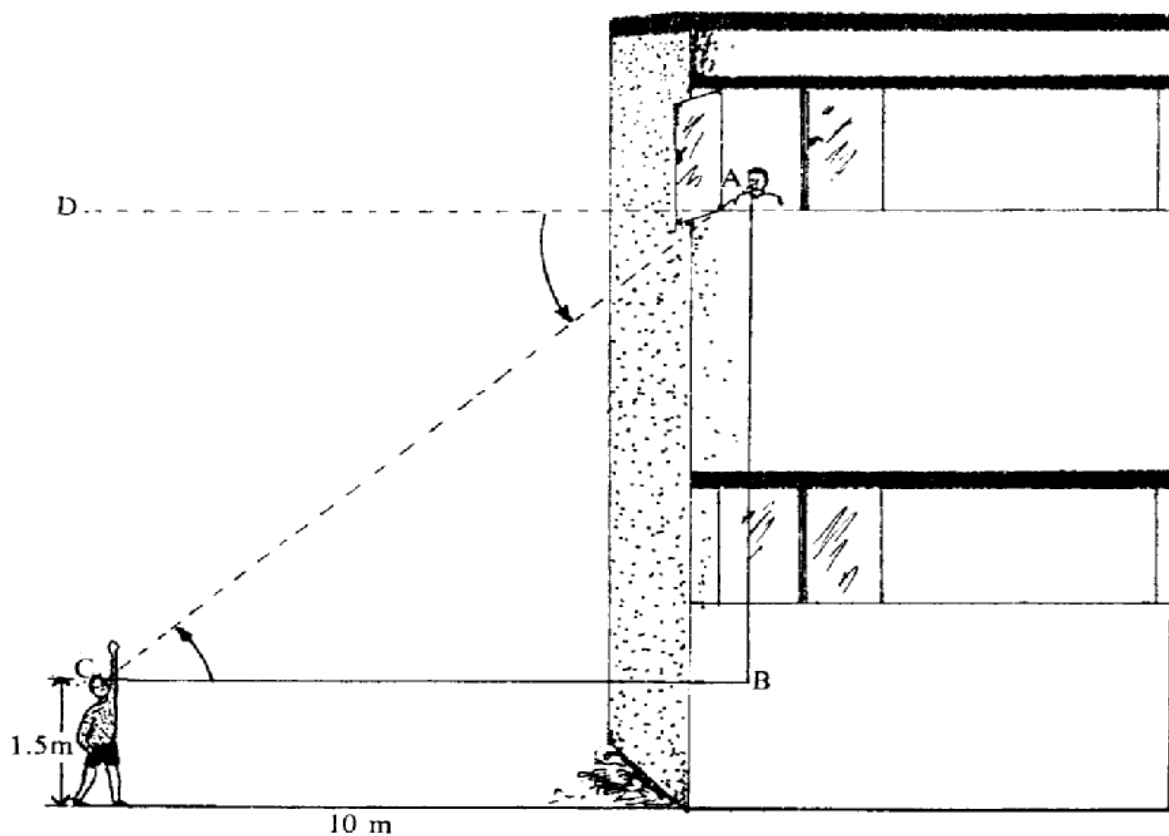


Fig. 22.11

Let CA be the line of sight of the boy on the ground and AC the line of sight of his friend in the building as they look at each other. The angle BCA through which the boy on the ground has to raise his line of sight from the horizontal is

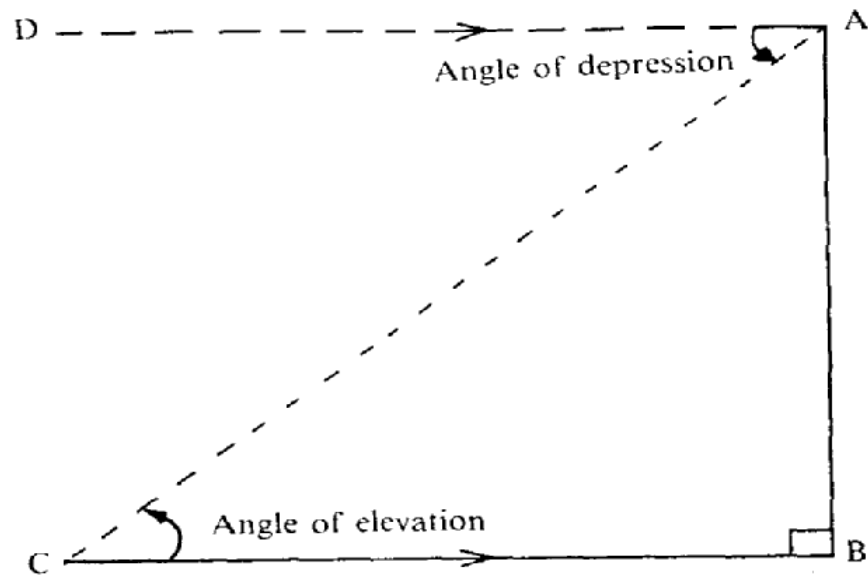


Fig. 22.12

called the **angle of elevation**. The angle DAC through which his friend in the storey building has to lower his line of sight from the horizontal is called the **angle of depression**.

In the figure, DA is parallel to CB and CA is a transversal. Therefore, $\angle DAC = \angle ACB$ (alternate angles). Hence, angles of elevation and depression are equal, see figure 22.12.

Angles of depression and elevation can be measured by use of an instrument called **clinometer**. A simple clinometer can be made from a cardboard in the shape of a protractor.

Project: Making a clinometer

- (i) Draw a semicircle with centre O and suitable radius on a hardboard.
- (ii) On the semicircle, mark the 0° point in the middle. From this point, make marks on either side at intervals of 10° , as in figure 22.13.
- (iii) Cut out the semicircle from the cardboard.
- (iv) Fix the semicircle to two pieces of wood whose edges should be in line with the straight edge of the semicircle.
- (v) From the point O, hang a small weight (about 25 g) by thin wire or thread to form a plumbline.
- (vi) Fasten a round straight stick with a cellotape along the straight edge of the semicircle as in the figure.

The instrument you have made is a clinometer.

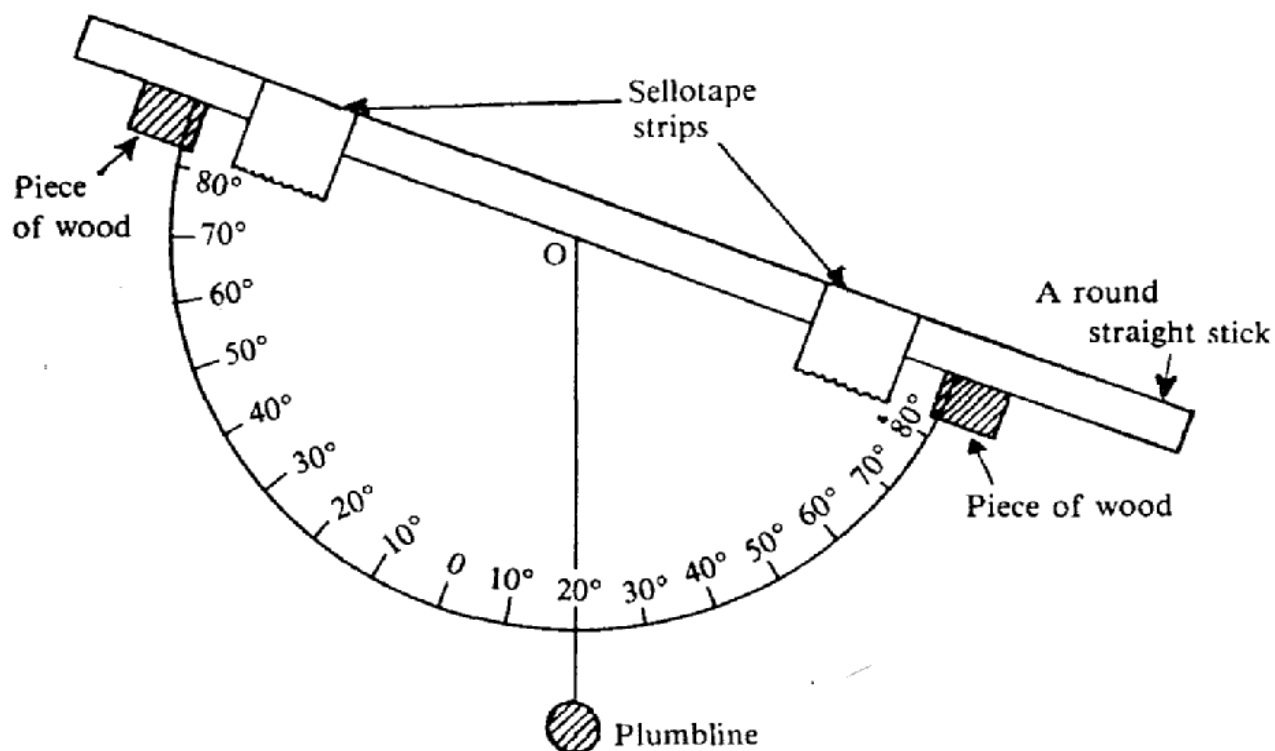


Fig. 22.13

To use a clinometer effectively, two people are required. One observes the top of the object whose angle of elevation is required and the other reads the angle between point O and the plumline.

The idea of complimentary angles is applied in the use of a clinometer, see figure 22.14.

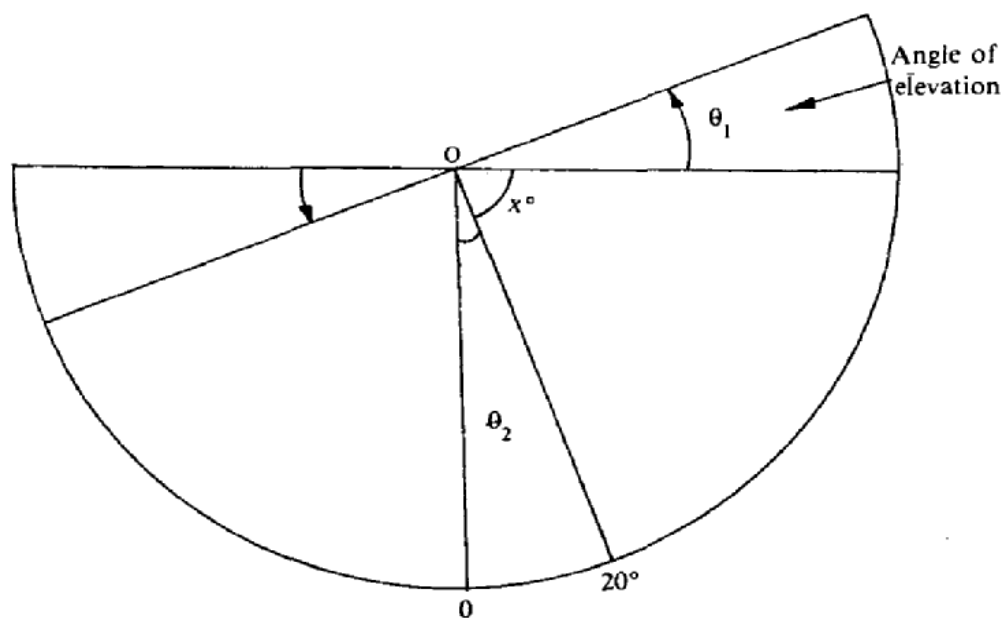


Fig. 22.14

$$x + \theta_1 = 90^\circ$$

$$x + \theta_2 = 90^\circ$$

$$\therefore \theta_1 = \theta_2$$

Therefore, θ_2 is the angle of elevation.

Example 3

A boy 1.5 m tall and 8 m from a tree finds that the angle of elevation to the top of the tree is 38° . Find the height of the tree by scale drawing.

Solution

Figure 22.15 shows the sketch and scale drawing.

Using a scale of 1 cm to 2 m.

The measurement of AB' on the scale is 3.9 cm.

1 cm represents 2 m

$\therefore 3.9$ cm represents (3.9×2) m = 7.8 m.

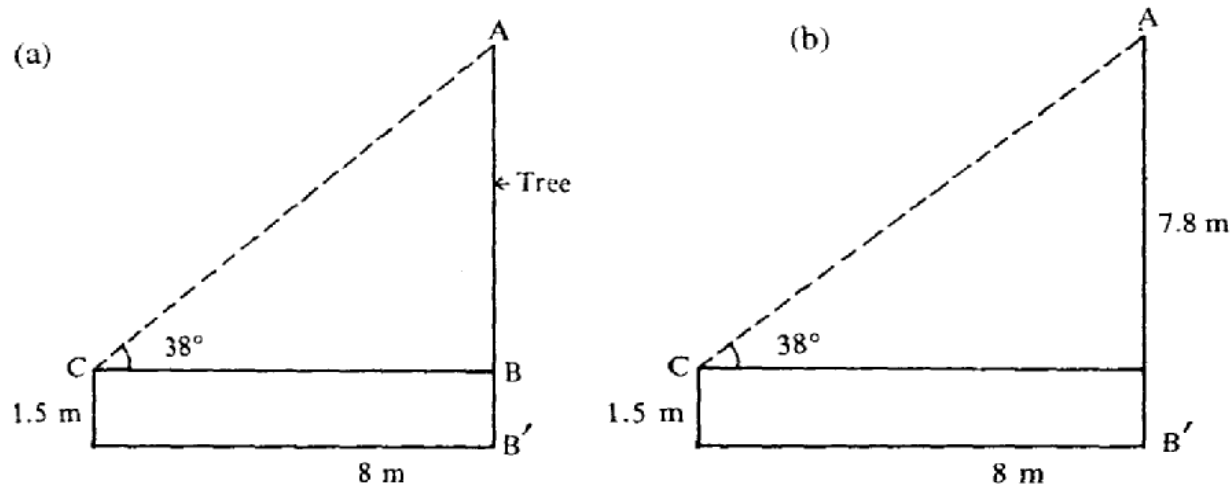


Fig. 22.15

Sketch

Scale drawing

Project

- Use a clinometer, tape measure and scale drawing to find:
 - the height of the tallest tree on the school compound.
 - the height of the staffroom.
 - the height of the flagpole.
- Using the clinometer and your height alone, find the width of the road nearest to your school.

Exercise 22.4

- The angle of elevation of the top of a flagpole from a point A, 14 metres away, is 36° . Use scale drawing to find the height of the flagpole.

2. The angle of depression from the top of a cliff to a stationary boat is 48° . Find the horizontal distance by accurate drawing if the height of the cliff is 80 metres. Measure the angle of elevation of the top of the cliff from the boat. What do you notice?
3. Find height RQ in figure 22.16 by scale drawing if the angle of elevation of S from P is 36° :

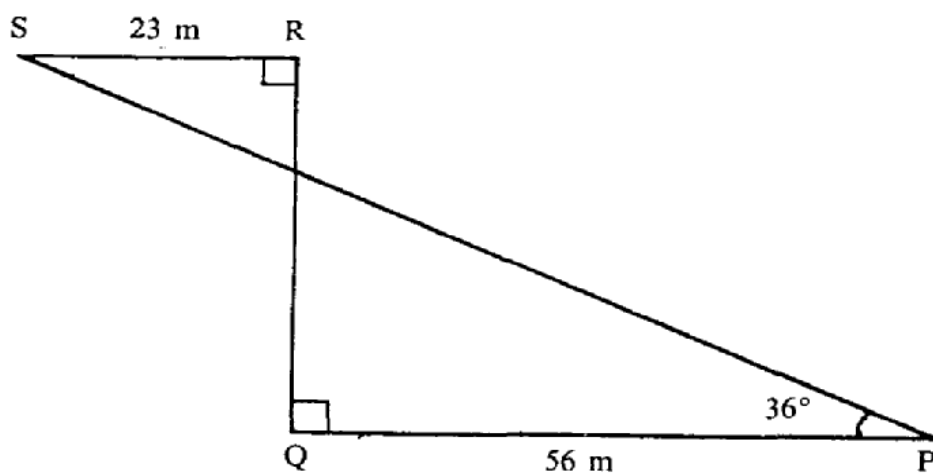


Fig. 22.16

4. A building tower casts a shadow 33 metres away. The angle of elevation of the tower from the tip of the shadow is 21° . Find the height of the tower by scale drawing.
5. In figure 22.17, Z is 50 m away from Y. Use scale drawing to find the distance from W to Y, given that the angle of elevation of X from Z and W are 24° and 35° respectively.

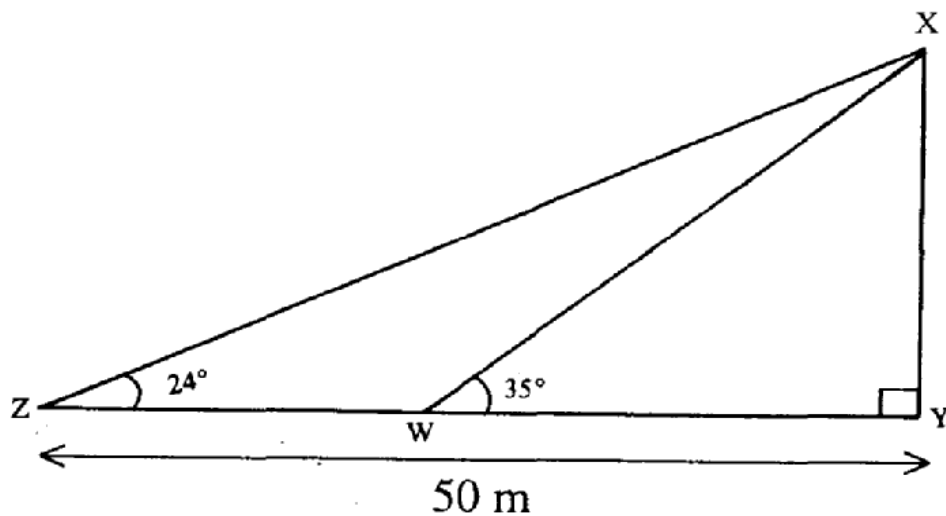


Fig. 22.17

- In figure 22.18, the angle of elevation of A from B is 42° and the angle of depression of C from B is 26° . Find the height AC by accurate drawing, given that CD is 13 metres.

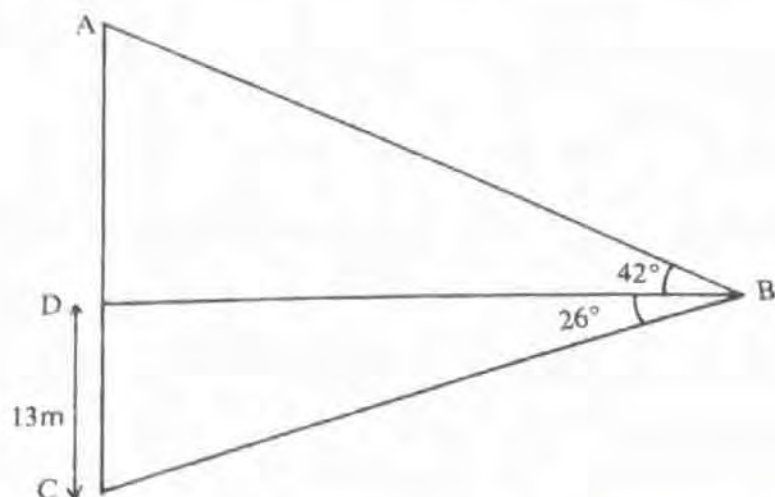


Fig. 22.18

- The angle of elevation of a church tower from a point A, 50 metres away from the foot of the church, is 24° . Find the distance between A and B if the angle of elevation of the tower from B is 20° .
8. From a viewing tower 15 metres above the ground, the angle of depression of an object on the ground is 30° and the angle of elevation of an aircraft vertically above the object is 42° . By choosing a suitable scale, find the height of the aircraft above the object.
9. Figure 22.19 represents a ladder 5 m long leaning against a wall. The angle of depression is 63° . Find, by scale drawing, the height of the wall and the horizontal distance between the foot of the wall and the ladder.

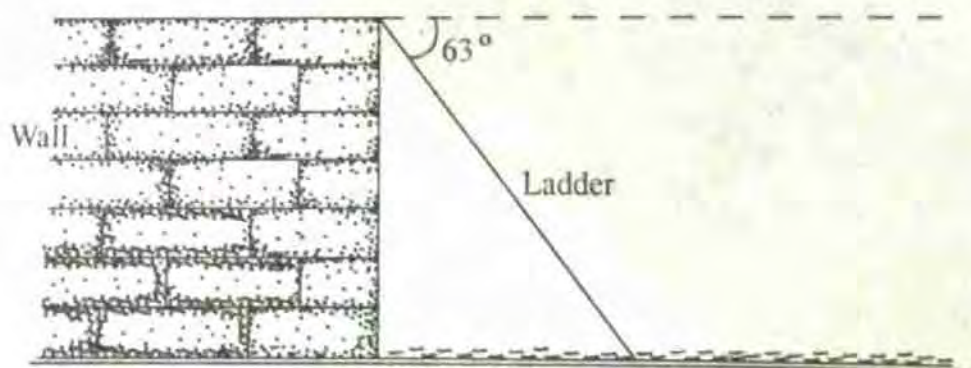


Fig. 22.19

10. A soldier standing on top of a cliff 100 m high notices two enemy boats in line, whose angles of depression are 10° and 23° . Find, by scale drawing, the distance between the boats.
11. The angles of elevation of an aircraft from two villages A and B 1 km apart are 67° and 53° respectively. Find the height of the aircraft in metres by scale drawing.
12. The angle of elevation of a stationary hot air balloon 50 m above the ground from a man on the ground is 17° . The balloon moves vertically upwards so that the angle of elevation from the man is 30° . Find, by scale drawing, the distance the balloon moves. How far above the ground is the balloon?

22.5: Simple Survey Techniques

Surveying an area of land involves taking field measurements of the area so that a map of the area can be drawn to scale. Pieces of land are usually surveyed in order to fix boundaries (land adjudication) of land for different owners, for town planning, road construction, water supplies, mineral development, e.t.c. There are two simple methods used in surveying.

(a) Triangulation

This is a method in which the area to be surveyed is divided into convenient geometrical figures, or is covered by a suitable geometrical figure.

The following examples illustrate the triangulation method.

- (i) The survey of a small forest was made by choosing a suitable triangle ABC round the forest such that $AB = 90$ m, $BC = 70$ m and $CA = 80$ m, as the sketch in figure 22.20 shows.

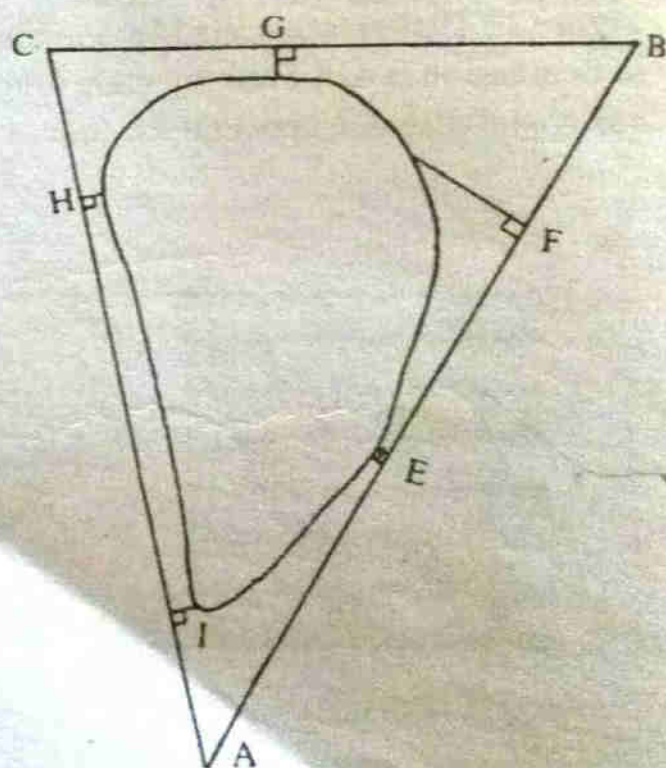


Fig 22.20

The lines AB, BC and AC are called **base lines** of the survey. The lines perpendicular to the base lines and joining the points E, F, G, H and I at the edge of the forest are called **offsets**. Using the baseline AB, we measure the lengths of AE and AF and these are 35 m and 65 m respectively. Similarly, using the baselines BC and CA, $BG = 39$ m, $CH = 20$ m and $CI = 65$ m. The lengths of the offsets at E, F, G, H and I are 5, 12, 5, 5 and 5 metres respectively. The measurements are recorded in a field book in a tabular form as follows:

	B		C		A
F12	65			15	65
E5	35	G5	39	H5	20
	A		B		C

Using a suitable scale and the measurements in the field book, the map of the forest can be drawn by joining the points E, F, G, H and I.

- (ii) A survey of a small island was made by using a triangle PQR in which $PQ = 75$ m, $QR = 70$ m and $RP = 60$ m, as shown in the sketch of figure 22.21:

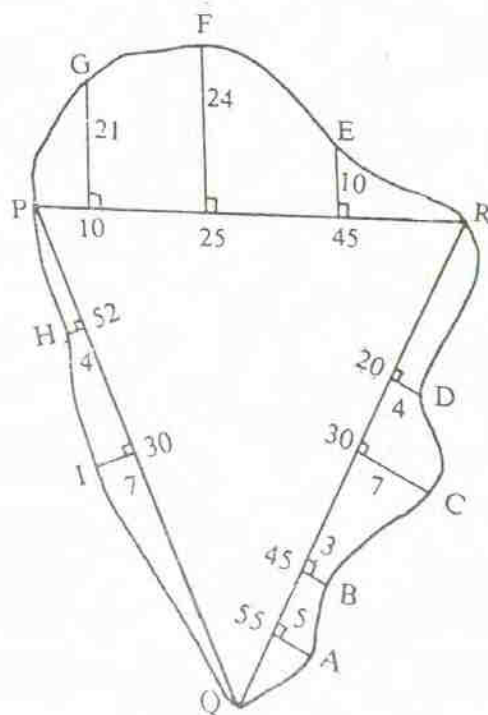


Fig 22.21

From the sketch:

- Name all the base lines.
- How many offsets can you see in the diagram?
- Record the measurements shown in the figure in a tabular form in a field book.
- Using a suitable scale, draw the map of the island.

- (iii) A river can also be surveyed and its map made using base line AB as shown in the sketch of figure 22.22:

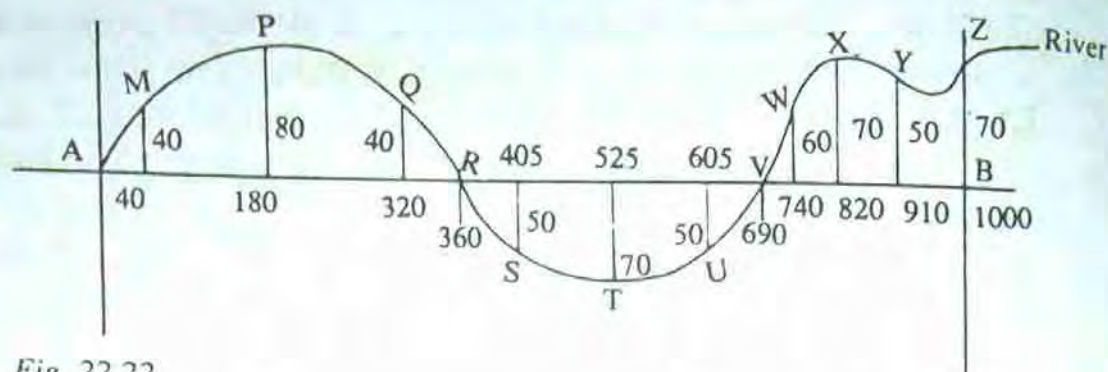


Fig. 22.22

- How many offsets can you see in figure 22.22?
- What can you say about the points A, R and V?
- Record the measurements from the sketch in a field book and use a suitable scale to draw the map of the river.

The type of survey that we have dealt with in this example is called **transverse survey** and is confined to surveying of small areas.

(b) *Survey of an area by use of Compass Bearing and Distances*

In this survey method, only bearings and distances from a chosen point are considered. Figure 22.23 shows a sketch of a piece of land. A map of the piece of land is required.

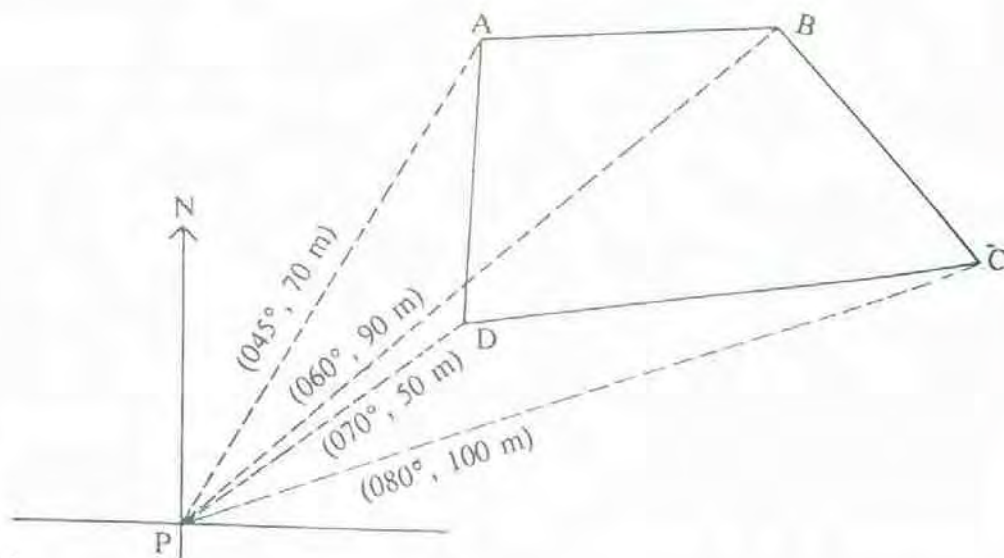


Fig. 22.23

Choose an external point P from where the bearings of A, B, C and D are obtained. Measure the distances PA, PB, PC and PD. Write the bearings and the distances of each point from P as ordered pairs, i.e., A (045°, 70 m),

B (060° , 90 m), C (080° , 100 m) and D (070° , 50 m). Draw figure 22.23 to scale to get the map of the area marked ABCD. Use the map to get the actual distances AB, BC, CD and AD. What is the bearing of A, B and D from C?

Project

In this section, some practical work outside the classroom will be carried out by groups of 10 pupils. Each group needs to carry out a separate survey and will also require the following items:

- (i) A metre rule and enough string.
- (ii) Short strings for offsets.
- (iii) Pegs for marking positions.
- (iv) Compasses.

1. Carry out the survey of a farm (coffee, tea, maize, e.t.c) near your school by using triangulation method:
 - (i) Choose a suitable polygon to cover the area to be surveyed. This will help in setting up a framework of base lines to be used.
 - (ii) Make suitable offsets
 - (iii) Record all measurements in a field book.
 - (iv) In the classroom, accurately draw the map of the area you have surveyed using a suitable scale.
2. By using compass bearings and distances from a point of your choice, carry out a survey of a road within the school compound. Record the measurements as ordered pairs and hence make a map of the surveyed area by the use of a suitable scale.

Exercise 22.5

1. Figure 22.24 shows the sketch of a school orchard. The bearings and distances of the points on its boundaries marked A, B, C, D, E, F, G and H from a point P are tabulated:

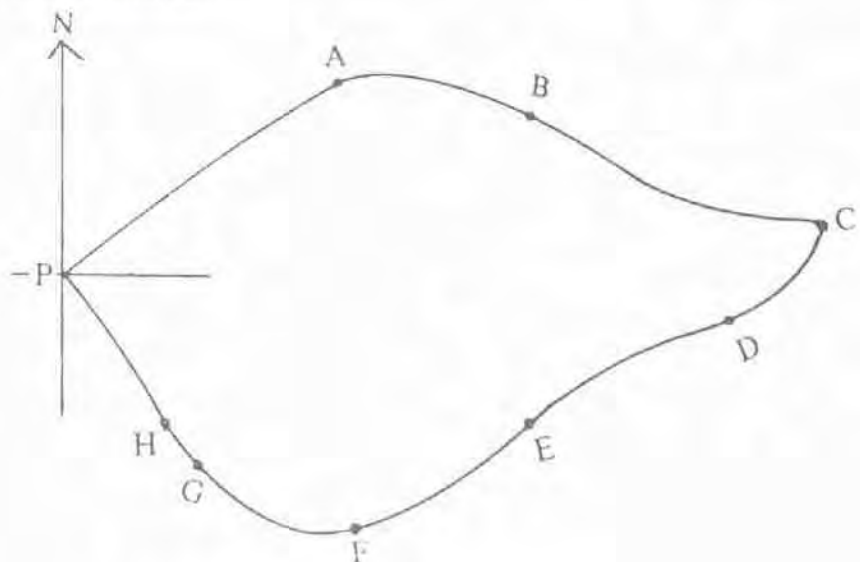


Fig. 22.24

<i>Point</i>	<i>Bearing</i>	<i>Distance in metres</i>
A	035°	55 m
B	050°	30 m
C	080°	120 m
D	090°	105 m
E	110°	70 m
F	135°	60 m
G	140°	30 m
H	140°	25 m

Use a suitable scale to draw accurate plan of the orchard. Hence, find:

- the bearing of A from C.
 - the bearing and the distance of C from F.
2. A vegetable garden, QRSTV, is adjacent to a straight concrete pavement of width 4.5 m. From a point O, observations of bearings and distances of Q, R, S, T, and V are made as follows:
- Q (270°, 80 m)
 - R (260°, 60 m)
 - S (240°, 130 m)
 - T (220°, 120 m)
 - V (110°, 75 m)

Use a suitable scale to draw the plan of the garden and the concrete pavement accurately.

3. A group of students in a school presented a sketch and notes of a section of a road 6 m wide in relation to the school's flag pole (FP), as in figure 22.25 from their viewpoint O. Use a suitable scale to draw a map of the road in relation to the flag pole.

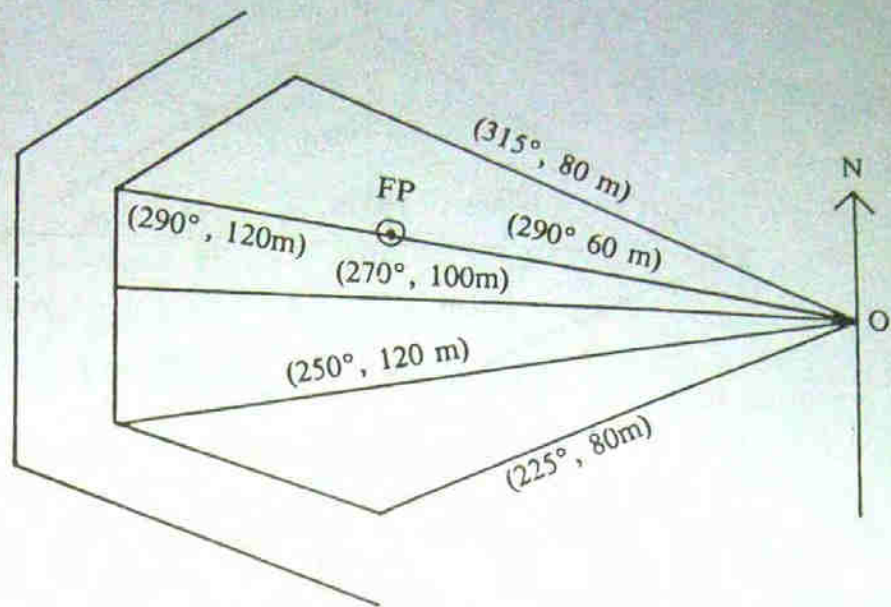


Fig. 22. 25

4. A surveyor recorded the measurements of a small field in a field book using base lines $AB = 75$ cm, $BC = 100$ cm and $CA = 100$ cm, as shown below:

	B		C		A
R9	55	W5	80	Z17	70
Q7	42	V6	70	Y5	50
P15	30	U7	60	X6	25
A	A	S10	20		C
			B		

Use a suitable scale to draw the map of the field.

22.6: Areas of Irregular Shapes

Areas of pieces of land which have irregular shapes can be obtained by subdividing them into convenient geometrical shapes, e.g., triangles, rectangles or trapezia. This is done by the use of base line and offsets of the area required.

For example, in figure 22.26, the area in hectares of the field can be found by the help of a base line and offsets as shown.

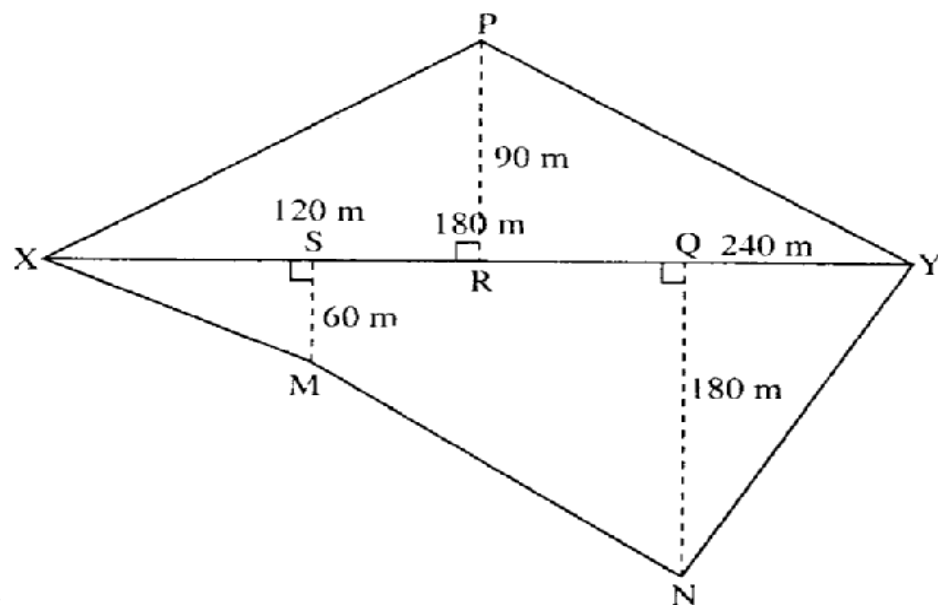


Fig. 22. 26

XY is the base line 360 m. SM, RP and QN are the offsets.

Taking X as the starting point of the survey, the information can be entered in a field book as follows:

	Y	
	240	180 to N
To R90	180	
	120	60 to M
	X	

Area of:

$$\text{Triangle XPR is } \frac{1}{2} \times 180 \times 90 \text{ m}^2 = 8\,100 \text{ m}^2$$

$$\text{Triangle PRY is } \frac{1}{2} \times 180 \times 90 \text{ m}^2 = 8\,100 \text{ m}^2$$

$$\text{Triangle XSM is } \frac{1}{2} \times 120 \times 60 \text{ m}^2 = 3\,600 \text{ m}^2$$

$$\text{Triangle QNY is } \frac{1}{2} \times 120 \times 180 \text{ m}^2 = 10\,800 \text{ m}^2$$

$$\begin{aligned} \text{Trapezium SQNM} &= \frac{1}{2} (QN + SM) \times SQ \text{ m}^2 \\ &= \frac{1}{2} (180 + 60) \times 120 \text{ m}^2 \\ &= 14\,400 \text{ m}^2 \end{aligned}$$

$$\text{Total area} = 45\,000 \text{ m}^2$$

Therefore, the area of the field is 4.5 ha.

Exercise 22.6

1. Find the area in hectares of a coffee field whose measurements are entered in a field book as follows. (Take $XY = 400$ m as the base line)

	Y	
	360	80 to Q
To R80	280	
To S160	200	
	80	200 to P
	X	

Use a scale of 1 cm to 40 m to draw the map of the coffee field.

2. (a) Find the area in hectares of a maize farm XABCYD in figure 22.27 which is drawn to a scale of 1 cm to 50 m.

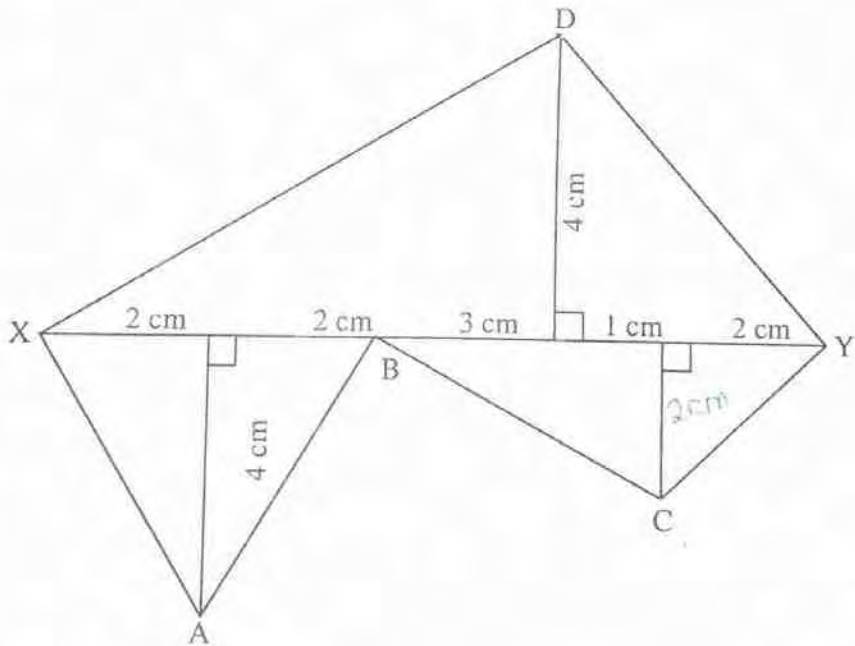


Fig. 22. 27

- (b) Taking XY as the base line and that the survey is from X to Y, enter the actual measurements of the farm in a field book.
3. Find the area in hectares of farms whose measurements are shown in field book as in the tables below. $AB = 600$ m and $XY = 500$ m:

(a)			(b)		
	B			Y	
	550	120 to L	To B100	450	
To K150	450			300	20 to C
	250	90 to M		120	70 to D
To J60	40		To A40	60	
	A			X	

Use the scale of 1 cm to 60 m to draw the maps of the farms.

Chapter Twenty Three

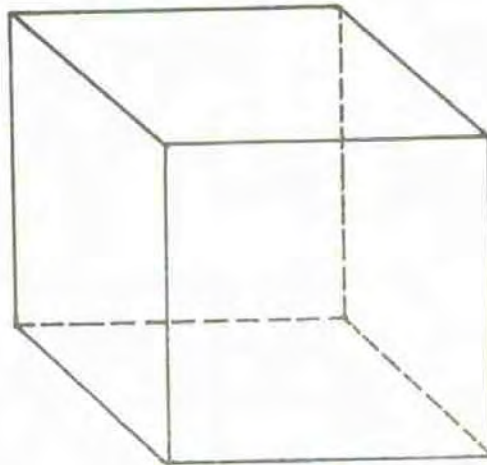
COMMON SOLIDS

23.1: Introduction

A solid is an object which occupies space and has a definite or fixed shape. Solids are either regular or irregular.

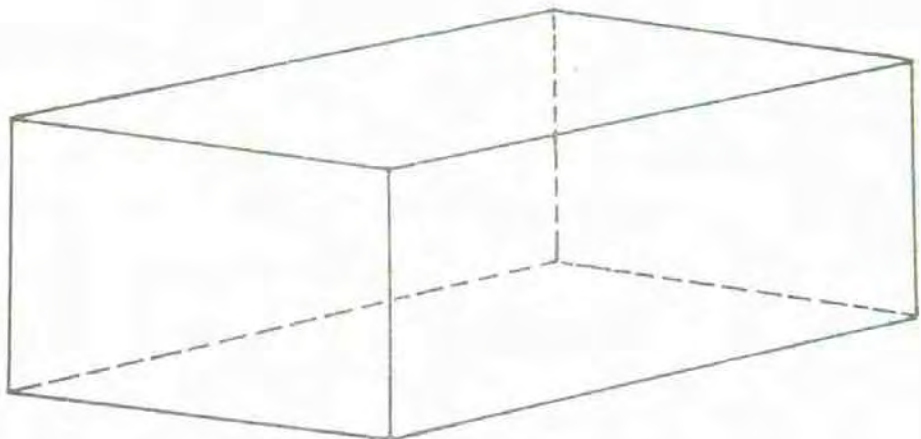
Below are drawings of some common regular solids.

(a)



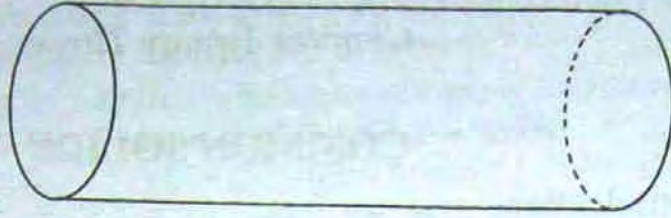
Cube

(b)

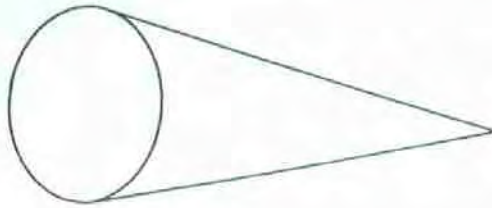


Cuboid

(c)

*Cylinder*

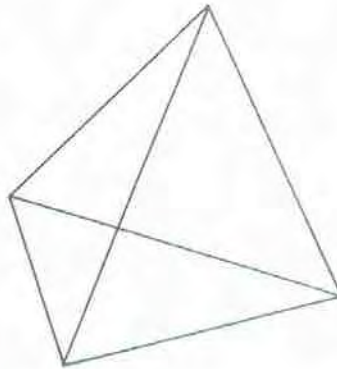
(d)

*Cone*

(e)

*Sphere*

(f)

*Tetrahedron**Fig. 23.1*

Give as many examples as possible of regular solids which you are familiar with.

All solids have surfaces. Some have faces, edges and vertices. Such solids are called **polyhedra (singular polyhedron)**. Intersections of faces are called **edges**. The point where three or more edges meet is called a **vertex**. Figure 23.2 shows a cuboid ABCDEFGH:

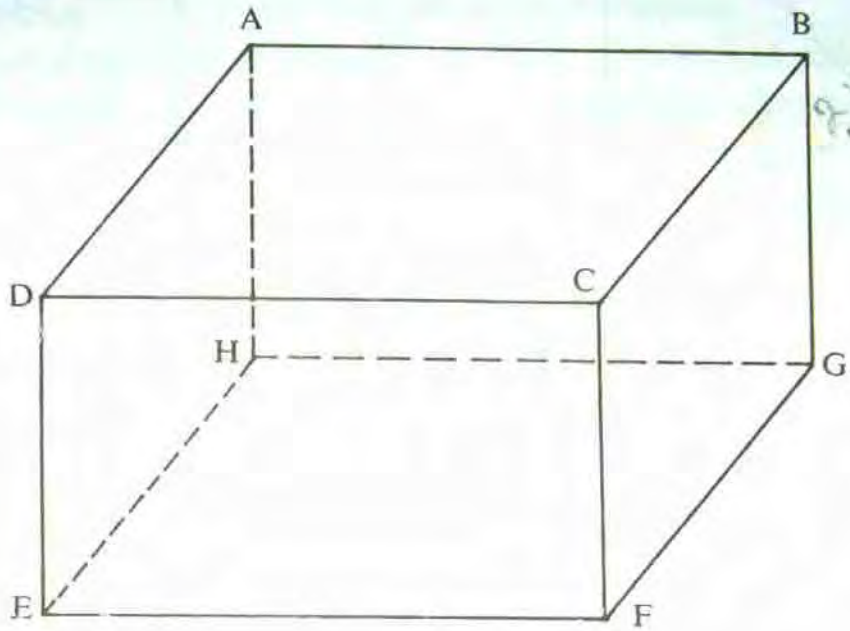


Fig. 23.2

In the figure:

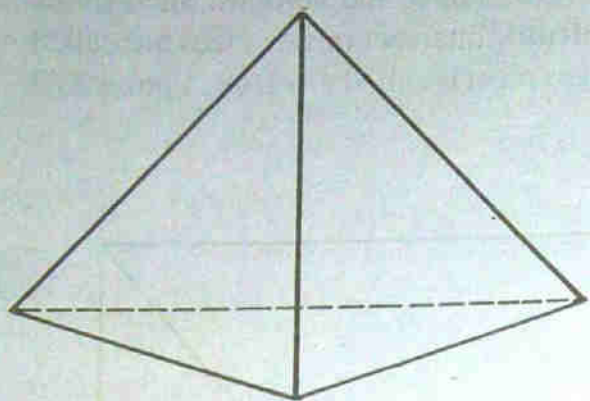
- ABCD and CBGF are of faces of the cuboid. Name other faces of the cuboid.
- DC and AH are edges of the cuboid. Name the other edges of the cuboid.
- H and D are vertices of the cuboid. Name the other vertices of the cuboid.

Exercise 23.1

- How many faces, edges and vertices does:
 - a cuboid,
 - a triangular prism,
 - a cone have?
- Name some common solid that have no vertices.
- Name a common solid that has neither a vertex nor an edge. How many faces does that solid have?

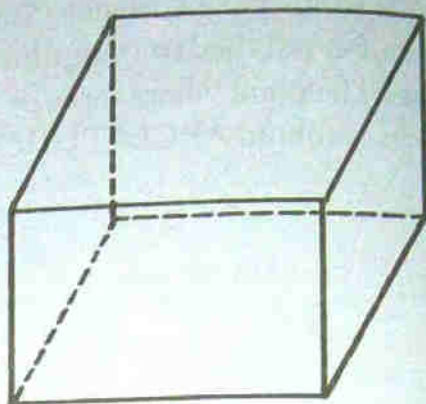
Polyhedra are named according to the number of faces they have. Figure 23.3 shows some examples of polyhedra.

(a)



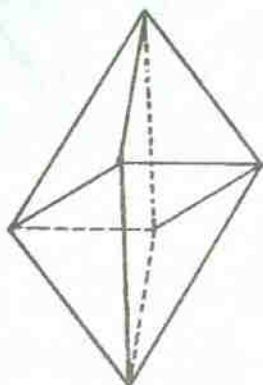
Faces: 4, Name: Tetraheron

(b)



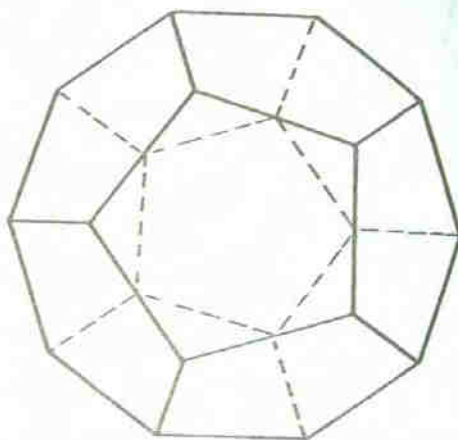
Faces: 6, Name: Hexahedron

(c)



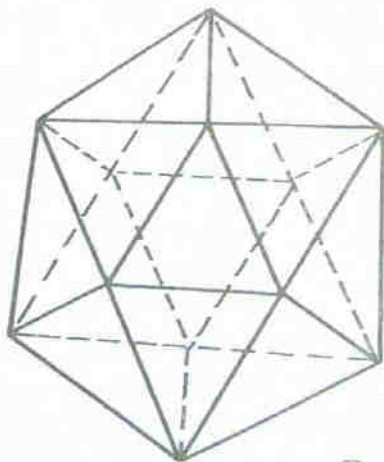
Faces: 8, Name: Octahedron

(d)



Faces: 12, Name: Dodecahedron

(e)



Faces: 20, Name: Icosahedron

Fig. 23.3

23.2: Sketching of Solids

To draw a reasonable sketch of a solid on a plain paper, the following ideas are helpful:

1. *Use of Isometric Projection*

In this method, the following points should be observed:

- (i) Each edge should be drawn to the correct length.
- (ii) All rectangular faces must be drawn as parallelograms.
- (iii) Horizontal and vertical edges must be drawn accurately to scale.
- (iv) The base edges are drawn at an angle of 30° with the horizontal lines.
- (v) Parallel lines are drawn parallel.

An isometric projection of a cuboid 5 cm long, 4 cm wide and 3 cm high is shown in figure 23.4.

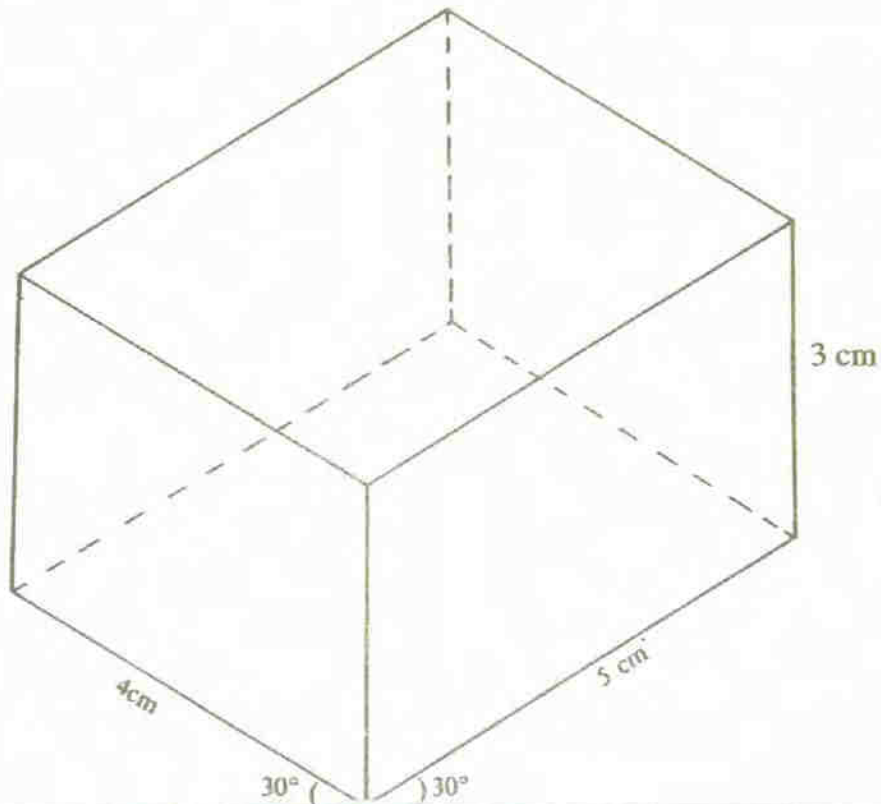


Fig. 23.4

2. *The use of Perspective Projection*

In this method, solids are drawn bearing the following points in mind:

- (i) Parallel lines are not drawn parallel. Horizontal parallel lines appear to meet at a **vanishing point**.
- (ii) Vertical lines are drawn vertical.

- (iii) For a front view of a solid, the measurements of the visible face are accurately drawn to scale.

A perspective projection of a cuboid 5 cm and 4 cm by 3 cm is shown in figure 23.5.

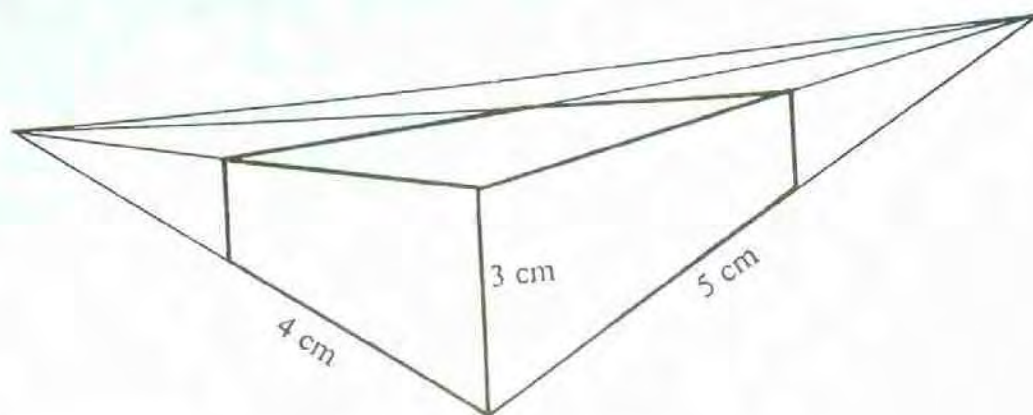


Fig. 23.5

3. Oblique Projection

To obtain an oblique drawing of a cuboid ABCDEFGH as in figure 23.6 in which $AB = 5$ cm, $BC = 3$ cm and $BG = 4$ cm:

- (i) Draw the horizontal line $AB = 5$ cm.
- (ii) Draw the vertical line $AF = 4$ cm.
- (iii) Draw AD and BC reduced to about $\frac{2}{3}$ of their true lengths, so that they make angles of 45° with AB .
- (iv) Draw the vertical lines BG , CH and DE accurately.
- (v) Join EF , FG , GH and HE .

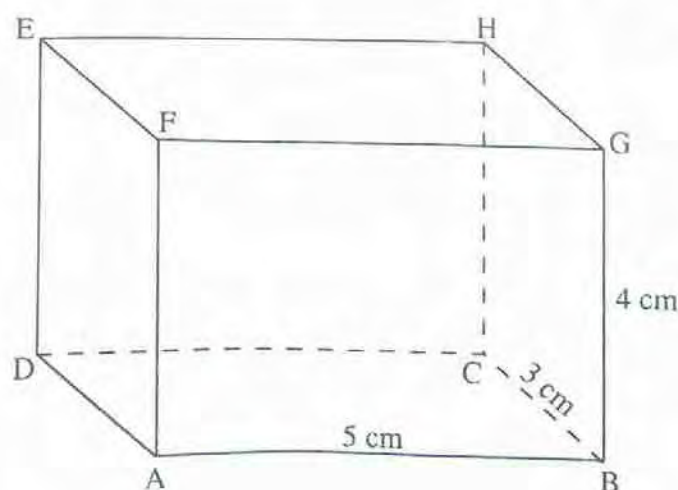


Fig. 23.6

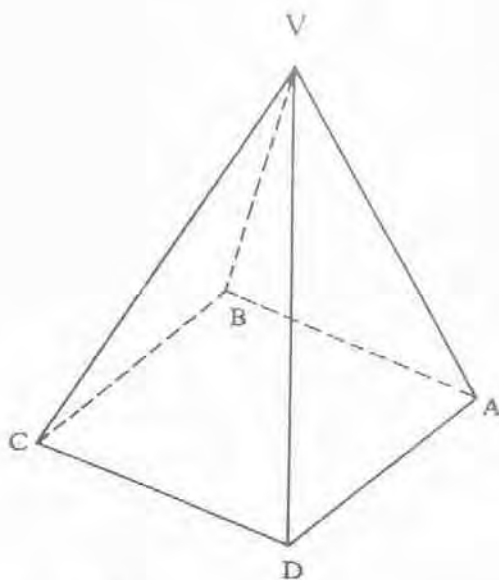
Exercise 23.2

1. Draw an isometric projection of a pyramid 7 cm high on a square base of side 4 cm.
2. A water tank is in the shape of a cuboid 3 m long, 2 m long wide and 3 m high. Draw:
 - (a) an isometric projection of the tank using a scale of 2 cm for 1 m.
 - (b) an oblique projection of the tank.
 - (c) a perspective drawing of the tank.
3. Draw an oblique projection of a cube of edge 4 cm.
4. Make a perspective drawing of a rail tunnel.
5. Make a perspective drawing of a classroom door half open, as viewed from outside.
6. Draw an oblique view of a long line of coffee trees, showing the vanishing points clearly.

23.3: Nets of Solids

Figure 23.7 (a) shows a sketch of a cardboard model of a right pyramid on a square base. If the pyramid is cut along the edges VA, VB, VC and VD, the faces can be laid out flat. The flat shape formed, see figure 23.7 (b), is called the net of the pyramid.

(a)



(b)

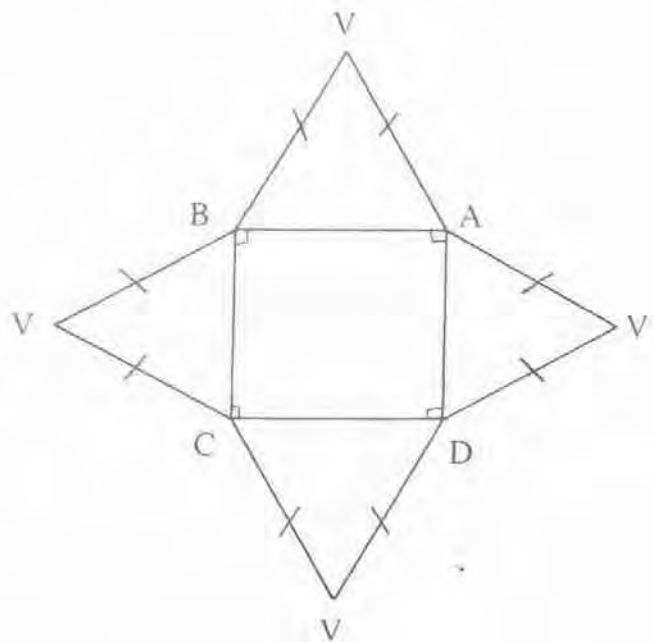


Fig. 23.7

Sketching Nets of Solids

The nets of common solids can be sketched. For example, the net of a cone with a base whose radius is 3 cm and slant height 5 cm can be sketched as in figure 23.8. Where else can the circle be drawn so that the new figure forms the net of the cone?

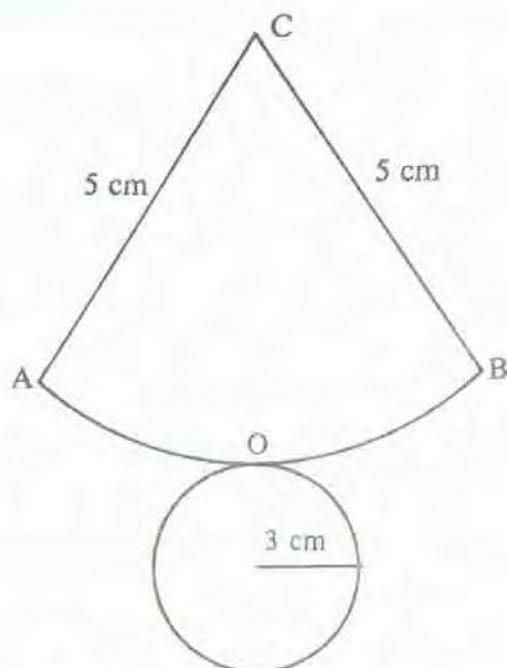


Fig 23.8

Infinite patterns like nets of models are called *tessellations*. A regular tessellation is a pattern of congruent regular polygons, all of one kind, filling a whole space, e.g., a squared paper. Tessellations are therefore widely used in the construction of models of solids.

Figure 23.9 is an example of a tessellation composed of parallelograms and triangles:

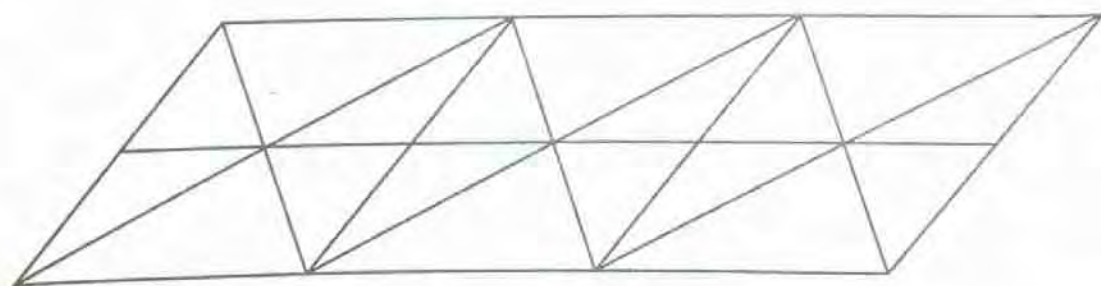


Fig 23.9

How many different nets of solids can you obtain from the tessellation above?

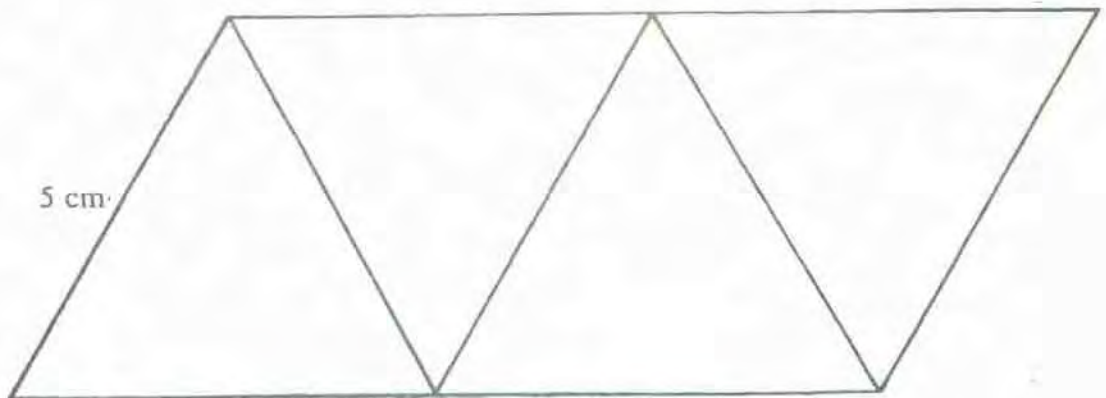
Exercise 23.3

1. (a) Draw accurately the nets of the following solids, where possible:
 - (i) Cylinder
 - (ii) Triangular prism
 - (iii) Pentagonal prism
 - (iv) Hexagonal prism
 - (v) A tetrahedron
 - (vi) A sphere
 - (vii) All the possible different nets of a cube
 - (b) Two right pyramids are joined together at the open base to form an eight-faced polyhedron. Sketch the net of the polyhedron.
2. Draw the solids of each of each of the nets below:

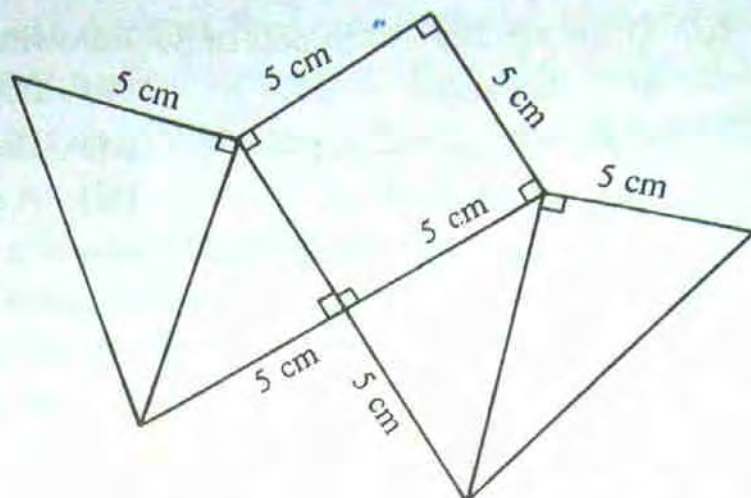
(a)



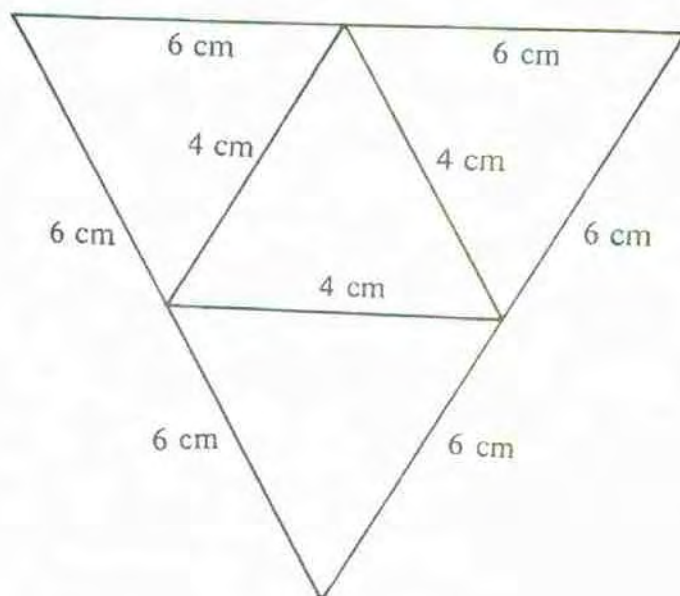
(b)



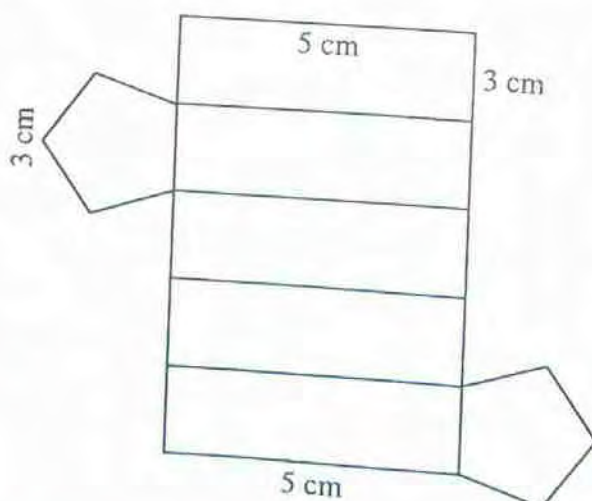
(c)



(d)



(e)



(f)

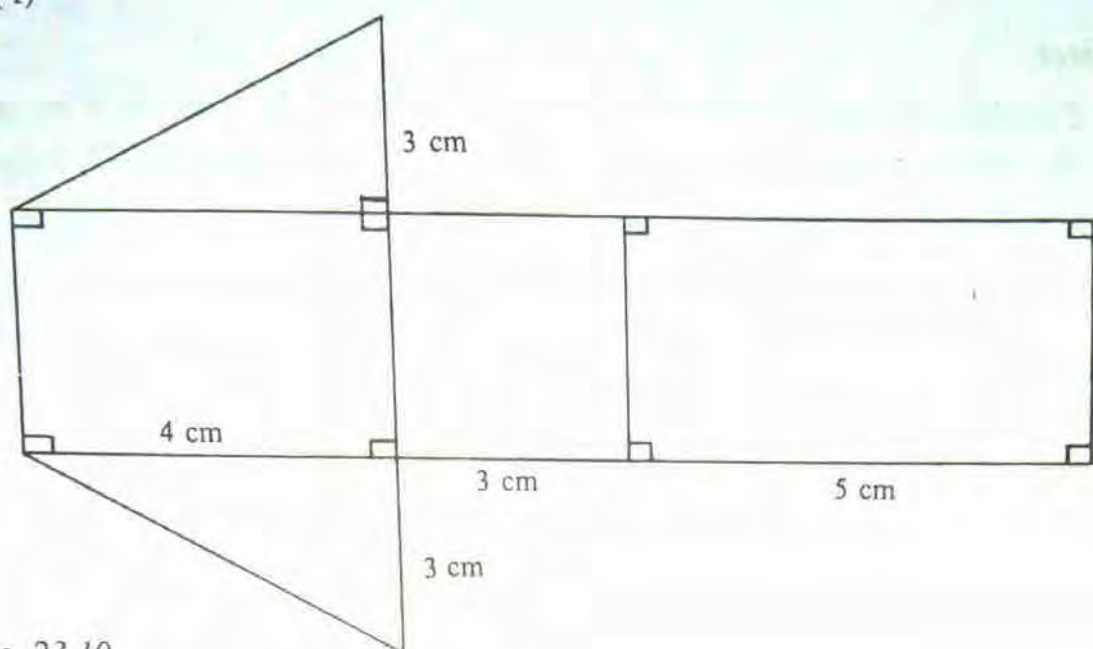


Fig. 23.10

23.4: Models of Solids

Cardboard models

Use of manilla papers is recommended in this section.

Draw accurately the net of a pyramid on whose base is a square of side 10 cm and slant edges are each 15 cm. Cut out the net, fold it to form a pyramid. Secure the edges using a cello tape.

The net of another pyramid can be cut out as shown in the figure 23.11 with tabs. Construct the net. Cut it out with tabs on alternate edges. The tabs help in joining the edges of the solid firmly using glue. Make two more similar solids. Join the three solids to form another solid. What name do we give to this solid?

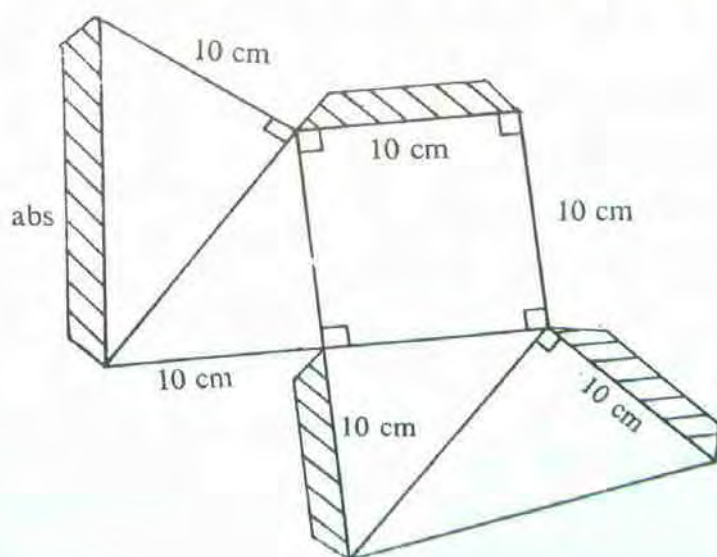


Fig. 23.11

Project**(a) Cardboard models**

- (i) Make a model of a cube of side 10 cm.
- (ii) Make a model of the following solids:
 - Tetrahedron.
 - Cylinder.
 - Octahedron.
 - Triangular prism.
- (iii) Draw a regular pentagon of side 5 cm. From each side, draw other regular pentagons so that you have six pentagons in total. Similarly obtain another set of six pentagons. Join any side of one set to the other. The net so formed is of a regular dodecahedron. Join all the sides to obtain the model of the solid.
- (iv) Copy and complete the table below with the help of the models you have made.

Solid	Number of faces (F)	Number of edges (E)	Number of vertices (V)
1. Cuboid	6	12	8
2. Tetrahedron	—	—	—
3. Triangular prism	—	—	—
4. Pentagonal prism	—	—	—
5. Hexagonal prism	—	—	—
6. Octahedron	—	—	—
7. Icosahedron	—	—	—

For each of the solids in the table, you should notice that the relationship between faces (F), edges (E) and vertices (V) is $F + V = E + 2$. This relationship is called **Euler's formula**. Does the relationship work for a sphere and a cylinder? Give reasons for your answer.

(b) Skeleton models

To make skeleton models, the following can be used:

- (i) Wire of suitable thickness.
- (ii) Plastic straws with an appropriate wire or string.
- (iii) Pair of pliers for cutting and bending the wires.

These models are advantageous over cardboard models because it is easier to see all the angles and edges.

Cube

With the use of wire, pair of pliers and solder or insulating tape, a cube as shown in figure 23.12 can be made:

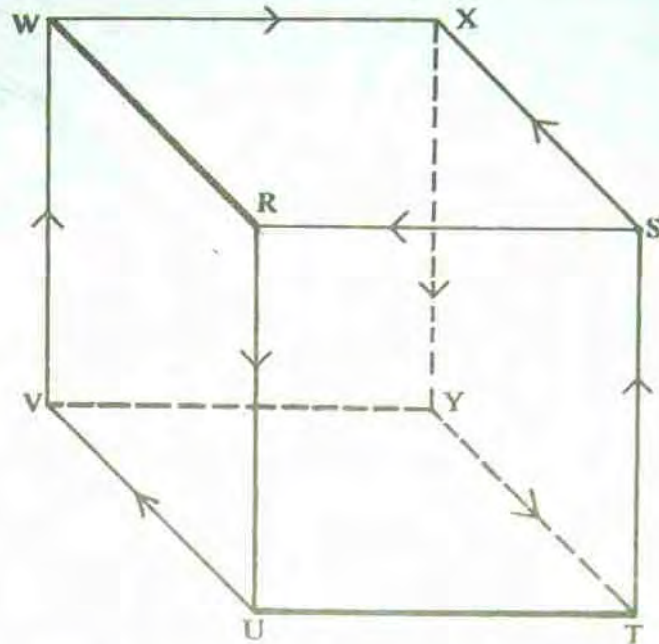


Fig 23.12

One way of making the model is by starting at the point marked S and moving in the directions indicated by the arrows. Three wires shown in bold lines can be joined to the rest of the skeleton using solder or insulating tap. Find out other possible ways of making the model.

Tetrahedron

A model of a tetrahedron as shown in figure 23.13 can be made using six equal plastic straws, each 15 cm long and a wire:

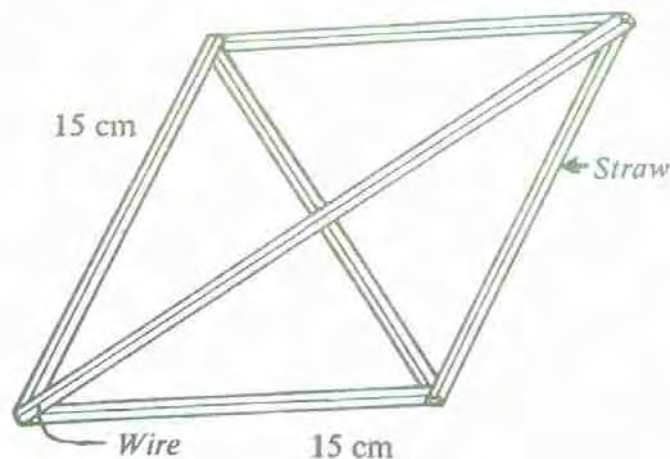


Fig 23.13

Pyramid with a square base

Eight equal plastic straws, each 15 cm long and enough thread to go through all the straws as shown in figure 23.14 are required.

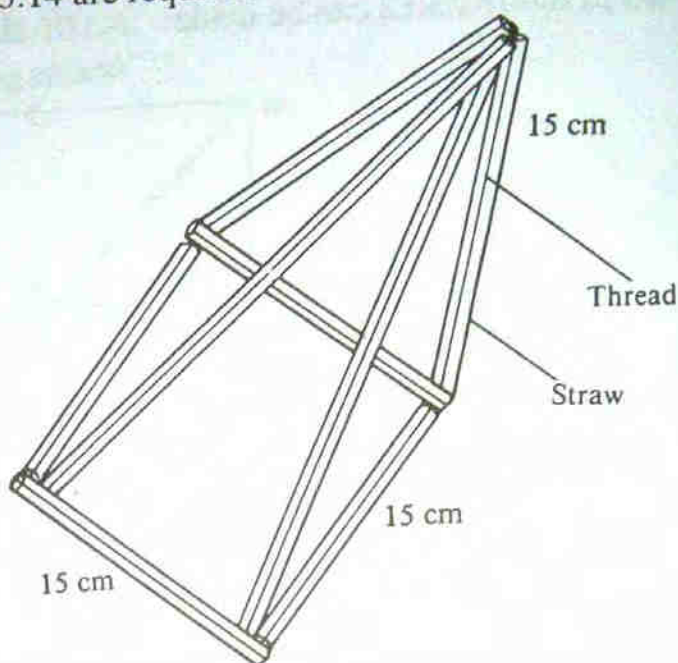


Fig 23.14

Make a skeleton model of a reasonable:

- (i) octahedron
- (ii) wedge

23.5: Surface Area of Solids from Nets

The surface area of a solid may be calculated by finding the area of its net.

Example 1

Figure 23.15 shows a right pyramid whose base is a square of side 10 cm and its slant side 15 cm long. Calculate its surface area:

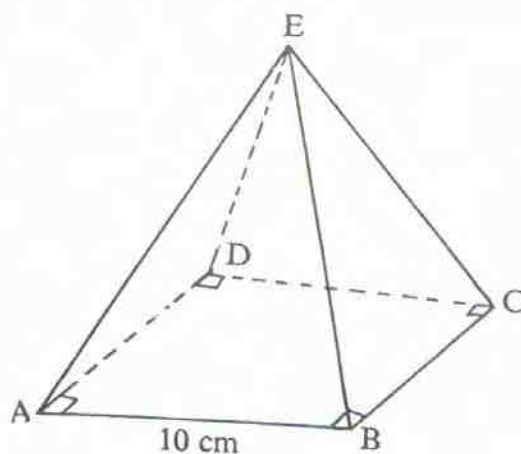


Fig 23.15

Solution

The net of the pyramid is shown in figure 23.16:

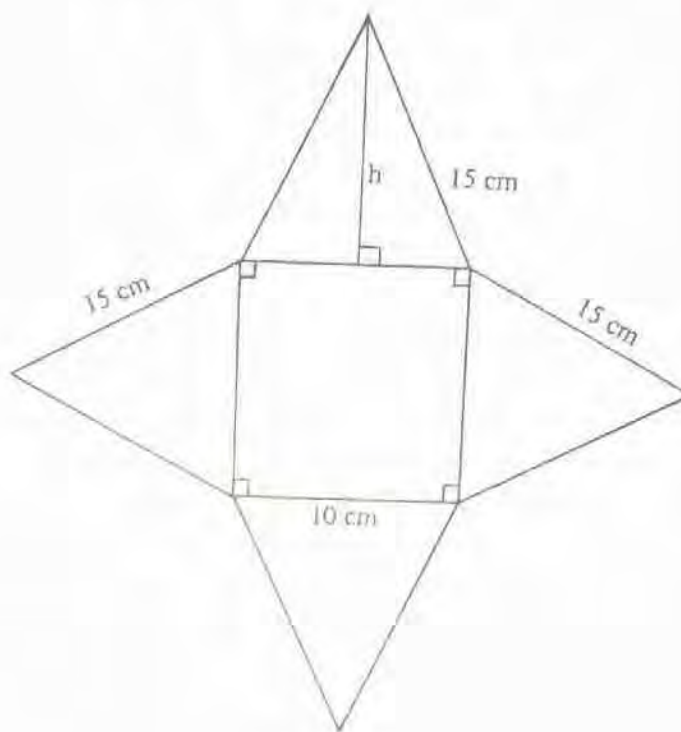


Fig 23.16

$$\begin{aligned}\text{Area of square} &= 10 \times 10 \\ &= 100 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Height of each triangle} &= \sqrt{15^2 - 5^2} \\ &= 14.1 \text{ (to 1 d.p.)}\end{aligned}$$

$$\begin{aligned}\text{Area of each triangle} &= \frac{1}{2} \times 10 \times 14.1 \\ &= 70.5 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{Surface area of the pyramid} &= 100 + 70.5 \times 4 \\ &= 382 \text{ cm}^2\end{aligned}$$

Exercise 23.4

For each of the following solids:

- (i) draw the net.
 - (ii) use the net to calculate the surface area of the solid.
1. A cube of side 8 cm.
 2. A cuboid measuring 12 cm by 6 cm by 8 cm.
 3. A tetrahedron whose faces are equilateral triangles of side 10 cm.
 4. A cylinder whose radius and height are 7 cm and 20 cm respectively.
 5. A polyhedron made up of a pyramid with isosceles triangles and a cuboid as shown in figure 23.17.

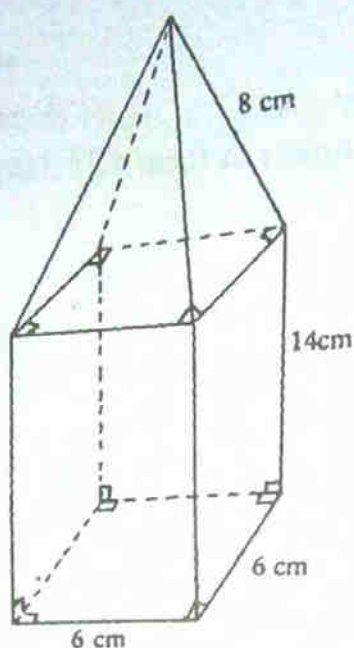


Fig 23.17

6. A triangular prism, as shown in figure 23.18:

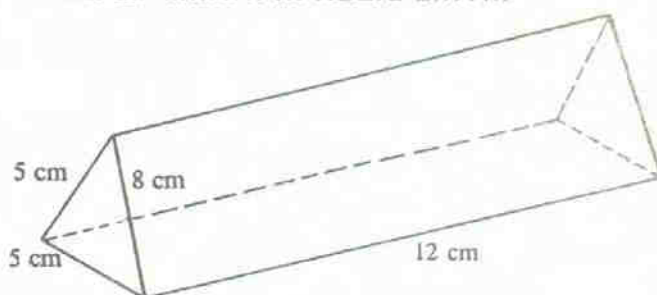


Fig 23.18

7. The wedge shown in figure 23.19:

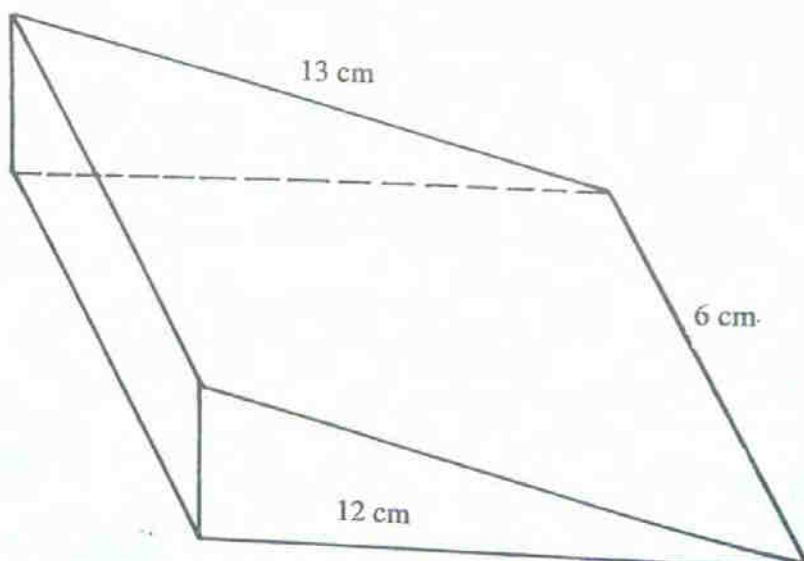


Fig. 23.19

8. A cone of radius 7 cm and height 10 cm.

23.6: Distance between Two Points on the Surface of a Solid

To find the distance between two points on the surface of a solid, first open up the solid into its net.

Example 2

Find the distance between B and X through G and F in figure 23.20, if $BA = 5$ cm, $AD = 3$ cm and $DE = 4$ cm.

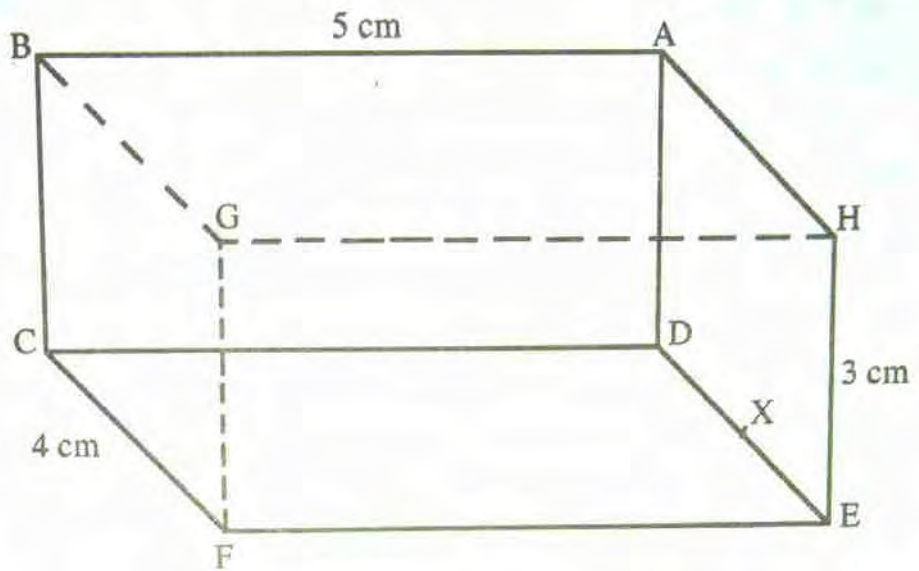


Fig. 23.20

Solution

Open the cuboid into a net:

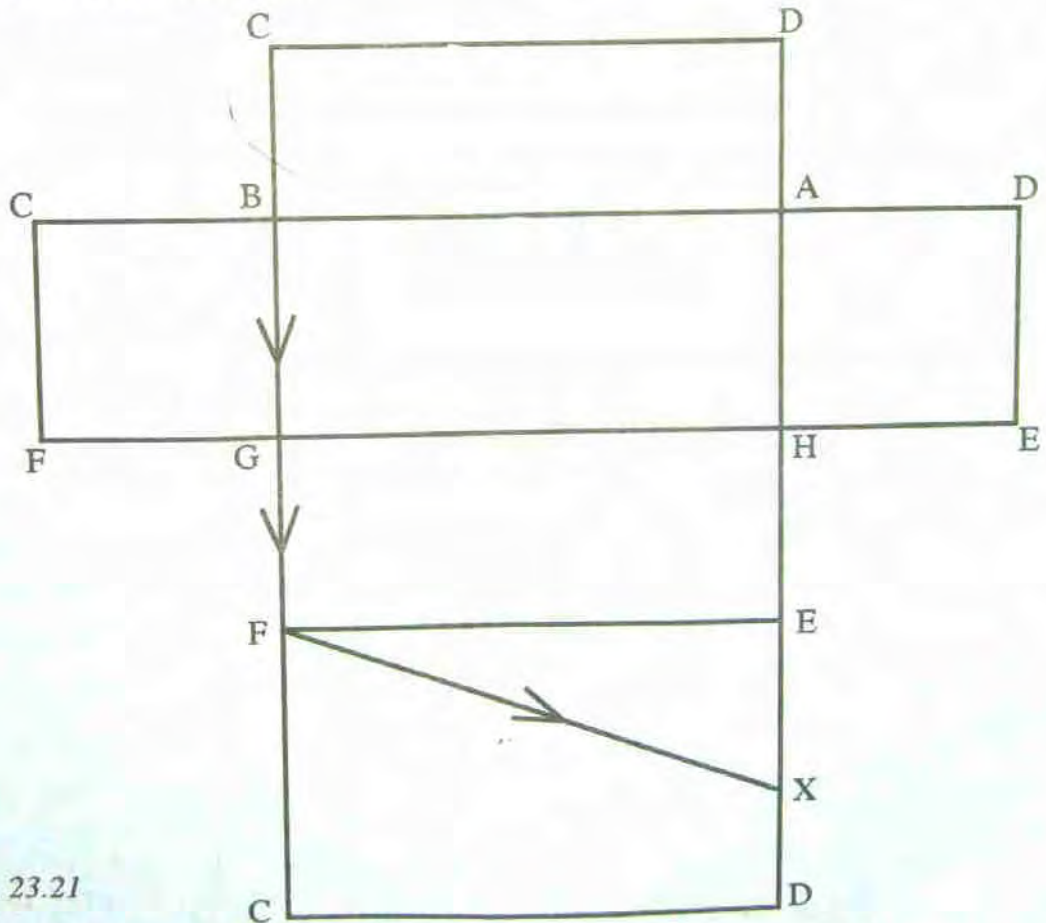


Fig. 23.21

$$BA = BG + GF + FX$$

$$= 4 + 3 + \sqrt{(5^2 + 2^2)}$$

$$= 7 + \sqrt{29}$$

$$= 7 + 5.385$$

$$= 12.385$$

$$\therefore BX = 12.4 \text{ cm (to 1 d.p.)}$$

Exercise 23.5

- Figure 23.22 shows a cube of side 8 cm. The points Q, R and S are midpoints of EH, HC and BC respectively. A string runs from F to Q on face EFGH, Q to R on face CDEH, R to S on face BCHG and S to A on face ABCD. Along what edges should the cube be opened so that the points F, Q, R, S and A lie on a straight line? What is the length of the line?

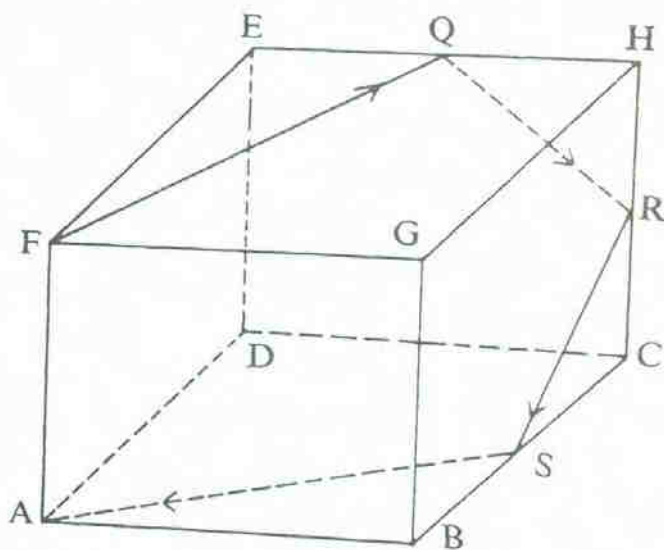


Fig. 23.22

- Figure 23.23 shows a triangular prism ABCDEF. Its cross section is an equilateral triangle of side 10 cm and its length is 20 cm. A string runs from F to Q through R and D. Along what edges should the cube be opened so that F, Q, R, and S lie on a straight line? What is the length of the straight line?

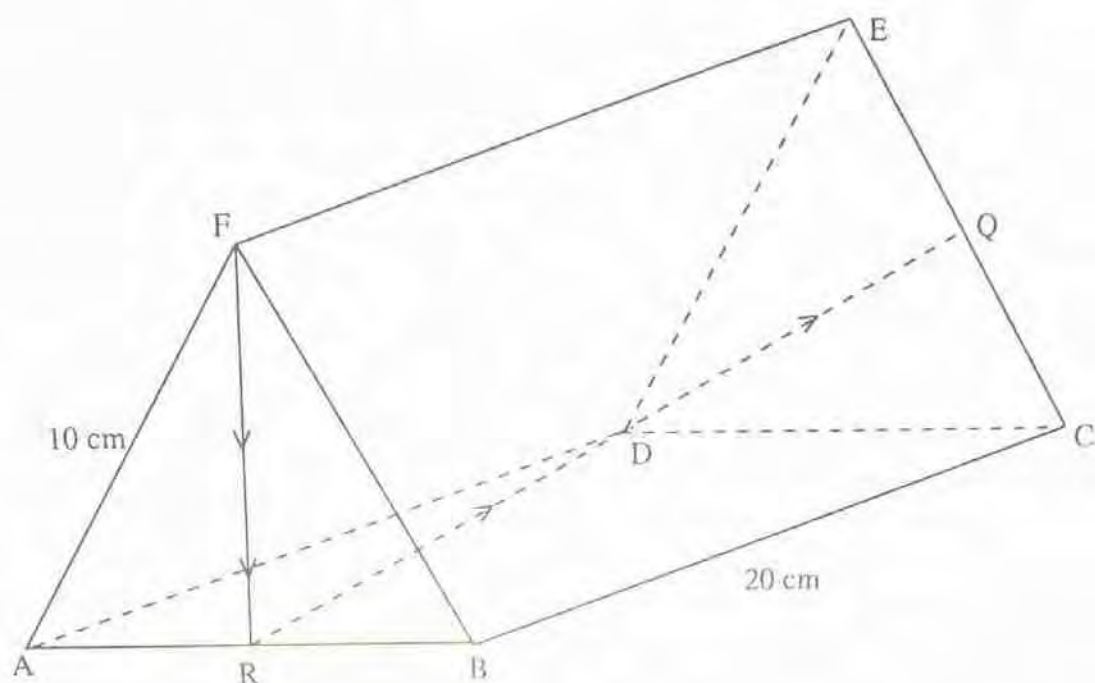
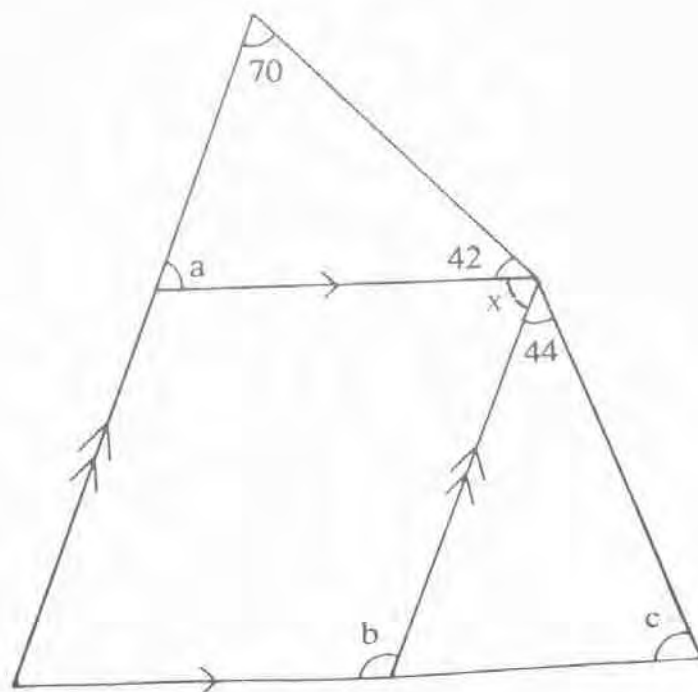


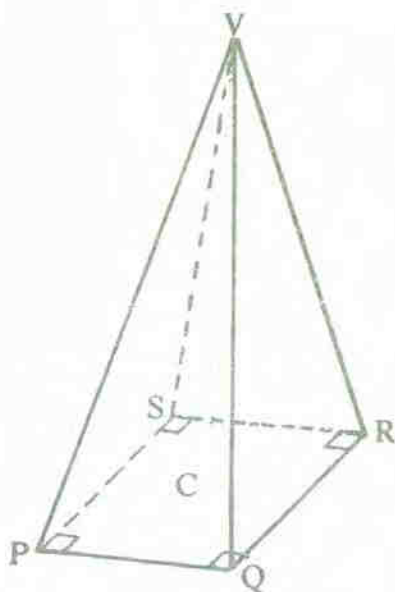
Fig. 23.23

Mixed Exercise 3

- Draw graph of each of the following equations:
 - $2x + 3y = 8$
 - $y = 2x - 4$
 - $x - y = 4$
 - $\frac{2y}{5} + 2x - 4 = 0$
- Find a , x , b and c in the figure below



3. Draw a triangle PQR, in which $PQ = 8$ cm, $QR = 7$ cm and $PR = 6$ cm. Using a pair of compasses and a ruler only, construct the bisector of $\angle PQR$ to meet PR at S. Through S, draw a line parallel to PQ to intersect QR at T. Measure $\angle QST$ and $\angle RTS$.
4. Find the length of a square whose area is 0.0084 m^2 .
5. It takes a tap $2\frac{1}{2}$ minutes to fill a container which holds 190 litres of water. Calculate the rate per minute at which the water is flowing.
6. Find the surface area of a rectangular glass block whose volume is $1\,524 \text{ cm}^3$, if it is 72 cm long and 48 cm wide.
7. Calculate the mass (in grams) and the volume of a metal bar 168 cm long, 4.6 cm wide and 2.8 cm high, given that its density is 8.5 g/cm^3 .
8. The figure below shows a pyramid on a square base PQRS. Given that $PV = QV = RV = SV = 5$ cm, draw accurately the net of the pyramid. Use the net to calculate the surface area of the pyramid.



9. Solve the following simultaneous equations using the graphical method:

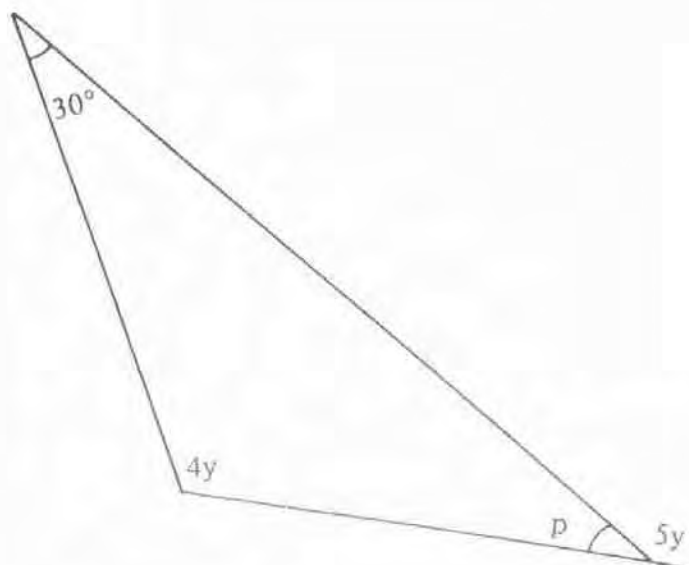
(a) $3x + 2y = 13$	(b) $2x + y = 22$
$5x - 3y = 15$	$x + 8 = 15$
(c) $x + y = 1$	(d) $5x + 2y = 0$
$4x + y = 7$	$x + y = 3$

10. Town A is 12 km to the north of town B while town C is 21 km to the south-east of town A. Find by scale drawing:
- the bearing of town B from town C.
 - the distance between B and C.
11. The number of cattle, goats and sheep in a farm are in the ratio 6 : 2 : 3. If the animals on the farm are to be reduced by $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{3}$ respectively, find how many are left if initially there were 1 450 animals on the farm.
12. A solution contains methylated spirit and water in the ratio 3 : 14 by volume. The initial volume was 1 850 cm³ and a litre of water is added to the solution.
- Calculate the new ratio of spirit to water.
 - If 370 cm³ of the new solution is poured in a measuring cylinder, how much of this is methylated spirit?
13. Power in watts dissipated in an electric circuit is given by the formula $W = \frac{V^2}{R}$ where W, V, and R are power in watts, voltage in volts and resistance ohms respectively. Below is a table for the three:

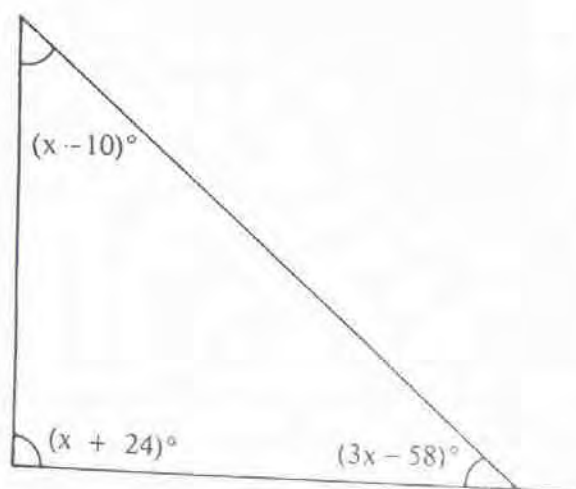
V	1	2	3	4	5	6	7	8	9	10
R	2	3	4	5	6	7	8	9	10	11
$W = \frac{V^2}{R}$	0.5	—	2.25	—	—	5.1	—	—	8.1	—

- Complete the tabulation of watts.
 - Draw a graph of W against V.
 - Use the graph to find:
 - wattage when the voltage is 2.5 volts, 7.5 volts.
 - the voltage when the wattage is 3 watts, 6 watts.
14. What acute angle is a fifth of its supplement?
15. The sum of the interior angles of two regular polygons of sides $n - 1$ and n are in the ratio 2 : 3. Calculate:
- the value of n .
 - the interior angle of each polygon.
16. Find the value of the unknown angle in each of the following figures:

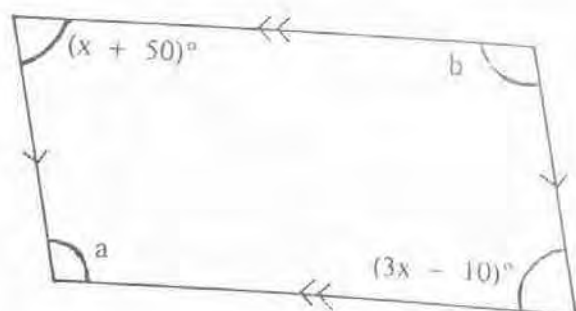
(a)



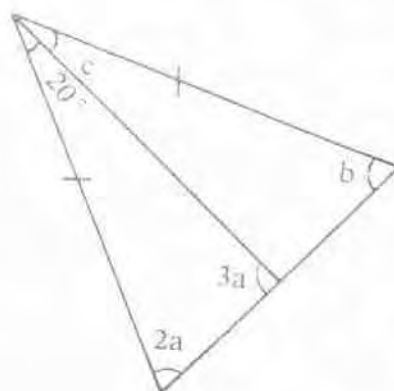
(b)



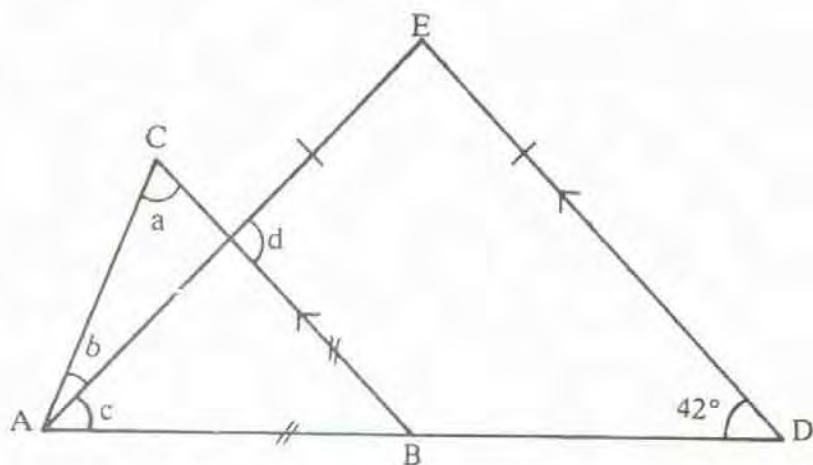
(c)



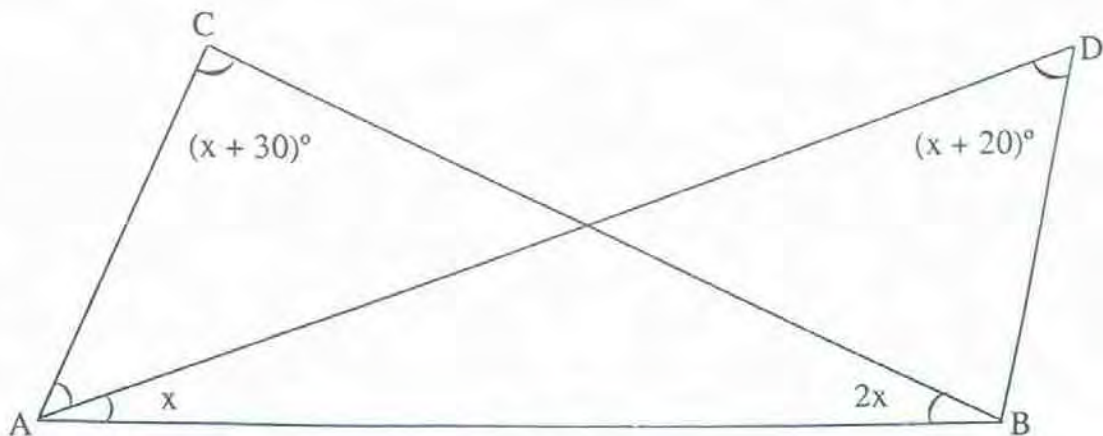
(d)



17. Four interior angles of a hexagon are 100° , 140° , 125° and 105° . The fifth interior angle is four times the sixth. Find, in degrees, the fifth interior angle.
18. In the figure below, $AB = BC$, $AE = ED$ and $\angle ADE = 42^\circ$. If BC is parallel to DE , find the angles marked a , b , c and d .



19. ABCDE is a regular pentagon. AE and CD produced meet at F. Find the angles of triangle DEF.
20. In the figure below, $\angle CBD = 2 \angle CAD$. Find the value of x .



21. In triangle ABC, $AB = 5$ cm, $AC = 7$ cm and, $\angle BAC = 20^\circ$. Find, by construction, the length of BC.

22. ABCD is a trapezium in which $BC \parallel AD$ and $\angle BAD = \angle ABC = 90^\circ$. $AB = 7$ cm, $AD = 4$ cm and $\angle ADC = 150^\circ$. Using ruler and pair of compasses only, construct the trapezium. Measure CD and BC. Hence, calculate its area.
23. (a) The line $y = -x$ intersects the line $2y = -3x + 10$ at a point A. Find the co-ordinates of A.
 (b) A line $y = 3x - 4$ intersects the line $y = x$ at S and $y = -x$ at T. Find the co-ordinates of S and T.
24. In a triangle ABC, $\angle ABC = 42^\circ$, $AB = 6$ cm and $BC = 5$ cm. P is a point such that $AP = 7$ cm and $CP = 8$ cm. Draw a circle of centre O, passing through A, C and P. Measure $\angle AOC$, $\angle APO$ and the length of OP.
25. (a) Find the surface area of a rectangular glass block whose volume is $1\,524\text{ cm}^3$, if it is 72 cm long and 48 cm wide.
 (b) The diameter of a cylindrical container, closed at both ends, is 0.28 m and its height is 14 m. Find its:
 (i) surface area.
 (ii) volume (4 s.f.).

Revision Exercises

Revision Exercise 1

- Find the L.C.M. of the following sets of numbers. Leave your answers in power form:
 (a) 20, 30, 40 (b) 28, 63, 100
- Patrick spent $\frac{2}{5}$ of his salary on food, $\frac{1}{3}$ of the remainder on electricity and saved the rest. What fraction of his salary did he save? If he spent sh. 1 200 on food, how much did he spend on electricity?
- Write the following as single fractions in their lowest forms:
 (a) $\frac{x}{2} + \frac{2x-3}{3} + \frac{2x+2}{4}$ (b) $\frac{pr}{p} + r^2 - \frac{pr}{p} + r^2$
- A man bought 10 mangoes at sh. 9.00 each. He ate four of the mangoes and sold the remainder, making an overall profit of sh. 8.00. Calculate:
 (a) his selling price per mango.
 (b) the percentage profit on each mango.
- Five men, each working 10 hours a day, take two days to cultivate one acre of land. How long will two men, each working six hours a day, take to cultivate three acres of land?
- The sum of interior angles of a regular n -sided polygon is $1\,080^\circ$. Find the size of each interior angle of the polygon. What is the name of the polygon?

7. The diameter of a cylindrical container of height 40 cm and open at one end is 0.28 m. Find:
- the surface area in cm^2 .
 - the volume of the container in cm^3 (to 4 s.f.).
8. Nafula broke three beakers and four test tubes and Munyao broke two beakers and five test tubes during practicals in the laboratory. If Nafula was charged sh. 560 and Munyao sh. 490 for the breakages, find the cost of one beaker and one test tube.
9. Ali travels a distance of 5 km from village A to village B in the direction 060° . He then changes course and travels a distance of 4 km in the direction 135° to village C. Find by scale drawing:
- the distance in km between A and C.
 - the bearing of A from C.

Revision Exercise 2

1. Evaluate:

(a) $\frac{1}{2}$ of $\frac{1}{4} \div \frac{1}{8} + \frac{3}{4} - \frac{1}{8}$

(b) $\frac{\frac{2}{3} \div \frac{4}{3}}{\frac{1}{3} + \frac{5}{6}} \div \frac{1}{2}$

2. Evaluate:

(a) $\frac{-5(23+41-85)}{40-75}$

(b) $\sqrt{\frac{0.032+0.608}{0.0016 \times 0.25}}$

3. Evaluate:

(a) $p^2 - r^2$

(b) $(p + r)$, if $p = 6$ and $r = 2$.

What do you notice?

4. Find the ratio $p : r$ if:

(a) $p : q = 1 : 4$, $q : r = 3 : 2$

(b) $p : y = 4 : 3$, $y : 2 = 5 : 7$, $2 : r = 1 : 2$

(c) $p : a = 1 : 2$, $a : b = 2 : 3$, $b : c = 5 : 1$, $c : r = 3 : 2$

5. Solve the following simultaneous equations using an appropriate method:

(a) $\frac{1}{4}x + y = 2\frac{1}{4}$

(b) $3t = s$

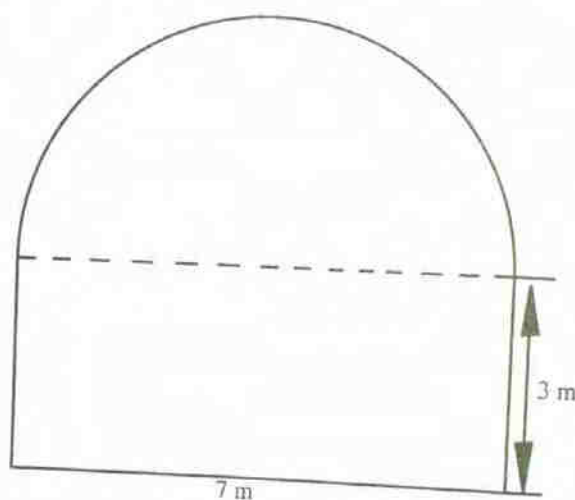
$\frac{1}{2}y - \frac{1}{4}x = 3\frac{3}{4}$

$t + 2s = 5$

6. Each exterior angle of a polygon is 40° . Find the number of sides of the polygon.

7. (a) Water and alcohol are mixed in the ratio 1 : 4. Find the density of the mixture if the density of water is 1 g/cm^3 and that of alcohol is 0.8 g/cm^3 .
- (b) 40 cm^3 of water is poured into an empty measuring cylinder. A stone of mass 129 g is put into the cylinder. If the density of the stone is 8.6 g/cm^3 , find the new reading of the cylinder.

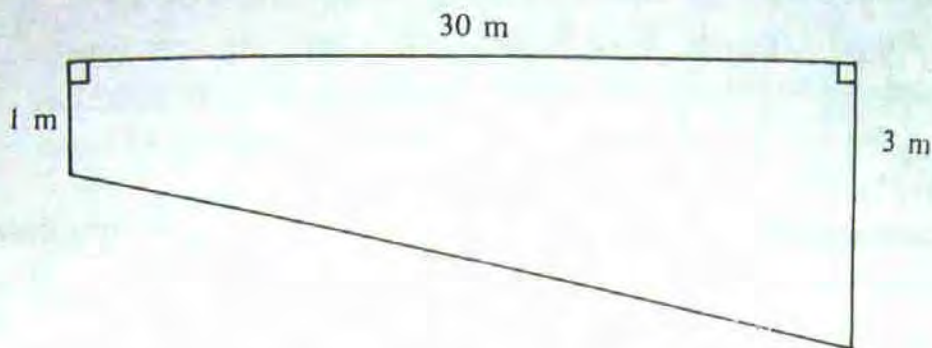
8. Use a ruler and a pair of compasses only to construct a triangle ABC in which $AB = 4.6$ cm, $BC = 5.4$ cm and $\angle ABC = 75^\circ$. Measure AC. Drop a perpendicular from B to meet AC at N. Measure BN. Hence, calculate the area of triangle ABC.
9. The angles of elevation of the top of a cliff from two boats A and B on the same side of the cliff and on the same horizontal level with the foot of the cliff are 30° and 50° respectively. If the distance from the foot of the cliff to boat B is 30 m, find by scale drawing:
- the height of the cliff.
 - the distance between A and B.
10. The figure below is a uniform cross-section of a hall 20 m long and 7 m wide. Assuming that each person requires an average of 5 m^3 of space, how many people would be accommodated in the hall with this arrangement?



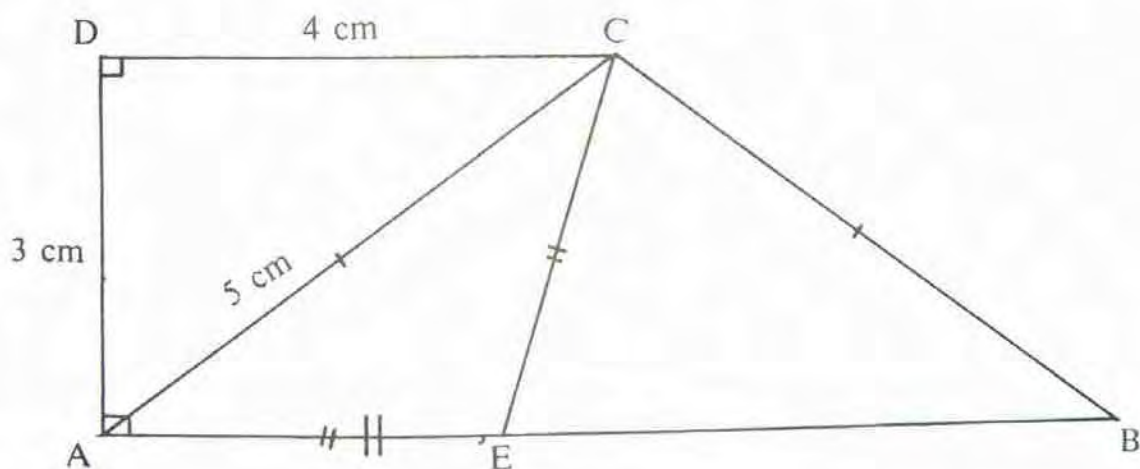
Revision Exercise 3

- Find a correct to 2 d.p. if $\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$, $b = 2$ and $c = 3.5$.
- Factorise each of the following expressions:
 - $3px - py + 3qx - qy$
 - $a^2 - 4ap - 4p + a$
- A farmer has three containers of capacity 12 l, 15 l, and 21 l. Calculate the capacity of:
 - the smallest container which can be filled by each one of them an exact number of times,
 - the largest container which can fill each one of them an exact number of times.

4. The following figure is a cross-section of a swimming pool 8 m wide: Calculate the capacity of the pool in litres.



5. Juma, Ali and Hassan share the profit of their business in the ratio 3 : 7 : 9 respectively. If Juma receives sh. 60 000, how much profit did the business yield?
6. The figure below is a trapezium in which DC is parallel to AB:
- Given that $AC = BC$, $AE = EC$ and $\angle ACE = 37^\circ$, find $\angle BCE$.
 - Find the area of the trapezium if $AC = 5$ cm, $DC = 4$ cm, $EB = 7$ cm and $AD = AE = 3$ cm.



7. Use graphical method to solve:
- $y = \frac{1}{2}x + 1$ and $y = \frac{1}{3}x + 2$
 - $y = 2x + 3$ and $y = 3x + 2$

Revision Exercise 4

1. Evaluate:

$$(a) \frac{1}{2} + \frac{1}{6} \text{ of } \left(\frac{13}{18} - \frac{5}{9} \right) \div \frac{1}{3}$$

$$(b) \frac{\frac{1}{4} + \frac{1}{5} \div \frac{1}{2} \text{ of } \frac{1}{3}}{\frac{1}{2} \text{ of } \left(\frac{4}{5} - \frac{3}{4} + \frac{1}{2} \right)}$$

2. Express each of the following as a single fraction in its simplest form:

$$(a) \frac{x+y}{3} - \frac{2x-y}{2}$$

$$(b) \frac{1}{x+1} - \frac{1}{x-1}$$

3. The sum of the interior angles of a polygon is 1440° . Find the number of sides of the polygon.
4. Take a number n , double it and add five to the result. If this result is doubled again, the new number is 22. Find n .
5. The external radius of culvert is 0.5 m and it is 0.1 m thick. If the culvert is 2.8 m long, find the volume of the material used to make it. (Take $\pi = \frac{22}{7}$)
6. The ratio of the cost of a commodity X to that of a commodity Y is 2 : 3 and the ratio of the cost of Y to the cost of a commodity Z is 6 : 1. If the total cost of the three commodities is sh. 1100, find the cost of X. Express the cost of Z as a percentage of the cost of Y.
7. Two people X and Y have goats. X has more goats than Y and if Y gives X one of his goats, X will have twice as many goats as Y. If X gives Y one of his goats, they will have an equal number of goats. How many goats does each have?

Revision Exercise 5

1. The price of a commodity was increased in the ratio 5 : 4. After one month, the price of the same commodity was reduced in the ratio 7 : 8 to attract more customers. If the new price was sh. 35, calculate the price of the commodity before the increase.
2. Solve the following simultaneous equations:

$$(a) \quad 3x + 4y = 18$$

$$2x - y = 1$$

$$(b) \quad -\frac{1}{2}x + y = 5$$

$$\frac{1}{3}y = 5 - x$$

3. Use a ruler and a pair of compasses only to construct a triangle ABC in which $AB = 7.5$ cm, $BC = 6$ cm and $AC = 4.5$ cm.
 - (a) Measure the angles of the triangle.
 - (b) Draw the circle passing through the points A, B and C. Measure the radius of the circle. Hence, estimate the area of the circle.
4. Mary was allowed a discount of 11% for goods worth sh. 8 000 and a discount of 8.6% for goods worth sh. 17 000. What percentage discount was she allowed altogether?
5. A Canadian on a tour in Kenya converted 5 600 Canadian Dollars to Kenya shillings for hotel accommodation and other miscellaneous expenses while in the country. He was in Kenya for 20 days and stayed in a hotel in which he was paying sh. 3 500 per day full board. He also hired a self-drive car for which he was paying sh. 7 000 per day and bought *curios* worth sh. 15 000. He donated the balance to a children's home in Nairobi. Calculate in Kenya shillings:
 - (a) his total expenditure on accommodation and car hire.
 - (b) the amount of money he donated to the children's home.
6. The length of an arc of a circle is $\frac{1}{10}$ of the circumference of the circle. If the area of the circle is 13.86 cm^2 , find:
 - (a) the angle subtended by the arc at the centre of the circle.
 - (b) the area of the sector enclosed by this arc.
7. There are unspecified number of cows and hens in a den. If there are total of 30 heads and 80 legs in the den, find the number of cows and hens in the den.
8. The table below shows the amount of money charged for hiring a car for a given distance:

<i>Distance covered (km)</i>	10	20	30	40	50
<i>Charges (sh)</i>	75	100	125	150	175

- (a) Draw a graph of the charges against the distance covered.
- (b) Use your graph to find:
 - (i) the standing charge.
 - (ii) how much money is charged for covering a distance of 28 km, 33 km and 42 km.
 - (iii) the distance covered if sh. 131.00, shs. 140.00 and sh. 190.00 is charged.

9. Measurements of a maize field using a base line XY were recorded as shown below. (Measurements are in metres)

	Y	
	240	
TO R 60	190	
	180	75 TO Q
	150	50 TO P
TO S 100	120	
	100	100 TO N
TO T 30	50	
	20	20 TO M
	X	

- Use a suitable scale to draw the map of the maize field.
 - Find the area of the field in hectares.
10. A model of a tent consists of a cube and a pyramid on a square base, see the figure below:
- Draw accurately the net of the model. Use the net to calculate the total surface area of the model.
 - If the ratio of the area of the model to the area of the actual tent is 1 : 10 000, find the area of the material required to make the tent (floor area inclusive).
 - If 1 m² of the tent material costs sh. 25.00, find the total cost of the material required to make the tent.

